

Georg Cantor: The Personal Matrix of His Mathematics

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HISTORIANS OF MATHEMATICS are generally accustomed to discussing ideas rather than individuals. A mathematician's personal life and his mathematics are frequently regarded as wholly separate, the former providing human interest while the latter comprises the heart of the matter. But the analysis of personality, in particular that of creative individuals, can reveal a great deal about the nature of discovery. In Georg Cantor's case this is especially true. If one were to consult only the published record of his research, the factors influencing his discovery, development, and subsequent defense of set theory and the transfinite numbers would remain obscure. At best, one could only hope for a partial, and probably inaccurate, view of what Cantor accomplished.

Why should he have been the mathematician most likely to defend transfinite set theory, despite tremendous opposition, not only from mathematicians, but from philosophers and theologians as well? How did his character and scientific temperament shape the earliest development of set theory? Satisfying answers to such questions cannot be expected without going beyond the record of Cantor's printed papers. Some hints can be found in letters and in the memoirs and reminiscences of those who knew him. But without surveying the larger context of Cantor's family history, including details of his personal and medical history, it is impossible to

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Unless otherwise noted, all page references to works of Georg Cantor referred to herein correspond to the pagination of the edition by Ernst Zermelo: *Georg Cantor, Gesammelte Abhandlungen mathematischen und philosophischen Inhalts* (Berlin: J. Springer, 1932; reprinted Hildesheim: Olms, 1966). For the sake of economy, the following abbreviations are used in references to the three surviving *Briefbüchern* in which Cantor drafted his letters before writing out final copies:

Cantor (I): Cantor's letter-book for 1884 through 1888

Cantor (II): Cantor's letter-book for 1890 through 1895

Cantor (III): Cantor's letter-book for 1895 and 1896

These letter-books are part of Cantor's surviving *Nachlass*, which is now kept in the Archives of the Akademie der Wissenschaften, Göttingen. I am grateful to Oberstudienrat Wilhelm Stahl for allowing me access to these materials before they were given to the Akademie. It should be noted that references to various parts of Cantor's *Nachlass* follow the system recently given to the collection by Ivor Grattan-Guinness. For details, see his article, "Towards a Biography of Georg Cantor," *Annals of Science*, 1971, 27:345-391.

appreciate the reasons why set theory, and in particular transfinite set theory, persisted in the face of continuing opposition to thrive in the hands of its creator.

BIOGRAPHY, GENEALOGY, AND MATHEMATICS

What follows is not an attempt to prepare the way for any sort of Freudian psychoanalysis of Cantor's personality. Still, it must be said that much of Cantor's personality was shaped under the very strong influence of his father. To understand why Cantor approached certain problems and harbored specific anxieties as he did, it is necessary to survey what little is known of the family history and to add some relevant new facts from hitherto-unpublished sources.

Of Cantor's grandparents, the family history is unclear, but there is agreement that his grandfather, Jakob Cantor, lived in Copenhagen.¹ Exactly when Cantor's father, Georg Woldemar Cantor, was born is uncertain; his death certificate lists 1814, whereas a Danish passport gives 1809. The latter date accords more closely with traditional family history, which has the Cantors leaving Copenhagen during the English siege of 1807. The family settled in St. Petersburg, where the upbringing and education of the child Georg Woldemar was entrusted to an Evangelical Lutheran mission.² In 1842 Georg Woldemar Cantor married Maria Anna Böhm, a Roman Catholic, in St. Petersburg.³ Her entire family was gifted musically and included several classical violinists, the most noted being Cantor's great-uncle Joseph Böhm, director of the conservatory in Vienna and founder of the great school for violinists which produced many a virtuoso at the time. The fact that Georg Woldemar and Maria Anna Böhm were married in the Evangelical Lutheran Church again reflects the close links that Cantor's father felt to the mission where he had been raised. All of the couple's six children, of which Georg was the eldest, were baptized as Lutherans and given very rigorous instruction by their father in matters of religion.

The serious and pervasive presence of religion in Georg Woldemar's daily life was strongly reflected in nearly every letter he addressed to his son—letters the young Cantor saved during his school days in Darmstadt and later at the Polytechnic Institute in Zurich. One letter in particular, written in 1860 on the occasion of his confirmation at the age of fifteen, Georg kept with him always. It deserves to be cited almost entirely, for in so many ways it is an uncanny prediction of the successes and failures of Cantor's personal life and public career.

Dearest Georg!

May this day be of blessed influence upon your entire future life, through the goodness of the Almighty, the Creator of the universe and Father of all living creatures. May you

¹For biographical details of Cantor's family history, refer to Adolf Fraenkel, "Georg Cantor," *Jahresbericht der Deutschen Mathematiker-Vereinigung*, 1930, 39:189–266. This biography also appeared separately as *Georg Cantor* (Leipzig: Teubner, 1930) and is reprinted in an abridged version in Cantor, *Gesammelte Abhandlungen*, pp. 452–483. Also see M. Peters, *Lied eines Lebens*, a biography of Cantor's daughter Else, privately printed in Halle (1961); Herbert Meschkowski, *Probleme des Unendlichen. Werk und Leben Georg Cantors* (Braunschweig: Vieweg, 1967); and Ivor Grattan-Guinness, "Towards a Biography," pp. 351–352.

²From a German translation of the Cantor family history, prepared in 1937 by Th. Hauch-Tausböll of the Danish Genealogical Institute, Copenhagen, and preserved in *Nachlass Cantor I*, as listed in Grattan-Guinness, "Towards a Biography," p. 348.

³Cantor related much of his family history in the course of his correspondence. The information given here has been gathered from numerous letters found throughout Cantor (I), (II), and (III). See in particular the biographical details that Cantor wrote out on visiting cards in 1899, transcribed in Grattan-Guinness, "Towards a Biography," pp. 379–380.

constantly and unremittingly keep before your eyes the virtuous resolutions which you have no doubt made today in silence as a solemn vow! . . . The future of one's *life* and the fate of the individual lie hidden from him in the most profound darkness. And it is good that it is so. No one knows beforehand into what unbelievably difficult conditions and occupational circumstances he will fall by chance, against what unforeseen and *unforeseeable* calamities and difficulties he will have to fight in the various situations of life. . . .

Often the most promising individuals are defeated after a tenuous, weak resistance in their first serious struggle following their entry into practical affairs. Their courage broken, they atrophy completely thereafter, and even in the *best* case they will still be nothing more than a so-called ruined genius! . . . Indeed, it is *not seldom* that young men come to such an end, even those who are *seemingly* endowed with the most promising features of mind and body, and whose prospects for the future through ability and family connections in their youth were also the rosiest in the world!

But they *lacked a steady heart* [*feste Kern*], upon which *everything* depends! Now, my dear son! *Believe me*, your *sincerest, truest, and most experienced* friend—this sure heart, *which must live in us*, is: a *true* religious spirit [*Gemüt*]! This reveals *itself to us* through a *sincere, humble feeling of the most grateful reverence for God*, from which feeling grows as well the *victorious, unshakable, enduring faith in God*, and which keeps and maintains us throughout our entire life in that silent, undoubting exchange with our heavenly Father! . . .

But in order to prevent as well *all those other* hardships and difficulties which inevitably rise against us [*entgegentürmen*] through jealousy and slander of open or secret enemies in our eager aspiration for success in the activity of our *own specialty or business*; in order to combat these *with success* one needs above all to acquire and to appropriate the greatest amount possible of the most basic, diverse technical knowledge and skills. Nowadays these are an *absolute necessity* if the industrious and ambitious man does not want to see himself pushed aside by his enemies and forced to stand in the second or third rank.

To the procurement of diverse, thorough, scientific and practical knowledge; to the *perfect* acquisition of foreign languages and literatures; to the many-sided development of the mind in many humanistic disciplines—in order by means of all this first to *equip yourself with dignity for those struggles yet to come*—and of this you must always be thoroughly conscious!—*to all this the second period* of your life, your *youth*, now just beginning, is *destined*. Whatever one neglects in *this period* or through *premature* extravagance of his best strength, health and time, he *squanders* [*verludert*], so to speak, that is irretrievably and *irreplaceably lost for ever*; just as innocence, *once lost*, is for ever and eternally, irretrievably lost. . . .

I close with these words: Your father, or rather your parents and all other members of the family both in Germany and in Russia and in Denmark have their eyes on *you* as the eldest, and expect you to be nothing *less* than a Theodor Schaeffer [his teacher] and, God willing, later perhaps a *shining star* on the horizon of science.

May God give you *strength*, persistence, health, sound character, and His best blessings! And therefore *you* should follow only on His course. Amen!

Your father, . . .⁴

Georg Woldemar Cantor's advice to his son reflected not only his strong religious beliefs but also his own experience. As a young man he had gone into business in St. Petersburg, establishing a wholesaling firm in 1834 under the title Cantor and Company. The firm's dealings were international, with interests in Hamburg, Copenhagen, and London; transactions reached as far as New York, Rio, and Bahia. But for reasons unknown, Cantor and Company failed in December of 1838, and by the following December liquidation was complete.⁵ Nevertheless, Georg Woldemar

⁴Reproduced in Fraenkel, "Georg Cantor," pp. 191–192. In the next to last paragraph, however, Fraenkel transcribes "ein leuchtendes *Gestirn* am Horizonte der *Ingenieure*," while the typescript version in *Nachlass Cantor I* reads "Wissenschaft" instead of "Ingenieure." Part of the confirmation letter is also reproduced in Meschkowski, *Probleme*, p. 3.

⁵From the Hauch-Tausböll history, *Nachlass Cantor I*; see also Grattan-Guinness, "Towards a Biography," p. 351.

Cantor still had the “inner resources and determination” to make a success of his business career, and he turned his talents to good use as a broker on the stock exchange in St. Petersburg.

When he died of tuberculosis in June 1863, Georg Woldemar left his family nearly half a million marks.⁶ But more than money, he left his eldest son with a compulsive desire to succeed professionally and with a firm belief in the ability of his own inner resources to overcome even the most distressing episodes of life. Cantor was to follow his father’s advice very seriously, and this, as much as any other aspect of his personal life, helps to explain why he was always determined to prevail and why he eventually fought with such tenacity and confidence in support of his theory of sets and transfinite numbers.

After leaving Russia the family lived shortly in Wiesbaden before settling permanently in Frankfurt. Young Georg, however, boarded at the Realschule in Darmstadt, from which he graduated in 1859 with the report that “his diligence and fervor are exemplary; his knowledge in basic mathematics including trigonometry is very good; performance praiseworthy.”⁷ A year later he was at the Höheren Gewerbeschule (trade school) and again, upon leaving in 1862, was given the highest praise: “He showed himself to be very gifted and a highly industrious pupil.”⁸ Georg Woldemar always took a special interest in his son’s education and was careful to direct young Georg’s personal and intellectual development, offering a good deal of encouragement. The closeness of their relationship was particularly noticeable in the letters Georg Woldemar sent to his son during the years he was away at school. The careful, emphatic admonitions to work diligently and thereby to insure success were never lost on young Georg.

The fact that Cantor spent two years in the Darmstädter Gewerbeschule suggests that his father’s original hopes for his son’s future—as a “shining star on the horizon of science”—did not include a devotion to *pure* science. But by the spring of 1862 the young Cantor had come to a decision: to devote his life to mathematics. Despite his initial uncertainty as to whether it was the right choice, he was wholly supported by his father. Cantor’s joy at his father’s approval was echoed in every word of the following letter:

My Dear Papa!

You can imagine how very happy your letter made me; it determines my future. The last few days have left me in doubt and uncertainty. I could reach no decision. My sense of duty and my own wishes fought continuously one against the other. Now I am happy when I see that it will no longer distress you if I follow my own feelings in this decision. I hope that you will still be proud of me one day, dear Father, for my soul, my entire being lives in my calling; whatever one wants and is able to do, whatever it is toward which an unknown secret voice calls him, *that* he will carry through to success!⁹

From the very beginning, apparently, Cantor had felt some inner compulsion to study mathematics. Although his father’s original doubts had been set aside, Cantor was left with a sense of having to prove that his father would not be disappointed. Characteristically, Cantor reinforced the validity of his decision by saying that it was not his alone—an unknown, secret voice was making its impression even then.

Cantor’s father followed his son’s university career with enthusiasm. In August

⁶Grattan-Guinness, “Towards a Biography,” p. 352.

⁷Fraenkel, “Georg Cantor,” p. 192.

⁸*Ibid.*

⁹*Ibid.*, p. 193; quoted in Meschkowski, *Probleme*, p. 5.

1862, when Cantor was preparing to take the *Reifeprüfung*, which would qualify him for advanced study in the sciences, Georg Woldemar sent his best wishes and again emphasized the importance of religious faith, to which his son would later turn in periods of doubt and depression. His father's words were a firm call to success, but as Cantor sat down to write his examination on August 18, 1862, he had more than his father's prayer that God might grant him blessings and luck. He also had his father's reminder: "In critical moments of one's life an unwavering and joyous trust in God and a profound, moving prayer to the Almighty Giver of all goodness before the start of the day gives stability, courage, and self-confidence. And thus commit yourself to God! Be fresh, joyous and cheerful in work!"¹⁰

By the end of August, Cantor's examination had been read and was "Recognized as proof of first-level preparation [*Maturitäts Zeugnis Nr. 1*] for the study of the natural sciences."¹¹ Apparently Cantor had done less well in subjects like geography and history, because his father was prompted to write, stressing the importance of a diverse and thorough study of all branches of knowledge.¹² Before Cantor had even left Darmstadt, his father had counseled (for instance, in his confirmation letter) that Georg should not merely concentrate on a single subject but should study everything from languages to music as thoroughly as practicable. Thus he was pleased to hear of his son's interest in hearing a series of lectures by the renowned Theodor Vischers on Shakespeare. When Cantor formed his own string quartet, his family was delighted, and his father invited the entire group to celebrate the New Year, 1863, in Frankfurt.¹³

Contrary to the unfounded claims made by E. T. Bell that Georg Woldemar had a thoroughly ruinous effect upon his son's psychological health,¹⁴ the surviving evidence of their relationship indicates quite the opposite. Cantor's father was a sensitive and gifted man, who loved his children deeply and wanted them to live successful and rewarding lives. From the time he was a child, young Georg was expected to be a credit to his family, and as he approached a career in mathematics, he wanted nothing more than to fulfill the hopes of his parents. But underlying even his earliest decisions was a sense of destiny, a feeling that unknown forces were at work, propelling him toward a career which he felt he could not deny.

DEPRESSION AND MENTAL ILLNESS

After finishing his doctorate in mathematics at the University of Berlin, Cantor began in 1869 to teach at the university in Halle, where he was to remain for the rest of his life. His earliest publications dealt with number theory and trigonometric series, from which he gained an early reputation as one of Germany's best young mathematicians. In 1874 he published his celebrated discovery that the set of all real numbers was nondenumerable, thus providing the first examples of infinite sets of different magnitudes. While sets like the integers, rational, or algebraic numbers were countable, the set of real numbers represented a distinctly higher order of infinity. This was an early, essential advance toward Cantor's creation of transfinite set theory. In 1878 he published another bold discovery—that the number of points in the two-dimensional plane could be corresponded in a one-to-one fashion with the points of a one-dimensional line. This raised serious and difficult questions related to the

¹⁰Georg Woldemar Cantor to Georg Cantor, Aug. 18, 1862, in *Nachlass Cantor I*.

¹¹Fraenkel, "Georg Cantor," p. 193.

¹²See Georg Woldemar Cantor to Georg Cantor, Aug. 26, 1862, and May 9, 1861, in *Nachlass Cantor I*.

¹³Georg Woldemar Cantor to Georg Cantor, Nov. 3, 1862, in *Nachlass Cantor I*.

¹⁴E. T. Bell, *Men of Mathematics* (New York: Simon and Schuster, 1937), Ch. 29.

invariance of dimension; it also provoked the first overt opposition to Cantor's research concerning the infinite, for Leopold Kronecker, an editor of *Crelle's Journal*, tried to prevent the publication of Cantor's paper, and actually succeeded in delaying its appearance for more than a year.¹⁵

Kronecker, one of Germany's most renowned and influential mathematicians, was well known for his disavowal of the infinite. Cantor, as a student, had studied with Kronecker at the University of Berlin, and was well acquainted with Kronecker's suspicions of the Bolzano-Weierstrass theorem, upper and lower limits, and the irrational numbers in general. In fact, by the mid-1880s, when Cantor had begun systematically to publish the elements of his transfinite set theory, Kronecker had publicly criticized Cantor's theory of irrational numbers (as published by Cantor's colleague at Halle, Eduard Heine), and was apparently denouncing transfinite set theory during his lectures in Berlin.¹⁶

Thus, early in his career, Cantor was already having to confront the strong opposition of one of the most eminent mathematicians of his day. Worse yet, transfinite set theory was still in its infancy. It was an untried if not suspect newcomer to most mathematicians who had been conditioned to reject the absolute infinite in mathematics in favor of arguments involving finite values only. Consequently, Cantor's new ideas were particularly vulnerable to opposition like Kronecker's, and Cantor resented what he regarded as unfair, premature criticism.

Meanwhile, Cantor was having his own difficulties with transfinite set theory. The primary goal of his *Grundlagen einer allgemeinen Mannigfaltigkeitslehre* of 1883 was the introduction of his transfinite numbers. These were actually infinite numbers by which one could count and identify the different magnitudes (or powers, as Cantor called them) of infinite sets. Among these, Cantor had already shown that apart from infinite sets (like the whole numbers) that were denumerable, there were also non-denumerable sets like the continuum of all real numbers. One of Cantor's most famous conjectures, the continuum hypothesis, asserted that there was *no* infinite set whose magnitude was greater than that of the integers but less than that of the reals. In an equivalent form, the continuum hypothesis claimed that the power of the continuum was equal to the second of Cantor's transfinite cardinal numbers \aleph_1 ; \aleph_0 , the first of Cantor's transfinite cardinal numbers, denoted the power of all denumerably infinite sets.¹⁷

¹⁵For discussion of Cantor's early publications and their importance in the creation of transfinite set theory, see Joseph Dauben, "The Trigonometric Background to Georg Cantor's Theory of Sets," *Archive for History of Exact Sciences*, 1971, 7:181–216; "Denumerability and Dimension: The Origins of Georg Cantor's Theory of Sets," *Revue*, 1974, 2:105–134; "The Invariance of Dimension: Problems in the Early Development of Set Theory and Topology," *Historia Mathematica*, 1975, 2:273–288.

¹⁶For details of Kronecker's antagonism against analysis in general, refer to Gösta Mittag-Leffler, "Une page de la vie de Weierstrass," *Compte rendu du deuxième congrès international des mathématiciens tenu à Paris du 6 au 12 août* (Paris: Gauthier-Villars, 1902), pp. 150–151; Meschkowski, *Probleme*, pp. 67–69; and Pierre Dugac, *Richard Dedekind et les fondements des mathématiques* (Paris: J. Vrin, 1976), pp. 141–142, 145–146, 161–163. Kronecker's hostility toward Cantor's work was often the subject of Cantor's correspondence with his friend Mittag-Leffler, the Swedish mathematician and founder of *Acta Mathematica*. For example, during a visit with Cantor in July of 1883, Kronecker laughingly said that he had been writing to the French mathematician Charles Hermite that Cantor's latest work was humbug. Cantor was not amused and complained about Kronecker to Mittag-Leffler in a letter of Sept. 9, 1883, in the Archives of the Institut Mittag-Leffler, Djursholm, Sweden. See also Cantor's letter to Mittag-Leffler of Nov. 7, 1883, as well as his letter to Johannes Thomae, Sept. 2, 1891: Cantor (II), p. 73, in which Cantor mentioned that Kronecker had been referring to his set theory as "mathematische Sophistik."

¹⁷For an analysis of the significance of the continuum hypothesis, see Kurt Gödel, "What is Cantor's Continuum Problem?" *American Mathematical Monthly*, 1947, 54:515–525; revised and expanded in Paul Benacerraf and Hilary Putnam, eds., *Philosophy of Mathematics, Selected Readings* (Englewood Cliffs, N.J.: Prentice-Hall, 1964).

Despite Cantor's repeated promises to prove the continuum hypothesis, he was never able to do so. For a time in 1884 he believed he had actually found a proof of the conjecture, but within days he reversed himself completely and thought that he could definitely show it to be false. Finally, however, he realized that he had made no progress at all, and that the cardinality of the continuum was still an open question.¹⁸ Thus, in the mid-1880s Cantor was not only faced with mounting criticism from Leopold Kronecker, but he was also plagued with his own failure to determine the power of the continuum. Such were the circumstances surrounding his hospitalization for what later proved to be recurring cycles of manic depression.

Cantor's first serious mental breakdown occurred shortly after his thirty-ninth birthday in the spring of 1884. The episode has always figured prominently in the accounts of his life given by popularizers, among whom none has treated Cantor's case with less reliability than E. T. Bell. Not content with the view advanced in 1927 by Arthur Schoenflies that Cantor's breakdown was the result of conflict with Kronecker and his inability to solve the continuum hypothesis,¹⁹ Bell turned to the popular psychology of his day and produced a predictable Freudian analysis.²⁰ He traced the roots of Cantor's mental instability to childhood and blamed Cantor's father for having left his son with deeply planted anxieties. These supposedly led to the breakdown which required hospitalization in 1884. But unless Bell had access to material unknown to historians before or since, there seems to be little justification for the scenario he describes. On the contrary, the actual details of Cantor's illness, as it is currently understood, are considerably more interesting than the analysis manufactured by Bell, and in fact they make a great deal of sense when keyed to other features of Cantor's personality.

When he suddenly suffered his first breakdown in May 1884, Cantor had just returned from an apparently successful, quite enjoyable trip to Paris.²¹ He had met a number of French mathematicians, including Hermite, Picard, and Appell, and was delighted to report to Gösta Mittag-Leffler that he had liked Poincaré very much and was happy to see that the Frenchman understood transfinite set theory and its applications in functional analysis. Since other French mathematicians were away from Paris on vacation, Cantor had a certain amount of free time to visit the galleries and museums, to indulge his love of music at the Paris Opéra, and to spend an evening at the Comédie Française. But after eight days, he was unexpectedly called back to Germany to deal with family matters (of an unknown nature) in Frankfurt, and shortly thereafter his breakdown occurred. It is not possible to conjecture the immediate cause, though in the tradition made popular by Schoenflies the collapse is

¹⁸Cantor described in considerable detail his reverses in affirming and denying the validity of his conjecture in letters to Mittag-Leffler of May 5, 1883, and Apr. 4, Aug. 26, Oct. 20, Nov. 14, 1884, all to be found in the Archives of the Institut Mittag-Leffler. See also Arthur Schoenflies, "Die Krisis in Cantor's mathematischen Schaffen," *Acta Mathematica*, 1927, 50:1–23, on pp. 15, 17–18; and Meschkowski, *Probleme*, p. 243. For studies of the undecidability and independence of the continuum hypothesis, refer to Kurt Gödel, "Ueber formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme," *Monatshfte für Mathematik und Physik*, 1931, 38:173–198; *The Consistency of the Continuum Hypothesis* (Princeton: Princeton University Press, 1940); and Paul Cohen, *Set Theory and the Continuum Hypothesis* (New York: W. A. Benjamin, 1966).

¹⁹Schoenflies, "Die Krisis," *loc. cit.*

²⁰Bell, *Men of Mathematics*, Ch. 29.

²¹For literature dealing with Cantor's nervous breakdowns, see Schoenflies, "Die Krisis," Fraenkel, "Georg Cantor," pp. 198, 207–210, Peters, *Lied eines Lebens*, and Grattan-Guinness, "Towards a Biography," pp. 355–358. Cantor mentioned his trip to Paris, shortly before his breakdown in 1884, in two letters: Cantor to Gösta Mittag-Leffler, May 4, 1884, letter 18 in the Archives of the Institut Mittag-Leffler; Cantor to Felix Klein, May 10, 1884, letter 437 in the Archives of the Universitätsbibliothek, Göttingen.

blamed on Kronecker and the intractability of the continuum hypothesis. Cantor's bitterness, Schoenflies argued, was the product of tremendous opposition to his work coupled with his failure to confirm his conjecture concerning the power of the continuum. Efforts to find a solution reached a fervent pitch; determination drove him on. Because of the strain, according to Schoenflies, the breakdown inevitably followed, and Cantor lapsed into an abyss of depression which left him weak and unsure of himself and of his transfinite set theory.

Schoenflies interpreted Cantor's attempted reconciliation with Kronecker later that summer as proof that this reconstruction of the causes and events leading up to the crucial moment of collapse was essentially correct. Cantor himself recalled the entire episode in very similar terms when he wrote to his friend Mittag-Leffler a few months later, on August 18, from the summer resort of Friedrichroda, where he was trying to recuperate.²² He felt that the breakdown had been his own fault, brought on not by overwork, he insisted, but by irritations and frictions with Kronecker. He wrote that he should never have let Kronecker upset him to such a degree, and consequently he decided that the best cure was to face Kronecker squarely and to attempt a reconciliation. On the same day that he wrote to Mittag-Leffler, Cantor posted a letter to Kronecker and tried to explain his unhappiness over the state of their rivalry.²³ Above all he hoped it was still possible to gain some semblance of equanimity. Whatever might come of the attempt, Cantor knew that writing to his rival would at least serve to lighten his heart. By the end of August, Kronecker had written a polite, even friendly response, recalling how much closer they had been when Cantor was a student in Berlin, and registering some surprise at Cantor's sudden claims to any hostility or animosity between them.²⁴ Cantor was greatly reassured and wrote off immediately to Mittag-Leffler to say how happy he was that he could work again without the dread of personal animosity.²⁵

In retrospect, the first major breakdown was relatively short and could not have lasted more than a month. His letter to Felix Klein of May 10 was apparently the last before the onset of the illness; by June 21 he was writing again to Mittag-Leffler saying he had not felt so "fresh" lately and could not be sure when he would get back to his research.²⁶ Despite the speed with which the depression seemed to pass, it left the entire family shaken. His eldest daughter, Else, only nine at the time, was bewildered by the incomprehensible change in her father, the swiftness with which his entire manner had been transformed.²⁷ Cantor himself emerged from his attack emotionally enervated, and he spent the following months trying to rebuild his strength and to regain his intellectual spontaneity.

Although he returned to mathematics in the autumn and attempted to work on the continuum hypothesis again, his attitudes generally had undergone substantial alteration. He began to emphasize other interests. The amount of time he devoted to various literary and historical problems steadily increased, and he read the history and documents of the Elizabethans with great attentiveness in hopes of proving that Francis Bacon was the true author of Shakespeare's plays. As time progressed, he

²²Cantor to Mittag-Leffler, Aug. 18, 1884, letter 22, Archives of the Institut Mittag-Leffler.

²³Mentioned in his letter to Mittag-Leffler (*ibid.*).

²⁴Leopold Kronecker to Cantor, Aug. 21, 1884, transcribed in Meschkowski, *Probleme*, pp. 237–239.

²⁵Cantor to Mittag-Leffler, Aug. 26, 1884, printed in Meschkowski, *Probleme*, pp. 242–243.

²⁶Cantor to Klein, May 10, 1884, cited in n. 21 above; Cantor to Mittag-Leffler, June 21, 1884, letter 19 in the Archives of the Institut Mittag-Leffler. See Schoenflies, "Die Krisis," p. 9; Grattan-Guinness, "Towards a Biography," pp. 376–377.

²⁷Peters, *Lied eines Lebens*, pp. 15, 27.

also began to intensify his study of the Scriptures and of the church fathers, and he developed new interests in Freemasonry, Rosicrucianism, and Theosophy.²⁸ Perhaps he felt that his earlier exclusive devotion to mathematics had been too concentrated, too painful, without the rewards he had expected to compensate and make it all worthwhile. Perhaps he recalled his father's exhortation to diversify his knowledge. Perhaps he hoped to dilute the intensity of his earlier devotion to mathematics and to take life more easily, thereby avoiding the chances of a relapse of his illness. Nothing reflects more directly Cantor's shift in attitude following his first breakdown than does his sudden desire to teach philosophy in Halle instead of mathematics.²⁹ It was as though he were striving for a certain balance as he began to take his extra-mathematical interests more and more seriously.

This strategy to divert his attention from an intensive preoccupation with mathematics to a number of alternative interests was apparently successful, at least for a time, but eventually the breakdowns began anew; they grew more intense, lasted longer, and occurred with greater frequency. In 1899 (the first year since 1884 in which any record survives of his hospitalization for reasons of mental instability) Cantor was again faced with a series of dilemmas, both professional and personal.³⁰ During the summer the antinomies of set theory were on his mind, as we know from the letters he sent to his friend the mathematician Richard Dedekind.³¹ Cantor was certainly unhappy at the unsatisfactory impasse he had reached with the old problems of the continuum hypothesis, of well-ordered sets, and the comparability of the transfinite numbers. He applied for a leave of absence from the university for the fall term, and his request was granted. But in November he sent an extraordinary letter to the Ministry of Culture, saying he was anxious to abandon completely his professorship in Halle.³² So long as his salary (6,000 marks annually) was not diminished, he would be content with a modest position in some library. Nor was a prominent title necessary; he was ready to accept virtually any alternative that might release him from the confines of the German university. Whether the Ministry could cooperate or not, he was determined to break away "by all means." He emphasized his qualifications, his knowledge of history and literature, his publications on the Bacon-

²⁸For details concerning the amount of correspondence Cantor devoted to such matters, consult, e.g., Cantor's letters to Constantin Gutberlet, Jan. 24, 1886, and Feb. 6, 1887, in Cantor (I), pp. 36–38, 94–95; Cantor to E. Schmidt, Mar. 26, 1887, in Cantor (I), pp. 98–102; Cantor to Grafen Vitzthum von Eckstadt, Aug. 29, 1887, in Cantor (I), p. 104; Cantor to K. Keipewetter, Sept. 9, 1891, in Cantor (II), pp. 79–80; and Cantor to Charles Hermite, Nov. 30, 1895, and Feb. 11, 1896, both in Cantor (III), pp. 45–50 and 144–145. Note also Cantor's work on the Goldbach theorem, discussed in Meschkowski, *Probleme*, pp. 168–172; Grattan-Guinness, "Towards a Biography," pp. 360–361; and Cantor's letter to Hermite, Nov. 30, 1895, transcribed in part in Meschkowski, *Probleme*, pp. 262–263.

²⁹Cantor to Mittag-Leffler, Oct. 20, 1884. In this letter Cantor first indicated his desire to abandon mathematics for the teaching of philosophy. Mittag-Leffler wrote back on Nov. 2, 1884, endorsing the idea but adding his hope that Cantor would continue to be the great mathematical *author*. That Cantor actually began to teach philosophy at Halle is clear from Sophie Kowalewski's letter to Mittag-Leffler of May 21, 1885, letter 35, Archives of the Institut Mittag-Leffler. Cantor wrote to Hermite on Jan. 22, 1894, explaining that for nearly twenty years mathematics had not been his sole interest, but that "Metaphysik und Theologie haben, ich will es bekennen, meine Seele in solchem Grade ergriffen, dass ich verhältnismässig wenig Zeit für meine *erste Flamme* übrig habe," in Cantor (II), p. 126; transcribed in Herbert Meschkowski, "Aus den Briefbüchern Georg Cantors," *Archive for History of Exact Sciences*, 1965, 2:514.

³⁰Grattan-Guinness, "Towards a Biography," pp. 368–369.

³¹Zermelo included these letters from Cantor to Dedekind in his edition of the *Gesammelte Abhandlungen*, pp. 443–450. Comparison with the originals shows that Zermelo did a very poor, sometimes surprisingly inaccurate, job of transcribing them. For a detailed analysis of corrections that ought to be made in Zermelo's edition, consult Ivor Grattan-Guinness, "The Rediscovery of the Cantor-Dedekind Correspondence," *Jahresber. Deut. Math.-Ver.*, 1974, 76:134–136.

³²Cantor to Graf, Nov. 10, 1899, transcribed in Grattan-Guinness, "Towards a Biography," pp. 378–379.

Shakespeare question, and even added the provocative news that he had come upon certain information in the course of his research concerning the first king of England, "which will not fail to terrify the English government as soon as the matter is published."³³ Cantor tried to inject a note of urgency into the matter by asking the Ministry to send him their reply within the next two days, for if they could offer him no alternative to teaching, then as a born Russian he would apply to the Russian diplomatic corps in hopes of being of service to Tzar Nicholas II.³⁴

Nothing seems to have come of Cantor's request, nor did he enter the Russian diplomatic service. The entire episode fit the pattern which had prompted him in 1884 to think seriously about giving up mathematics in order to teach philosophy. But he was no more successful in abandoning mathematics in 1899 than he had been in 1884, and consequently he remained at the university in Halle and, when his health permitted, continued to teach mathematics. For part of 1899 he was again hospitalized, and before the year was out, another tragic event occurred which produced much pain for the entire family: on December 16, 1899, while Cantor was delivering a special lecture on the Bacon-Shakespeare question in Leipzig, his youngest son died suddenly. Cantor did not return home until later that evening to discover the terrible news.

Cantor described his son Rudolf (who would have been thirteen on December 20) in a letter to Felix Klein a week later, and the tragedy was borne in every word he wrote.³⁵ The boy had been frail in childhood but then began to grow stronger. His spirit correspondingly strengthened; he was so loving, so amiable, that he was everyone's favorite. Rudolf had been gifted musically, and his father hoped he might follow in the family tradition and become a famous violinist. In writing to Klein, Cantor was prompted to recall his own early study of the violin and to rekindle his doubts about the wisdom of having given up his inherited talents and love of music to become a mathematician. His early enthusiasm for science had faded from memory; he could not even remember *why* he had ever given up music for mathematics. That early voice calling him to a new and challenging life in 1863 seemed entirely forgotten and buried under the weight of subsequent events. Disappointed and unhappy over his own choice of profession, Cantor had wanted something better for his son. As Cantor confided to Klein, Rudolf represented a means through which he might still live and enjoy a way of life he now regretted having abandoned.

Despite his hospitalization and his son's death in 1899, Cantor managed to maintain his equilibrium for another three years, until he was again hospitalized and relieved of his teaching duties for the winter term 1902–1903.³⁶ Barely a year later yet another shock sent him to the hospital again, and thereafter he was frequently in and out of the Halle Nervenlinik. The dramatic events of Jules König's paper, read during the Third International Congress for Mathematicians in Heidelberg, greatly upset him.³⁷ Cantor was present with his two daughters, Else and Anna-Marie, and was outraged at the humiliation he felt he had been made to suffer. He seemed less upset at the suggestion that his theory was somehow wrong, for he regarded that to

³³*Ibid.*

³⁴See Document VI, *ibid.*, pp. 380–381.

³⁵"Unser jüngster Kind Rudolf ist uns vier Tage vor seinem dreizehnten Geburtstage, auf dem Wege in einen Handarbeitsunterricht, dem er seit einiger Zeit besuchte, am 16ten Dec. durch Herzschlag genommen worden," in Cantor to Klein, Dec. 31, 1899, letter 455, in the Archives of the Universitätsbibliothek, Göttingen; transcribed in Grattan-Guinness, "Towards a Biography," p. 381.

³⁶Grattan-Guinness, "Towards a Biography," pp. 368–370.

³⁷See the discussion in Gerhard Kowalewski, *Bestand und Wandel* (Munich: Oldenbourg, 1950).

be impossible, unthinkable; instead, he was furious at the public way in which his work had been brought to question. As the congress drew to a close, he was still in a state of agitation, and Schoenflies' story of how Cantor boomed into breakfast a week later to describe in excited terms the error in König's proof, shows that his mind was racing to find ways of dispelling the doubts that lingered in the aftermath of the congress.³⁸ It is no surprise that the strain was too much for him; he was soon hospitalized once again and given a leave from teaching for the winter term 1904–1905. Thereafter he spent increasingly longer periods at the Nervenlinik in Halle, from October 22, 1907, to June 15, 1908, and from September 28, 1911, to June 18, 1912, when he was then moved to yet another sanatorium.³⁹ He was admitted to the Halle clinic for the last time on May 11, 1917. By then the end was near. He did not want to remain, and he wrote continuously to his family asking that they come to take him home.⁴⁰

Summer passed to autumn, but Cantor was not allowed to return home. World War I was still raging. Food was scarce, and the lack of nourishment shows in a surviving photograph—his face gaunt and tired, his eyes still a sparkling blue but no longer piercing, no longer full of questions. On January 6, 1918, he died, apparently of heart failure. But all that he represented would never die, wrote Edmund Landau to Cantor's wife. One had to be thankful, he observed, that mankind had been given a Georg Cantor, from whose works later generations would learn: "Never would anyone remain more alive."⁴¹

THE SIGNIFICANCE OF CANTOR'S NERVOUS BREAKDOWNS

With the above details of Cantor's life and his medical history in mind, it is now appropriate to consider their implications. Above all, there is reason to suspect that much more is involved than Schoenflies' conventional interpretation might suggest. After all, the account which Schoenflies published was concerned exclusively with Cantor's first major breakdown in 1884, and it was not difficult for him to draw explicit lines between Cantor's illness and specific anxieties which the climate of Cantor's research had produced. But what of the later episodes? At points, particularly in 1904, there seem to be links between periods of emotional upset and difficult impasses with his mathematics, yet it is by no means clear that the troubles with research and professional rivalries were the sole cause of his breakdowns. The traditional account, passed by word of mouth from mathematicians of Cantor's own generation to the next, was eventually codified in Schoenflies' article devoted to Cantor's "critical years," an interpretation elaborated upon and given great currency through the popular writings of E. T. Bell.⁴² A final, even more unfortunate stamp of authenticity, considering the source, came from Bertrand Russell, who shortly before his own death wrote in somewhat piquant terms that "Georg Cantor, the subject of the following letter, was in my opinion, one of the greatest intellects of the nineteenth

³⁸For details, refer to Arthur Schoenflies, "Georg Cantor," *Mitteldeutsche Lebensbilder 3* (Magdeburg: Selbstverlag der Historischen Kommission, 1928).

³⁹Grattan-Guinness, "Towards a Biography," p. 368. From time to time Cantor apparently visited other sanatoriums, though no official records are known by which exact dates of admission or duration of stay may be documented for periods other than those listed here.

⁴⁰See letters from Cantor to his wife, Vally Guttmann Cantor, in *Nachlass Cantor II*, as listed in "Towards a Biography," p. 348.

⁴¹Edmund Landau to Vally Guttmann Cantor, Jan. 8, 1918, in *Nachlass Cantor II*; transcribed in Meschkowski, *Probleme*, p. 270.

⁴²Schoenflies, "Die Krisis," and Bell, *Men of Mathematics*.

century. . . . After reading the following letter, no one will be surprised to learn that he spent a large part of his life in a lunatic asylum."⁴³

Fortunately it is possible to assess the nature and consequence of Cantor's nervous breakdowns with somewhat more objectivity than that shown by either Bell or Russell. Ivor Grattan-Guinness was the first to consider Cantor's illness from a clinical point of view by reviewing the records which survive in the Halle Nerven-klinik and by reconstructing the outline of a case history:

The attacks all began suddenly, and usually in an autumn season, and exhibited phases of excitement and exaltation: they ended suddenly in the following spring or summer, and were sometimes followed by what we now understand to be the depressive phase. In Cantor's day it was seen as a cure, and he would be sent home to sit silent and motionless for hours on end.

Grattan-Guinness then concluded, with the advice of a psychologist who examined Cantor's file in the Halle Nerven-klinik:

Cantor's illness was basically *endogenous*, and probably showed some form of manic depression: exogenous factors, such as the difficulties of his researches and the controversies in Halle University, are likely to have played only a small part in the genesis of his attacks, little more than the clap that starts the avalanche. Thus he would have suffered his attacks if he had pursued only an ordinary mundane career.⁴⁴

Moreover, there is good direct evidence which tends to confirm that this diagnosis is more realistic and in accord with the known facts of Cantor's history than any previous attempts to explain his mental illness. For example, well before his first nervous breakdown in 1884, Cantor had been prone to periods of depression. In 1863, when he was beginning his studies at the Polytechnicum in Zurich, he developed serious misgivings about his future—to a drastic degree. His father sent off a worried letter to his son on January 21 in an attempt to help him out of the doldrums. The letter began with a little quatrain, somewhat frivolous, but designed to coax Cantor from his moodiness.⁴⁵ Then followed a number of exhortations that might just as easily have applied to Cantor's later bouts of depression. Above all, Cantor's father wanted to dispel Georg's melancholy. If only he were home, his father was sure he could break his son's depression, cheer his state of mind to such a degree that he would be immune to any relapse for three years, at least. Apparently young Georg was anxious about the costs of his education, and he doubted his ability to succeed were he forced to hurry his studies and face examinations before he was fully prepared.⁴⁶ Such anxieties compelled him to study without interruption, even going without sleep if necessary, which eventually left him so tired and despondent that his father became concerned. Georg Woldemar wrote reassuringly, making it clear that means were not lacking to allow his son to take as much time as necessary to complete his studies. But within five months his father would be dead of tuberculosis,

⁴³Bertrand Russell, *The Autobiography of Bertrand Russell* (New York: Bantam, 1968), p. 217.

⁴⁴Grattan-Guinness, "Towards a Biography," pp. 368–369.

⁴⁵Georg Woldemar Cantor to Georg Cantor, Jan. 21, 1863, *Nachlass Cantor I*; cited in Meschkowski, *Probleme*, p. 2; "Grillen sind mir böse Gäste / immer mit leichtem Sinn / tanzen durch' Leben hin, / das nur ist Hochgewinn!" Note that *Grillen* may refer both to crickets and to a melancholy visit of the blues.

⁴⁶"Wenn du bis in die Nächte hinein arbeitest, so handelst du unrecht an dir selber und schlecht an mir. Wozu die Gewalt? Ich habe dir, glaube ich, schon bis zum überdruss wiederholt, dass wir die Mittel haben, (Gottlob, und ich glaube reichlicher, als du dir einbildest!) um dein Studium so lange auszudehnen als wir wollen." *Ibid.*

and Cantor would have to leave Zurich. Moreover, he would then be without the understanding support which his father had always given. Cantor had promised to make a success of his career in mathematics, but after June of 1863, with the death of his father, he was left to do so alone.

CANTOR'S MUSE

Too little information has been preserved to allow any detailed assessment of Cantor's personality, which leaves the historian either to say nothing on the subject or to conjecture as best he can. With this in mind, the analysis which follows can only plead reasonableness by virtue of its consonance with the known facts of Cantor's history; it makes no claims to being the only reading possible.

On September 17, 1904, Cantor was admitted to the Halle Nervenlinik, where he remained until March 1, 1905. When he was released, he did something quite remarkable, something which I believe throws an unsuspected light upon his periods of mental breakdown. At least in Cantor's own mind his periods of depression and seclusion served a unique and generative purpose, providing rest and quiet during which great progress might be made in many facets of study by mere reflection. The most surprising declaration Cantor made upon his release from the clinic in 1905 was that he had had an "inspiration from above, which suggested to me a renewed study of our *Bible* with opened eyes and with banishment of all previous preconceptions."⁴⁷ He was writing to the English mathematician Philip Jourdain and explained how the "captivity and solitude" of his confinement had produced unexpected insights from which Cantor produced his pamphlet *Ex oriente lux*.⁴⁸ It was written as a series of dialogues between a master and his pupil, and the major point of the piece was to argue that Christ was the natural son of Joseph of Arimathea.

The point of all this is not to show that Cantor was interested in somewhat peculiar aspects of "documentary Christianity,"⁴⁹ as he referred to the pamphlet *Ex oriente lux*, but rather to show how he regarded his periods of hospitalization. During the long months of seclusion his mind was left free to ponder many things, and in the silence he could perceive the workings of a divine Muse, he could hear a secret voice from above which brought him both inspiration and enlightenment. Moreover, these convictions also provided a similar link between Cantor's periods of contemplation and his mathematics.

⁴⁷Cantor to P. E. B. Jourdain, Mar. 29, 1905, in Notebook I of the two Jourdain Notebooks in the Archives of the Institut Mittag-Leffler. Cantor's letter appears at fol. 82. See the transcriptions of the document in Grattan-Guinness, "The Correspondence between Georg Cantor and Philip Jourdain," *Jahresber. Deut. Math.-Ver.*, 1971, 73:123-124.

⁴⁸Georg Cantor, *Ex oriente lux. Gespräche eines Meisters mit seinem Schüler über wesentliche Punkte des urkundlichen Christentums. Bericht vom Schüler selbst Georg Jacob Aaron, cand. sacr. theol. Erstes Gespräch* (Halle: C. E. M. Pfeffer, 1905). On Apr. 5, 1905, Cantor wrote to the English mathematician Grace Chisholm Young, describing as he had to Jourdain the results of his being isolated in the Halle Nervenlinik for several months. He wrote in his own English: "As you know, I had been hermetically secluded 5½ months (from 17. Sept. to 1. March) from the world, except few visites from my family. But I can not say, that I am by this long fire-baptism embittered because I do know the great pressure, that has been practiced by the 'Ministerium' and the 'amiable' german colleagues upon my wife and my children! Farther I had a great interest to study the quite unreasonable puerile treatment and soitsdisant cure of the lamentable patients. The Muse afforded to me I employed to a renewed study of our Bible with opened eyes and postponing all prejudices. The result has been highly remarkable, as you will see by a little pamphlet (anonymous) of half a sheet, what I will send you perhaps in a week; it is now in the printing office. The title is 'Ex Oriente Lux. . .'" The letter has been printed in Grattan-Guinness, "Towards a Biography," p. 385.

⁴⁹The full title referred to essential points "des urkundlichen Christentums" (see n. 48).

Following another long period at the Halle Nervenlinik in 1908, Cantor wrote to a friend in Göttingen, the English mathematician Grace Chisholm Young. Apparently Cantor's inner voice knew more than the details of Christian history; his Muse was also a mathematician. Here it is best to let Cantor speak for himself:

A peculiar fate, which thank goodness has in no way broken me, but in fact has made me stronger inwardly, happier and more expectantly joyful than I have been in a couple of years, has kept me far from home—I can also say far from the world—since October 22, 1907, until June 15, 1908, thus until last Monday. . . . In my lengthy isolation neither mathematics nor in particular the theory of transfinite numbers has slept or lain fallow in me; the first publication in years which I shall have to make in this area is designated for the "Proceedings of the London Mathematical Society," to which, for the great honor of having named me to its membership, I shall always remain grateful as well as to the Royal Society for conferring upon me the Sylvester Medal three and one-half years ago.⁵⁰

At the close of this same letter he lashed out against Poincaré's remarks condemning Cantorism at the International Congress in Rome in 1908. Poincaré believed that mathematics could be saved from the paradoxes of set theory by allowing only those concepts which could be completely defined in a *finite* number of words. Poincaré criticized Cantor for starting from the "genus supremum," or the assumed existence of the actual infinite, instead of generalizing, building more complicated theorems from simpler, established results. In his letter to Mrs. Young, Cantor made reference to the impossibility of any sort of "genus supremum" embracing *all* transfinite numbers. In fact, he was adamant in underscoring that point:

I have never proceeded from any "Genus supremum" of the actual infinite. Quite the contrary, I have rigorously proven that there is absolutely no "Genus supremum" of the actual infinite. What surpasses all that is finite and transfinite is no "Genus"; it is the single, completely individual unity in which everything is included, which includes the "Absolute," incomprehensible to the human understanding. This is the "Actus Purissimus" which by many is called "God."⁵¹

There can be no mistake about Cantor's identification of his mathematics with some greater absolute unity in God. This also paralleled his identification of transfinite set theory with divine inspiration. Even *before* his first nervous breakdown of 1884, Cantor had told Mittag-Leffler that his transfinite numbers had been communicated to him from a "more powerful energy," that he was only the means by which set theory might be made known. Just as he had been inspired to write *Ex oriente lux*, a "Dialogue of a master with his student . . . reported by the student himself," so too his mathematics could be inspired through a Muse from above. Thus his periods of isolation could be regarded as periods during which, as he told Mrs. Young, the transfinite numbers lay neither fallow nor forgotten but might be further elucidated by the grace of God sent to inspire new lines of research. All this was very much in keeping with the principles Cantor had learned from his father.

⁵⁰Cantor to Grace Chisholm Young, June 20, 1908, transcribed in Herbert Meschkowski, "Zwei unveröffentlichte Briefe Georg Cantors," *Der Mathematikunterricht*, 1971, 17(4):30–40; esp. pp. 32–33.

⁵¹*Ibid.*, p. 34. See also W. H. Young, "The Progress of Mathematical Analysis in the Twentieth Century," *Proceedings of the London Mathematical Society*, 1926, 24:422–423. For Poincaré's remarks made in Rome which were responsible for Cantor's reply to Mrs. Young, see Henri Poincaré, "L'avenir des mathématiques," *Atti del IV Congresso Internazionale dei Matematici*, Vol. I (Rome, 1909), pp. 167–182, translated into English as "The Future of Mathematics," *The Smithsonian Report for 1909* (Washington, D.C.: GPO, 1910), pp. 123–140, esp. p. 140.

Georg Woldemar Cantor had been careful to instill in his son a belief in the ability to succeed through hard work and faith in God. Without such confidence in his own abilities, Cantor might never have had the courage to face the relentless opposition he encountered to his work, almost from the start. Had he not been able to cast himself in the role of God's messenger, inspired by some higher source, he might never have asserted the unquestionable, indubitable correctness of his research. The religious dimension which Cantor attributed to his transfinite numbers should not be discounted as an aberration. Nor should it be forgotten or separated from his existence as a mathematician. The theological side of Cantor's set theory, though perhaps irrelevant for understanding its mathematical content, is nevertheless essential for the full understanding of his theory and why it developed in its early stages as it did. Cantor believed that God endowed the transfinite numbers with a reality making them very special. Despite all the opposition and misgivings of mathematicians, in Germany and elsewhere, he would never be persuaded that his results could be imperfect. This belief in the absolute and necessary truth of his theory constituted for Cantor an imperative of sorts. He could not allow the likes of Kronecker to beat him down; he felt a duty to keep on, in the face of all adversity, to bring the insights he had been given as God's messenger to mathematicians everywhere.

THE POSITIVE EFFECTS OF NEGATIVE REACTIONS TO TRANSFINITE SET THEORY

One has only to read the opening sections of Cantor's *Grundlagen einer allgemeinen Mannigfaltigkeitslehre* (1883) to see how his interest in philosophy affected his presentation of set theory. We know that he went into great detail concerning the philosophy of the infinite because he wanted to confront and dispel the long-standing objections traditionally levied against the idea of actual infinities. Cantor knew his work was revolutionary; he had found it difficult to overcome his own long-standing prejudice against completed infinities.⁵² But gradually he came to see the power of his new ideas, and he found that it was possible to eliminate even the objections of philosophers and theologians. Nevertheless, his heavy philosophical encasement of set theory in the *Grundlagen* was a disadvantage to its widespread acceptance. This was suggested by Felix Klein's appeal, and mentioned specifically in Mittag-Leffler's plea that Cantor remove all such nonmathematical material from his publications on the subject.⁵³

Cantor, however, continued to believe that his mathematics and his metaphysics went hand in hand. As a result, set theory was often considered as belonging more to philosophy than to serious mathematics, and for many years journals and mathematical indexes alike listed works in set theory under the rubric "Philosophy."⁵⁴ Only after publication of the "Beiträge zur Begründung der transfiniten Mengenlehre" (1895–1897), completely mathematical and devoid of any overtly philosophical

⁵²Georg Cantor, *Grundlagen einer allgemeinen Mannigfaltigkeitslehre. Ein mathematisch-philosophischer Versuch in der Lehre des Unendlichen* (Leipzig: Teubner, 1883); in Cantor, *Gesammelte Abhandlungen*, p. 175.

⁵³Cantor to Klein, Feb. 7, 1883, letter 432, Archives of the Universitätsbibliothek, Göttingen; Mittag-Leffler to Cantor, Mar. 11, 1883, Archives of the Institut Mittag-Leffler.

⁵⁴When the *Jahrbuch über die Fortschritte der Mathematik* first began to include the literature of set theory (at the behest of the Italian mathematician Giulio Vivanti) in 1894, it was reviewed under the heading "Philosophie"; in 1904 it was given a subsection between "Philosophie" and "Pädagogik"; only after the First World War was set theory given a separate, independent position in the *Jahrbuch*. For further details, see Fraenkel, "Georg Cantor," p. 215.

content, did his theory of transfinite numbers begin to receive its full share of study and recognition.

If his metaphysical bent affected the form and early acceptability of Cantor's set theory, his deeply religious convictions played a different role. His personality, as we have considered it in some detail, was such that success was very important to him, and he felt almost compelled to prove to his parents that his decision to study mathematics had been the proper one. But as the years passed, theoretical difficulties with the continuum hypothesis and mounting external criticism from opponents like Kronecker conspired to raise doubts and to feed Cantor's anxieties about the acceptability and success of his work. As a result of his breakdown in 1884, he was forced to deal with the problems that had seemed to provoke his illness. He attempted a reconciliation with Kronecker, but that was short-lived. He worked less fervidly at his mathematics, and for a time seemed rather detached from any interest in the success or promotion of set theory. Perhaps a weaker man would never have recovered from the experiences of 1884–1885. What seemed to keep his spirit alive, however, was faith in the absolute certainty of his theory of transfinite sets. Because he believed that set theory, having been divinely inspired by God, was therefore absolutely and necessarily true, nothing could succeed in shaking his confidence that it could never be flawed. It was exactly this point that he once made in a letter of 1888:

My theory stands as firm as a rock; every arrow directed against it will return quickly to its archer. How do I know this? Because I have studied it from all sides for many years; because I have examined all objections which have ever been made against the infinite numbers; and above all, because I have followed its roots, so to speak, to the first infallible cause of all created things.⁵⁵

Such unwavering convictions were Cantor's greatest allies in periods of stress and difficulty. Had it not been for the force of his own faith, supported by convictions entirely nonmathematical in nature, his theory of transfinite numbers might never have survived the criticism of finitists and intuitionists alike. Cantor did much more for set theory than merely discover its basic principles. He not only shaped its early character and formulated all of its most essential elements virtually single-handedly, but he also insured that once mathematicians were ready to consider the significance of the transfinite numbers, the entire theory would be ready to stand on the foundations which he had given it.

Though distressing and ultimately injurious to Cantor's health and well-being, the continuing conflict with Kronecker had its positive results. It forced Cantor to look more carefully at the foundational aspects of the new theory he was building. It caused him to search for more direct, mathematically acceptable, rigorous means by which to introduce, to define, to explain his transfinite numbers. It might well have prompted him to take all philosophical objections more seriously. And in the end, it stimulated his concern for the *Deutsche Mathematiker-Vereinigung*, and his earnest desire to promote not only a German forum for mathematicians, but to further as well the organization of mathematical congresses on an international scale.⁵⁶ Finally, it may be that without the keen and relentless opposition from Kronecker, Cantor

⁵⁵Cantor to K. F. Heman, June 21, 1888, Cantor (I), p. 179.

⁵⁶Details of Cantor's roles as founder and first president of the DMV, as well as his motives for encouraging the organization of international congresses for mathematicians, are given in Dauben, *Georg Cantor*, pp. 160–165.

might never have been driven to produce so detailed and carefully argued an exposition of his theory as the *Beiträge* of 1895 and 1897.

Later generations might overlook his metaphysical pronouncements and miss entirely the deeply religious roots of Cantor's faith in the veracity of his work. But these, as well as the early encouragement of his father, contributed to Cantor's resolve not to abandon his transfinite numbers for less controversial and more acceptable interests. Instead, his determination seems actually to have been strengthened in the face of opposition. His forbearance, as much as anything else he might have contributed, insured that set theory would survive the early years of doubt and denunciation to flourish eventually as a vigorous, revolutionary force in scientific thought of the twentieth century.