A Coherence-Preserving Modelling Framework for Multi-Scale Energy Systems

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Abstract

Conventional modelling of complex energy systems often focuses on efficiency, capacity, and stability. However, many failures and performance degradations arise not merely from material or energetic losses, but from **phase mismatch and coherence breakdown** across interacting subsystems. This paper proposes a coherence-preserving modelling framework, introducing a mathematical index to quantify synchronization across scales. Building on the Kuramoto model, we demonstrate how coherence functions as both a diagnostic and predictive tool, with applications to power grids, energy storage interfaces, and thermal transport systems.

1 Introduction

Energy storage and transmission are typically evaluated through capacity, efficiency, and degradation rates. Yet, the hidden bottleneck often lies in the loss of *coherence*—the inability of subsystems to maintain phase alignment. We argue that coherence should be treated as a fundamental modelling dimension, complementing conventional efficiency and stability analysis.

2 Theoretical Framework

2.1 Multi-Scale Definition of Coherence

- **Microscopic**: phase stability of individual particles or units (e.g., local ion migration).

- **Mesoscopic**: synchronization of groups of units (e.g., battery cell clusters).
- **Macroscopic**: global phase relations across entire systems (e.g., frequency synchronization in power grids).

2.2 Relation to Stability

Traditional stability theory (e.g., Lyapunov methods) focuses on convergence to equilibria. Coherence instead emphasizes *relative synchrony* among subsystems.

- A system may be globally stable but incoherent (e.g., local oscillations in a grid).
- High coherence enhances resilience, as phase mismatch amplifies perturbations.

Thus, coherence acts as a **leading indicator** of stability.

2.3 Coherence and Phase Transition

The Kuramoto model shows that coherence undergoes a transition as coupling strength K increases:

- $K < K_c$: incoherent state, $C \approx 0$.
- $K > K_c$: synchronized state, $C \to 1$.

The critical coupling is approximated by:

$$K_c = \frac{2}{\pi g(0)} \tag{1}$$

where $g(\omega)$ is the probability density of natural frequencies.

2.4 Extensions of Coherence Index

- **Local coherence**: synchronization within subgroups or subnetworks.
- **Weighted coherence**: incorporating heterogeneous coupling strengths.
- **Time-varying coherence**: capturing dynamic rather than steady-state behavior.

3 Mathematical Model

We adopt the Kuramoto model of coupled oscillators:

$$\frac{d\theta_i}{dt} = \omega_i + \frac{K}{N} \sum_{j=1}^{N} \sin(\theta_j - \theta_i) \tag{2}$$

where θ_i is the phase of oscillator i, ω_i its natural frequency, K the coupling constant, and N the system size.

The global coherence is measured by the order parameter:

$$C = \left| \frac{1}{N} \sum_{j=1}^{N} e^{i\theta_j} \right| \tag{3}$$

with $C \approx 1$ indicating high synchrony and $C \approx 0$ indicating incoherence.

4 Numerical Validation Method

To verify the effectiveness of the coherence index, we propose the following simulation scheme:

- **System size**: $N = 20$.
- **Natural frequencies**: sampled from a normal distribution $N(0, 1)$.
- **Initial phases**: uniform distribution in $[0, 2\pi]$.
- **Integration method**: 4th-order Runge–Kutta, $\Delta t = 0.01$, $T = 50$.
- **Coupling strengths**: $K = 0.5, 1.0, 2.0, 3.0, 5.0$.

Outputs:

1. Phase evolution of oscillators.
2. Time evolution of $C(t)$.
3. Average coherence $\langle C \rangle$ vs. coupling strength K.

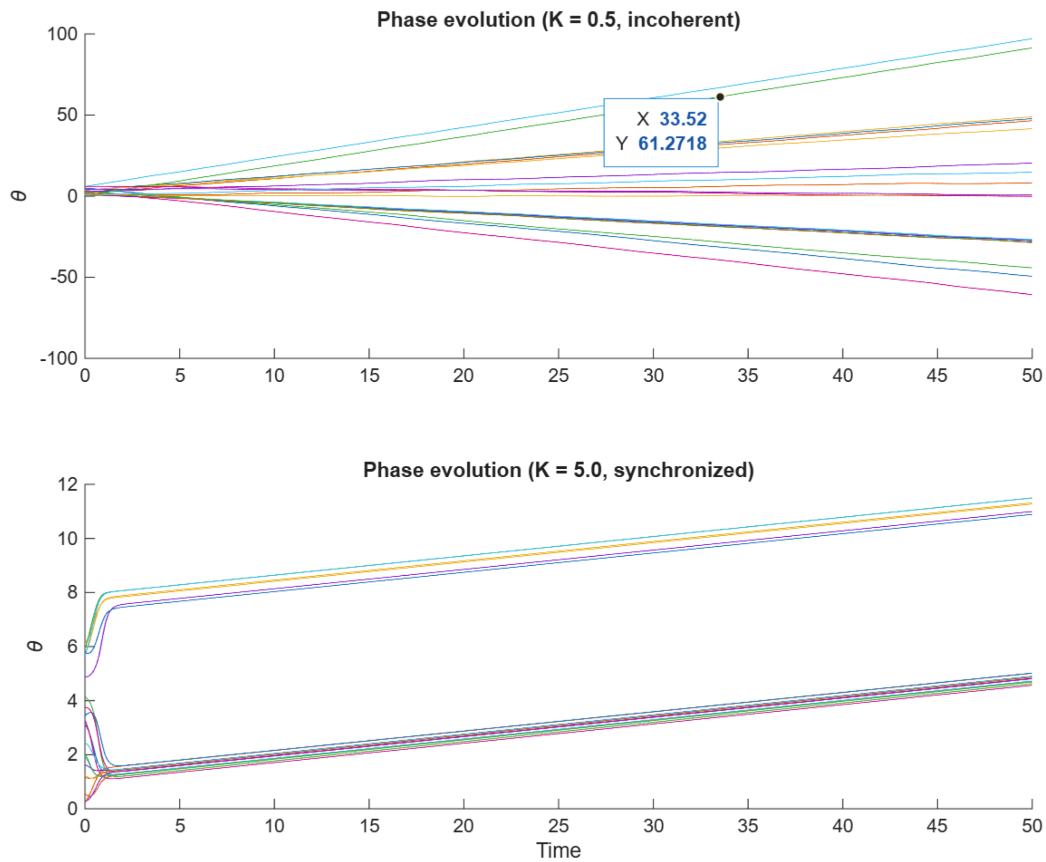

Figure 1: Phase evolution for $K = 0.5$ (top, incoherent) and $K = 5.0$ (bottom, synchronized).

5 Results

5.1 Phase Evolution

Figure 1 compares the phase trajectories of $N = 20$ oscillators. At $K = 0.5$, oscillators drift apart over time, showing incoherent dynamics. At $K = 5.0$, phases converge rapidly into a locked state, indicating strong synchronization.

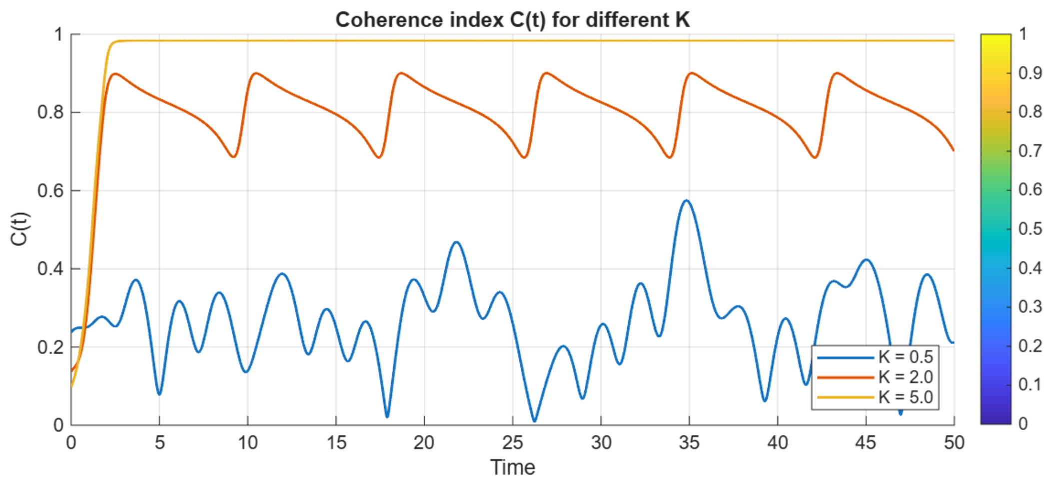

Figure 2: Time evolution of coherence index $C(t)$ under different coupling strengths.

5.2 Time Evolution of Coherence Index

Figure 2 plots $C(t)$ for different K. At $K = 0.5$, $C(t)$ fluctuates at low values (< 0.4). At $K = 2.0$, partial synchrony emerges with $C(t)$ in the 0.6–0.8 range. At $K = 5.0$, $C(t)$ saturates near 1.0, reflecting near-perfect coherence.

5.3 Synchronization Transition

Figure **??** shows $\langle C \rangle$ as a function of K. A sharp increase occurs near $K \approx 2.0$, consistent with the predicted synchronization threshold. Beyond this, $\langle C \rangle$ approaches unity.

6 Applications

- **Power grids**: generators as oscillators, coupling as line admittance. Low $K \Rightarrow$ loss of C, blackout risk.
- **Energy storage**: ion migration and interfacial dynamics mapped as phase coupling. Reduced $C \Rightarrow$ higher impedance.

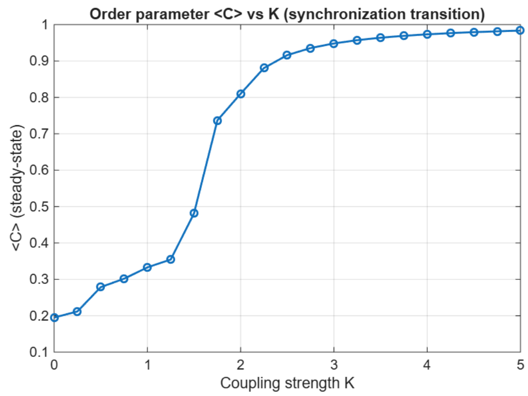

Figure 3: Enter Caption

- **Thermal transport**: lattice vibration mismatch lowers C, correlating with increased thermal resistance.

7 Discussion and Conclusion

This coherence-preserving framework extends conventional modelling of complex systems by introducing synchronization as a quantifiable dimension. Theoretical analysis shows that coherence is not only a complement to efficiency and stability but also a precursor of systemic resilience. Numerical validation demonstrates its ability to capture the transition from disorder to synchrony. Future work will integrate this framework with quantum simulation and high-performance computing for broader applications in energy and material sciences.

References

1. Kuramoto, Y. *Self-entrainment of a population of coupled nonlinear oscillators.* Lecture Notes in Physics, 1975.

2. Strogatz, S.H. *From Kuramoto to Crawford: exploring the onset of synchronization in populations of coupled oscillators.* Physica D, 2000.

3. Dörfler, F., Bullo, F. *Synchronization in complex networks of oscillators: A survey.* Automatica, 2014.

4. Motter, A.E., Myers, S.A., Anghel, M., Nishikawa, T. *Spontaneous synchrony in power-grid networks.* Nature Physics, 2013.