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# ARITHMETICAL COMPUTATIONS IN ROMAN NUMERALS

W. FRENCH ANDERSON

Historians of mathematics have always asserted that the Roman numerals are so cumbersome that it would be nearly impossible, if not impossible, to use them for arithmetical computations. Thus F. Cajori in *A History of Elementary Mathematics* (New York, 1950) states (p. 11):

> While the older notations served merely to record the answer of an arithmetic computation, the Hindu notation... assists with marvelous power in performing the computation itself. To verify this truth, try to multiply 723 by 364, by first expressing the numbers in the Roman notation; thus, multiply DCCXXIII by CCCLXIV. This notation offers little or no help; the Romans were compelled to invoke the aid of the abacus in calculations like this.[1]

Historians seem to be unanimous in their belief that the Romans did all their arithmetic either on the abacus or by finger-counting. This view has been further reinforced by the studies of D. Maher, who has shown that there is no record of computations actually made, using Roman numerals, in any classical or medieval source.[2]

My object in the present paper is primarily to show that, contrary to the long-held universal opinion, arithmetical computations can be carried out efficiently with Roman numerals. I do not attempt to prove that the Romans actually used the method described below, but I believe that any reader, once he discovers how simple the operations are, will be inclined to imagine that some Roman engineers and surveyors, in building their great projects, did occasionally do their computations very much in the way described below, even though they left no records of their work.

The essential processes required for mathematical calculations in any system are addition, subtraction, multiplication, division, involution (powers), and evolution (roots). If these operations can be successfully performed, then any problem, using finite numbers and not adversely affected by the absence of a symbol for zero, can be performed.

The processes of addition and subtraction with Roman numerals are simple; in fact, they are easier in this system than in our Hindu-Arabic system, since with Roman numerals the processes are practically mechanical.

Suppose the numbers to be added are MCDLXIX (1469) and DCCCXVII (817). Set the problem up as shown below and count the number of times each of the separate letters is used. Place these totals under the line; the final aggregation is the answer. The sole complication which keeps this operation from being entirely mechanical is when a "subtractive" numeral is present (e.g., IX [9], CD [400]) as in this example:

| | |
|---|---|
| MCDLXIX | 1,469 |
| DCCCXVII | 817 |
| MDDCCLXXXVI | 2,286 |
| or MMCCLXXXVI | |

*Explanation*: CD plus C equals D, IX plus I equals X.

Subtraction is the reverse of addition. Simply cancel out the letters on the bottom line (subtrahend) from the letters on the top line (minuend). In other words, for each letter in the lower line, cancel out a letter of equal value in the top line. The number left uncanceled in the top line after the lower line is completely canceled out is the answer. Using the same numbers as above, the first subtraction may be shown as follows:

MCDLXIX
DCCCXVII
D

The operation continues with successive cancellations until the answer is obtained:

| | |
|---|---|
| MCDLXIX | 1,469 |
| DCCCXVII | 817 |
| DCLII | 652 |

*Explanation*: D from M leaves D, CCC from CD leaves C, X from X cancels exactly, VII from IX leaves II.[3]

Multiplication is a little more complicated.

Let us multiply two simple numbers first, say XVII and LX. In the Roman system it will be seen that multiplication is performed from the highest order to the lowest—the reverse of our process. Another difference is that in setting up the problem the number with the fewest letters is used as the multiplier, regardless of value, thus:

| | |
|---|---|
| XVII | 17 |
| LX | 60 |
| D CCL L L | 1020 |
| C L X X | |
| D CCCLLLLXX | |
| or | |
| MXX | |

*Explanation*: multiplication proceeds as follows:

| | |
|---|---|
| L times X equals D<br>L times V equals CCL<br>L times I equals L<br>L times I equals L<br>X times X equals C<br>etc. | below the first line |
| Then collect the total as in addition; this result is the answer. | below the second line |

The Appendix contains a more difficult multiplication problem.

Division using Roman numerals is essentially the same as division using our Hindu-Arabic numerals. However, the Roman system does have one advantage: it is not necessary to find exactly how many times the dividend can be divided by the divisor in each operation. For example, when dividing 209 by 19 in our system it is necessary to use 1 (actually 10) as the first number of the quotient. But when dividing CCIX by XIX (see below), the first number of the quotient can be X, V, or I. In this example X should be used; but if V or I is used, it is simply necessary to repeat the division and collect the terms to obtain the final answer. This advantage eliminates the necessity, present in our system, of finding the exact number of times the divisor will go into the dividend at each step.

In dividing CCIX by XIX there are three steps:

```
                                 X
(1) XIX )CCIX          (2) XIX )CCIX
                                CXC
                                ----
                                 XIX

          XI                     11
(3) XIX )CCIX              19 )209
          CXC                  19
          ----                 --
           XIX                  19
           XIX                  19
           ---                  --
```

*Explanation*: (1) Set up the problem as in our Hindu-Arabic notation. (2) For the first number of the quotient use any number that, multiplied by the divisor, can be subtracted from CCIX. X is the highest such number. Multiply X times XIX and subtract this result (CXC) from CCIX. This gives XIX as the new dividend. (3) XIX can be subtracted from XIX one time. Subtraction leaves nothing. Thus the answer to the problem is XI. (A remainder offers no obstacle to the process.)

It is interesting to see what would happen if V were chosen as the first number of the quotient in the last problem instead of X. The solution would then work as follows:

```
          V                        VV
(1) XIX )CCIX          (2) XIX )CCIX
          XCV                     XCV
          ----                    ----
           CXIV                    CXIV
                                   XCV
                                   ----
                                    XIX

                VVI
      (3) XIX )CCIX
                XCV
                ----
                 CXIV
                 XCV
                 ----
                 XIX
                 XIX
                 ---
```

VVI equals XI and thus there is no obstacle in working the problem this way; it only takes a little longer to obtain the answer. This accommodation is not found in our place-value system.

The Appendix contains a more difficult division problem.

Involution using Roman numerals is identical with involution using our system: it simply involves repeated multiplication. Thus $5^5$ equals 5 times 5 times 5 times 5, etc.; likewise, $V^V$ equals V times V times V, etc.[4]

Evolution in Roman numerals is also very similar to our system; the Roman, again, has one advantage. Our Hindu-Arabic evolution, like division, requires the exact number for each place in the quotient, since it is a place-value system; whereas in the Roman system, as in division, any number can be used and then, if necessary, repeated. A disadvantage is that values cannot be carried out, as can be done in our system, to any desired degree of accuracy.

To find the square root of a number, for example DCXXV (625), proceed as follows:

```
       X                            XX
(1) X √DCXXV              (2) X √DCXXV
       C                            C
       -----                       -----
       DXXV          XXX           DXXV
                                   CCC
                                   -----
                                    CCXXV

                  XXV
            (3) X √DCXXV
                  C
                  -----
                  DXXV
          XXX     CCC
                  -----
                  CCXXV
          XLV     CCXXV
                  -----
```

*Explanation*: (1) Set up the problem. Find a number which, when squared, can be subtracted from the given number. X is such a number and X squared is C. Subtract C from DCXXV and obtain DXXV as the new dividend. (2) Double the present quotient (X), obtaining XX. Find a number which, when multiplied by

XX plus the number, can be subtracted from the new dividend. Again X is such a number and X times XXX is CCC. Subtract CCC from DXXV and obtain CCXXV as the new dividend. (3) Proceed as in step (2). V times XX doubled (or XL) plus V (or XLV) equals CCXV which, when subtracted from CCXXV, leaves nothing. Thus the square root of DCXXV is XXV.

The Appendix contains a computation of the square root of a prime number.

Clearly the present belief that Roman numerals cannot be used for arithmetical computations is incorrect. The main disadvantage of the Roman notation is, of course, that a lengthy problem involves a great many more symbols than does our system, but this does not make the Roman numerals impossible to use. For addition and subtraction, the numerals are actually simpler than our system, since with them these processes involve only the mechanical operations of counting or canceling.

Careful attention will show that a knowledge of the Hindu-Arabic system is not essential before one can use the Roman system. The reader might fear that perhaps he is unconsciously thinking in Hindu-Arabic while apparently working with the Roman notation. Actually this is not the case. The mind can think just as easily in the Roman notation as in the Arabic. Being used to our own system, we inadvertently feel that it is the only system worth using. But just as there are many different languages, so there are many different numerical systems, each of which can be used independently of all others.

It remains for us to try to explain why modern thinkers have so unanimously condemned the Roman system as unusable. Perhaps part of the reason lies, as has been said, in our feeling that our system is the only usable one. We seem to have forgotten that a system of notation merely symbolizes abstract qualities called numbers. Three units are three units whether designated by three, 3, III, *tres*, or *drei*. Numbers have universal and unchangeable relationships. Multiplication of three units by three units will produce nine units whether we designate the operation in Hindu-Arabic or in Roman notation. Thus any numerical system may be used for computations; some are simply more efficient than others.

Not only do we cling to our own familiar system, but we find another difficult in itself. In teaching others how to use Roman numerals in arithmetical computations, the author has discovered that there are three stages through which a student regularly goes. First the processes seem simple to him. Anyone can follow the operations as described by another. But when attempting to perform the operations by himself, the student finds the system complicated and cumbersome. Then, once the new method of notation is mastered, the processes seem easy again. The author went through this same pattern in working out the above procedures. At first it seemed only logical that they could be performed, but it was difficult to discover how they were actually done. Now the processes seem quite simple. Perhaps mathematical historians went only as far as the difficulties encountered in the second stage of the routine and so thought that the numerals were too cumbersome for use.

# APPENDIX

## *Chart 1: Multiplication*

Multiplication of DCCXXIII (723) by CCCLXIV (364), the "verification" problem instanced by Cajori (above, p. 145). The reader will recall that $\overline{L}$ means 50,000, $\overline{X}$ means 10,000, $\overline{V}$ means 5,000.

DCCXXIII
CCCLXIV

| Step | | |
|---|---|---|
| (1) | $\overline{L}\,\overline{X}\,\overline{X}$ M M C C C | |
| (2) | $\overline{L}\,\overline{X}\,\overline{X}$ M M C C C | 723 |
| (3) | $\overline{L}\,\overline{X}\,\overline{X}$ M M C C C | 364 |
| (4) | $\overline{XXV}\,\overline{V}\,\overline{V}$ D D L L L | 2892 |
| (5) | $\overline{V}$ M M C C X X X | 4338 |
| (6) | M M D C C C X C I I | 2169 |
| (7) | $\overline{C}\,\overline{C}$ $\overline{L}\,\overline{X}$ M M M C L X X I I | 263,172 |

*Explanation*:
(1) multiplication by C
(2) multiplication by C
(3) multiplication by C
(4) multiplication by L
(5) multiplication by X
(6) result of the multiplication by IV:

| | |
|---|---|
| DCCXXIII | DCCXXIII |
| I | V |
| DCCXXIII | MMDDDLLVVV |

subtraction:

MMDDDLLVVV
DCCXX III
MMDCCCXCII

The multiplication of IV, IX, XL, etc., can in many instances be done in the head by collecting the terms as they are being counted; the entire process has been shown here for clarity.

(7) totaling of the partial products (totaling of the terms is done while they are being collected).

## *Chart 2: Division*

Division of $\overline{CXX}$MMMCDLVI by 123,456

CDXXXII
432

| | |
|---|---|
| | C C L X X X V |
| CDXXXII ) | $\overline{C}\,\overline{\overline{X}}\,\overline{\overline{X}}$ M M M CD LV I |
| | $\overline{\overline{XL}}$ M M M C C |
| (1) | $\overline{\overline{L}}\,\overline{\overline{X}}\,\overline{\overline{X}}\,\overline{\overline{X}}$ C C LV I |
| | $\overline{\overline{XL}}$ M M M C C |
| (2) | $\overline{\overline{X}}\,\overline{\overline{X}}\,\overline{\overline{X}}\,\overline{\overline{V}}$ M M LV I |
| | $\overline{\overline{X}}\,\overline{\overline{X}}$ D D D L L |
| (3) | $\overline{\overline{X}}\,\overline{\overline{V}}$ CD LV I |
| | M$\overline{\overline{V}}$ C C C XX |
| (4) | $\overline{\overline{X}}$ M C X X XV I |
| | M$\overline{\overline{V}}$ C C C X X |
| (5) | $\overline{\overline{V}}$ M D C C C XV I |
| | M$\overline{\overline{V}}$ C C C X X |
| (6) | M M CD XC VI |
| | M M L L L V V |
| (7) | C C C X X X V I |

Answer: CCLXXXV $\frac{\text{CCCXXXVI}}{\text{CDXXXII}}$ $285\frac{336}{432}$

*Explanation*:
(1) First new dividend—formed by subtracting the product of C and the divisor (CDXXXII) from the dividend.
(2) Second new dividend—formed by subtracting the product of C and CDXXXII from (1).
(3) Third new dividend—formed by subtracting the product of L and CDXXXII from (2).
(4) Fourth new dividend—formed by subtracting the product of X and CDXXXII from (3).
(5) Fifth new dividend—formed by subtracting the product of X and CDXXXII from (4).
(6) Sixth new dividend—formed by subtracting the product of X and CDXXXII from (5).

(7) Remainder—formed by subtracting the product of V and CDXXXII from (6).

*Chart 3: Square Root of a Prime Number*

Obtaining the square root of M$\overline{\text{V}}$DXXIII
4,523

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | LXVII | | | | | | | | | |
| L | $\sqrt{\text{M}\overline{\text{V}}}$ | | | | D | X | X | I | I | I |
| | M | M | | | D | | | | | |
| CX | M | M | | | | X | X | I | I | I |
| | M | | C | | | | | | | |
| CXXV | CM | | | | | X | X | I | I | I |
| | | D | L | L | | XX | V | | | |
| CXXXI | C | C | | | XC | | V | I | I | I |
| | C | | | | X | XX | | | | I |
| CXXXIII | C | | L | X | | | | V | I | I |
| | C | | X | X | X | | | I | I | I |
| | | | X | X | X | IV | | | | |

Answer: LXVII$\frac{\text{XXXIV}}{\quad}$ 67$\frac{34}{\quad}$

*Explanation*: The procedure is the same as explained in the example of evolution shown above. Remainders may simply be placed with the square root to show that the exact square root is a fraction larger. Space does not permit a full discussion of these remainders; however, my experiments seem to indicate that it is possible to obtain a fraction accurate in value to two decimal places (one significant figure beyond the decimal point).

HARVARD COLLEGE

## NOTES

1. This problem of Cajori's, which he states cannot be solved using Roman numerals, has been worked in Chart 1 of the Appendix.

2. Mr. David Maher, of the Harvard Class of 1955, has written a senior honors thesis on the history of the Roman numerals. His chief study was made in the works of Varro, Vitruvius, Pliny the Elder, Columella, and Frontinus. Maher found no evidence of problems performed using the numerals. He shows that the operations of Roman arithmeticians, with the possible exception of Frontinus, could probably have been done on the abacus or with finger-counting. However, the work of Frontinus was too complex (having worked out several values which proved the value of *pi* to be 22/7) to have been done in any known way using either of these two methods, the abacus or finger-reckoning. Unfortunately, like the other arithmeticians, Frontinus did not show how he arrived at his conclusions.

3. When subtracting two numbers such as MCDLXIX from MMCCXXXII (in which the subtrahend has a greater number of, say, tens than has the minuend) the problem is simply set up so that a unit of higher denomination (here a C) is grouped with the tens (this grouping is in the minuend):

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| M | M | C | C | X | X | X | I | I |
| M | CD | | L | X | | IX | | |
| | DC | C | L | | X | I | I | I |

Thus, even in as extreme a case as III from MII, the answer would be:

| | | |
|---|---|---|
| M | I | I |
| I | I | I |
| IM | or | CM XC IX |

4. Once again the reader should understand that the author is not asserting that the Romans ever employed this method or the particular notation (i.e., $\text{V}^{\text{V}}$) used here. The purpose of this paper, as stated, is only to show how the Roman numerals might have been used in arithmetical operations.

To those who gave of their time and assistance in the preparation of this paper, I wish to express my deep and sincere gratitude first of all to my parents, who gave me help both with the ideas and with the exposition.

The problem presented itself originally in the Virgil class of Miss Mary J. Barnett when I was a senior in Central High School, Tulsa, Oklahoma in 1953/4. Mr. G. W. Hall, chemistry instructor at Central, gave me considerable tutoring in working with numerical systems. Dr. Ralph W. Veatch, professor of mathematics at the University of Tulsa, read and criticized the original paper.

On continuing work at Harvard I have been immeasurably helped by Professor Sterling Dow. At every turn Professor Dow has generously assisted not only with the structure but with the research of the paper. Without his interest and encouragement this work would never have been carried to completion. Professor I. Bernard Cohen of Harvard gave of his time to help with the research. Mr. David Maher, of the Class of 1955 of Harvard College, has kindly shared with me the results of his complementary studies. Professor A. W. Richeson, of the University of Maryland, did considerable research in mathematical journals.

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