The Option to Stock Volume Ratio and Future Returns

Travis L. Johnson *
Stanford University
Graduate School of Business

Eric C. So
Stanford University
Graduate School of Business

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Abstract

We examine the information content of option and equity volumes when trade direction is unobserved. In a multimarket asymmetric information model, we show that equity short-sale costs result in a negative relation between relative option volume and future firm value. In our empirical tests, firms in the lowest decile of the option to stock volume ratio (O/S) outperform the highest decile by 1.47% per month on a risk-adjusted basis. Our model and empirics both indicate that O/S is a stronger signal when short-sale costs are high or option leverage is low. O/S also predicts future firm-specific earnings news, consistent with O/S reflecting private information.

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1. Introduction

In recent decades, the availability of derivative securities has rapidly expanded. This expansion is not limited to equity options and now includes a vast array of securities ranging from currency options to credit default swaps. Derivatives contribute to price discovery because they allow traders to better align their strategies with the sign and magnitude of their information. The leverage in derivative securities, combined with this alignment, creates additional incentives to generate private information. In this way, trades in derivative markets may provide more refined and precise signals of the underlying asset's value than trades of the asset itself. Understanding how and why derivatives affect price discovery is therefore vital to understanding how information comes to be in asset prices.

This study focuses on the information content of trading volumes. Observed transactions play an important role in price discovery because order flow imbalances can reflect the sign and magnitude of private information. While market makers can observe these imbalances, most outside observers cannot, which makes the problem of inferring private information more complex. Techniques to empirically estimate order flow imbalances are computationally intensive, typically requiring the pairing of intraday trades and quotes. This problem is exacerbated when agents have access to multiple trading venues because the mapping between transactions and private information becomes more difficult to identify. In this paper, we address the inference problem of the outside observer by examining the information content of option and equity volumes when agents are privately informed but trade direction is unobserved.

We provide theoretical and empirical evidence that informed traders' private information is reflected in O/S, the ratio of total option market volume (aggregated across calls and puts) to total equity market volume. The O/S measure was first coined and studied by Roll, Schwartz, and Subrahmanyam (2010), whose findings suggest that cross-sectional and time-series variation in O/S may be driven by informed trade. As a natural extension of these findings, we examine the relation between O/S and future returns. Empirically, we find that contrasting publicly available totals of firm-specific option and equity volume portends directional prices changes, in particular that low O/S firms outperform the market while high O/S firms underperform. At the end of each month we sort firms by O/S and compute the average return of a portfolio consisting of a short position in stocks with high O/S and a long position in stocks with low O/S. This portfolio provides an average risk-adjusted hedge return of 1.47% in the month following the formation date.

If option volume is concentrated among risky firms with higher return volatility, one might anticipate the opposite result, namely that firms with higher O/S earn higher future returns. While our finding is inconsistent with this risk-based explanation, we take several steps to mitigate concerns that exposure to other forms of risk, for example liquidity risk, explains the O/S-return relation. First, we show that the relation holds after controlling for exposure to the three Fama-French, momentum, and Pastor-Stambaugh liquidity factors. Second, we show that the predictive power of O/S for future returns is relatively short-lived, failing to predict returns beyond the second month following portfolio formation. Third, we show that a decile strategy using changes in O/S also generates positive risk-adjusted alphas.

We argue that the negative relation between O/S and future returns is driven by short-sale costs in equity markets, which make option markets an attractive venue for traders with negative news. Motivated by this story, we model the capital allocation decision of privately informed traders who can trade in option and equity markets. Capital constraints and equity short-sale costs lead informed agents to trade options more frequently for negative signals than positive ones, thus predicting a negative relation between relative option volume and future equity value. An important innovation of our paper is that this relation does not require classifying trades as being buyer- versus seller-initiated. Instead, our theoretical predictions and empirical tests rely on publicly available volume totals.

Having established the negative cross-sectional relation between O/S and future returns, we next test our model's prediction that this relation is stronger when short-sale costs are high. As short-sale costs increase, informed traders are more likely to switch from equities to options for negative signals, which strengthens the O/S-return relation. We test this prediction using three different measures of short-sale costs. The first measure is derived from institutional ownership, as in Nagel (2005), and is available throughout our 1996-2008 sample window. We also use two direct measures of short-sale costs, transacted loan fees and available loan supply, from a proprietary database of institutional lending that is available on a monthly basis from 2002 through 2008. Across all three measures, we find that portfolio alphas associated with O/S are increasing in the cost of shorting.

An additional empirical prediction arising from our model is that the O/S-return relation is weaker when option leverage is high. As option leverage increases, bid-ask spreads in options markets increase, which weakens the O/S-return relation because the bid-ask spread acts like a switching cost for traders considering the use of options to avoid short-sale costs. When the bid-ask spread is larger, fewer traders switch from equities to options for negative signals, and the O/S-return relation is therefore weaker. Empirically, we find that the portfolio alphas associated with O/S are monotonically decreasing across option leverage quintiles.

It may be initially puzzling why we do not find a relation between the ratio of call to put volume and future returns. Our model demonstrates that O/S provides a clearer signal of private information than the ratio of call to put volume because call volume could be good news (if informed traders are buying) or bad news (if informed traders are selling), and put volume is similarly ambiguous. Thus, in the absence of information about the sign of each trade (i.e. buy vs. sell), O/S is an indication of the sign of private information while the ratio of call to put volume is not. Our model does, however, predict a positive relation between call-put volume differences and future return skewness because informed traders buy calls (puts) for extremely good (bad) news and sell calls (puts) for moderately bad (good) news. Consistent with this prediction, we show empirically that the ratio of call volume to put volume predicts return skewness in the subsequent month.

We also find that O/S predicts the sign and magnitude of earnings surprises, standardized unexplained earnings, and abnormal returns at quarterly earnings announcements in the following month. These tests show that the same O/S measure we use to predict monthly returns also contains information about future earnings announcements that occur in the subsequent month. This is consistent with O/S reflecting private information that is incorporated into equity prices following a subsequent public disclosure of the news.

The rest of the paper is organized as follows. We begin in Section 2 by discussing our results in the context of existing literature. We model the multimarket price discovery process and formalize the equilibrium strategy of informed traders in Section 3. The model results in four main findings regarding the information content of relative trading volumes, which we translate into empirical predictions. In Section 4, we describe the data, methodology, empirical results, and robustness checks. Finally, we present results pertaining to quarterly earnings announcements in Section 5 and conclude in Section 6.

2. Relation to literature

The two immediate antecedents of our work are Easley, O'Hara, and Srinivas (1998), hereafter referred to as EOS, and Roll, Schwartz, and Subrahmanyam (2010), hereafter RSS. EOS contains a multimarket equilibrium model wherein privately informed traders are allowed to trade in both option and equity markets.[1] The EOS model highlights conditions under which informed traders transact in both option and equity markets, and predicts that directional option volume signals private information not yet reflected in equity prices. Specifically, their model predicts that positive trades (i.e. buying calls and selling puts) are positive signals of equity value and that negative trades (i.e. selling calls and buying puts) are negative signals of equity value. An interesting but otherwise unexplored empirical finding in EOS is that negative option market activity carries greater predictive power for

[1] The authors point out that asymmetric information violates the standard assumptions underlying complete markets and, therefore, the option trading process is not redundant. Consistent with this idea, Bakshi, Cao, and Chen (2000) find that S&P500 options frequently move in the opposite direction of equity prices.

future price changes. EOS comment on this finding in the following excerpt:

> An interesting feature of our results is the asymmetry between the negative- and positive-position effects ... suggesting that options markets may be relatively more attractive venues for traders acting on 'bad' news. An often-conjectured role for options markets is to provide a means of avoiding short-sales constraints in equity markets ... Our results support this conjecture, suggesting a greater complexity to the mechanism through which negative information is impounded into stock prices [Page 458].

We provide a formal means of understanding their finding by introducing short-sale costs into a microstructure framework with asymmetric information. Like EOS, informed agents trade with a risk neutral market maker, and can buy or sell shares of stock, buy or sell calls, or buy or sell puts. Unlike EOS, we impose capital constraints and short-sale costs that play a central role in determining which assets informed traders choose to trade. It is comparatively cheaper to capitalize on bearish private signals in option markets because traders can buy puts or sell calls, and in both cases they can create new option contracts without first borrowing them from a third party. In our model's equilibrium, the costs associated with short-selling make informed traders more likely to use options for bad signals than for good ones and, as a result, high O/S indicates negative private information and low O/S indicates positive private information.

Like EOS, we solve a static model and therefore need the additional assumption that some friction prevents equity prices from immediately reflecting the information in option volumes in order for the model's prediction about the conditional mean equity value to translate into return predictability. Our main empirical prediction, that O/S is a negative cross-sectional signal of future returns, differs from EOS in that it can be tested empirically without signing the direction of trades. We predict and confirm that contrasting publicly available totals of firm-specific option and equity volume portends directional price changes.

Empirically, our study of the relation between O/S and future returns is a natural extension of the work in RSS, which introduces the option to stock volume ratio, and coins it O/S. The authors find substantial intertemporal and cross-sectional variation in O/S, and explain a significant part this variation in a regression framework. In particular, O/S is increasing in firm size and implied volatility but decreasing in option bid-ask spreads and institutional holdings. Our results shed additional light on the variation in O/S by examining the theoretical determinants of relative option volume when a subset of market participants are privately informed, and the empirical relation between O/S and future returns. RSS also shows that O/S in the days immediately prior to announcement predicts the magnitude of returns at earnings announcements, consistent with O/S reflecting traders' private information. Conditional on there being positive (negative) earnings news, they find that O/S predicts higher (lower) announcement returns (see Section 5 for more details). Our analysis builds upon this finding by demonstrating an unconditional predictive relation between the prior month's O/S and earnings surprises.

Another recent paper examining option volume is Roll, Schwartz, and Subrahmanyam (2009), which shows a positive cross-sectional relation between Tobin's q and unscaled option volume. The authors interpret this as evidence that liquid option markets increase firm value because they help complete markets and generate informed trade. Our model and empirical tests support this intuition by demonstrating that option markets are an attractive venue for informed traders.

The results of this paper also relate to the literature on price discovery and information flow in multiple markets.[2] Pan and Poteshman (2006) use proprietary CBOE option market data and document strong evidence of informed trading in option markets. The authors find that sorting stocks by the amount of newly initiated positions in puts relative to calls foreshadows future returns but they conclude the predictability is not due to market inefficiencies and instead reflects the fact that their volume measure is not publicly observable. A key innovation of our paper is demonstrating that publicly available, non-directional, volume

[2]Whether option markets lead equity markets or vice versa remains an open question. Anthony (1988) examines the interrelation of stock and option market volume and finds that call-option market activity predicts volume in the underlying equity with a one-day lag. Similar findings are reported in Manaster and Rendleman Jr. (1982). In contrast, Stephan and Whaley (1990) find no evidence that option markets lead equity markets. While our results do little to resolve this debate, they demonstrate that combined signals from option and equity markets serve as a leading indicator of future equity market returns.

totals predict future returns. Similarly, Cremers and Weinbaum (2010) and Zhang, Zhao, and Xing (2010) find that publicly available asymmetries in implied volatility across calls and puts predict future returns.

Prior research establishes that equity volume, the denominator of our primary return predictor O/S, is useful by itself in predicting future returns, though the direction depends on the way volume is measured (see, e.g., Gervais, Kaniel, and Mingelgrin (2001); Lee and Swaminathan (2000); Brennan, Chordia, and Subrahmanyam (1998)). We decouple O/S into separate measures of equity and option volume and show that past option volume is negatively related to future returns incremental to past equity volume. Other extant work uses equity volume as a conditioning variable for examining the relation between past and future returns. Specifically, Lee and Swaminathan (2000) shows that high (low) volume winners (losers) experience faster momentum reversals, and Llorente et al. (2002) show that the relation between equity volume and return autocorrelation changes sign depending on the amount of informed trading for a given equity.

During the 2008 financial crisis, the SEC banned short sales for 797 'financial' stocks, providing an interesting case study of the impact of short-sale costs on options markets. Both Battalio and Schultz (2011) and Grundy, Lim, and Verwijmeren (2010) find that option market spreads increased and option market volume decreased for firms subject to the ban relative to those exempt from it. A key component of our model is that option markets serve as an alternative venue for negative news when shorting is costly, and at first glance the 2008 episode contradicts this premise. However, as emphasized in Battalio and Schultz (2011), the short-sale ban also imposed costs on option market makers who short equity, making it more difficult for them to hedge when selling puts or buying calls. In our model, increasing costs for market makers who write puts or buy calls will increase spreads and *decrease* volume in option markets, while banning shorts will increase both spreads and volume in options markets. Our model therefore suggests we interpret the decrease in option market volume during the 2008 ban as a result of the added costs of shorting for market makers outweighing the relocation of trades stemming from negative information to option markets.

In modelling the relation between short-sale costs and informed trading, our paper is also related to Diamond and Verrecchia (1987), who model the impact of short-sale constraints on the speed of adjustment of security prices to private information when informed traders only have access to equity markets. In their model, short-sale constraints cause some informed parties with negative information not to trade. Thus, the absence of trade in their model is a negative signal of future firm value. In our model, trading options is an alternative to abstaining from trade when the cost of shorting is high. As a result, a high option volume ratio, rather than the absence of trade, reflects negative private information. Diamond and Verrecchia (1987) also suggest that the introduction of traded options would effectively lower short-sale costs by providing a lower cost venue to capitalize on negative news. Our model and empirical results support this intuition by predicting and empirically confirming a negative relation between O/S and future equity values.

3. The model

We present a model of informed trading in both equity and options markets in the presence of short-sale costs and a borrowing constraint. Informed traders build a portfolio by trading sequentially with a competitive, risk neutral market maker. As argued in Black (1975), privately informed traders may prefer to trade in options markets because of the additional leverage they provide. The borrowing constraint makes it impossible for informed traders to match the leverage in options by using a combination of the risk-free asset and the stock. As a result, we find that option trades are more likely to originate from an informed trader than stock trades regardless of parameter choices. A key feature of our model is that traders must pay a lending fee to a third party when shorting stock. Because it is costly to trade on bad news in the stock market, in equilibrium the mean equity value conditional on an option trade is lower than the mean equity value conditional on a stock trade.

There are three tradable assets in the model: an equity, a call option, and a put option.

The stock liquidates for $\tilde{V}$ at some time $t = 2$ in the future. The value of $\tilde{V}$ is unknown prior to $t = 2$, but it is common knowledge that

$$\tilde{V} = \mu + \tilde{\epsilon} + \tilde{\eta}, \tag{1}$$

where μ is the exogenous mean equity value, and $\tilde{\epsilon}$ and $\tilde{\eta}$ are independent, normally distributed, shocks with zero mean and variances σ_ϵ^2 and σ_η^2. The call and put are both struck at μ, and both expire at time $t = 2$. We define $\tilde{C}$ and $\tilde{P}$ as the value of the call and put at $t = 2$, and note that

$$\tilde{C} = \max(\tilde{V} - \mu, 0) = \max(\tilde{\epsilon} + \tilde{\eta}, 0) \tag{2}$$

$$\tilde{P} = \max(\mu - \tilde{V}, 0) = \max(-(\tilde{\epsilon} + \tilde{\eta}), 0). \tag{3}$$

We focus on the case of a single strike price because our aim is to model the choice between trading options and trading equities, as opposed to the choice amongst options of different strikes. We use μ as a strike price so that calls and puts have the same leverage. To avoid complexities arising from early exercise, we assume that all options are European.

Trade occurs at time $t = 1$, at which point a fraction θ of the traders (henceforth "informed traders") know the realization of $\tilde{\epsilon}$ but the remaining traders, and the market maker, do not. Informed traders only have a noisy signal of the true value $\tilde{V}$ since they do not know the realization of $\tilde{\eta}$. The distribution of $\tilde{V}$ conditional on the information that $\tilde{\epsilon} = \epsilon$ is:

$$\tilde{V}|(\tilde{\epsilon} = \epsilon) \sim N(\mu + \epsilon, \sigma_\eta^2) \tag{4}$$

Informed traders are risk neutral, and therefore value the stock, call, and put using:

$$E(\tilde{V}|\tilde{\epsilon} = \epsilon) = \mu + \epsilon \tag{5}$$

$$E(\tilde{C}|\tilde{\epsilon} = \epsilon) = \Phi\left(\frac{\epsilon}{\sigma_\eta}\right)\epsilon + \phi\left(\frac{\epsilon}{\sigma_\eta}\right)\sigma_\eta \tag{6}$$

$$E(\tilde{P}|\tilde{\epsilon} = \epsilon) = -\Phi\left(\frac{-\epsilon}{\sigma_\eta}\right)\epsilon + \phi\left(\frac{\epsilon}{\sigma_\eta}\right)\sigma_\eta \tag{7}$$

respectively, where Φ is the standard normal's cumulative distribution function, and ϕ is its probability distribution function. The first term of the informed trader's option valuation, given in equations (6) and (7), represents the contribution of the conditional mean of $\tilde{V}$, while the second represents the contribution of the remaining volatility σ_η. The informed trader's valuation function for a call is increasing in both ϵ and σ_η, while for a put it is decreasing in ϵ and increasing in σ_η.

Informed traders have no endowment and select the portfolio of risky assets and a risk-free asset that maximizes their expected final period wealth. Because they have no endowment, traders need to borrow to take long positions. All investors have the same maximum borrowing $\$\kappa$.[3] Traders can borrow and short the same number of shares they can buy, and write φ options contracts for every share they can short, where $\varphi > 1$. The exact value of φ is determined by the margin requirements imposed on the trader.

We require that each trade be in exactly one type of risky asset, resulting in six possible trades: buy or sell stock, buy or sell calls, and buy or sell puts. At equilibrium prices, the informed traders have a strict preference among the assets except at 6 cutoff points. Allowing trades in bundles of multiple assets, for example 1 call and 2 shares, complicates the analysis without changing our results or providing additional insight. Bundles serve as "intermediate" portfolios used by the informed trader upon receiving a signal near their indifference point between the two bundled assets. As long as the bundle trades that include a short position in equities require traders to pay short-sale costs, our model still predicts that equity volume

[3] Alternatively, we could allow a different maximum borrowing amount for traders buying equities and traders buying options. In this case, informed trader demand would shift toward whichever asset allowed more borrowing, but none of our main results would change.

reflects positive private information and option volume negative private information.

A fraction $1-\theta$ of the traders are uninformed and trade for reasons outside the model, possibly a desire for liquidity, the need to hedge other investments or human capital, or a false belief that they have information. As noted in EOS, hedging is a commonly cited reason to trade in options markets. Regardless of their motivation, uninformed traders choose among the same possible portfolios as the informed traders. Uninformed traders' volume is distributed among these trades (buy and sell stock, calls, and puts) with the probabilities γ_1, γ_2, $\ldots \gamma_6$, where $\sum_{i=1}^{6} \gamma_i = 1$.

A competitive and risk neutral market maker posts bid and ask prices for all three assets, along with fixed order sizes, that result in zero expected profit for each trade.[4] For notation, we write a_s, b_s, a_c b_c, a_p, and b_p for the ask and bid prices of the stock, call, and put, respectively. Similarly, q_{bs}, q_{ss}, q_{bc}, q_{sc}, q_{bp}, and q_{sp} denote the quantities available for each type of trade. For example, q_{bs} gives the quantity of stock available for the trader to buy. As in EOS and Glosten and Milgrom (1985), trades occur sequentially and at fixed order sizes. The key difference, however, is that in our model these order sizes are endogenously determined. The market maker is aware of the traders' budget constraint, and determines order sizes from the exogenous budget parameter κ and the endogenously determined prices. With only one chance to trade and in possession of valuable information, informed traders take the largest position allowed by the capital constraint when choosing to trade. The traders make their portfolio choice, the market maker decides on prices, and jointly these determine the quantities traded.

This approach is different from EOS, in which the fixed order sizes are exogenous parameters used to capture the leverage available in each asset. In the EOS model and using their notation, stock trades are in round lots of γ shares, and option trades are in a single contract controlling θ shares. When $\theta > \gamma$, options are more levered in the sense that trading options

[4] In the model, additional market makers have no impact as long as they are risk-neutral and competitive. In reality, the return predictability evidence in this paper suggests there is some segmentation between option and equity markets, perhaps because they have different market makers.

provides more exposure to the underlying than trading the stock itself. When $\theta < \gamma$, the opposite is true and stocks have more leverage than options. Confirming the intuition in Black (1975), they show that trades carry more information in whichever market has greater leverage. However, the EOS model provides no guidance on the relation between γ and θ. We specify a relation between these two by noting that leverage in options is only relevant when there is a capital constraint or some other cost of borrowing. Modelling this relation allows us to pin down the order sizes because the total value of each trade must satisfy the budget constraint; specifically, it tells us that at equilibrium trade quantities, options have more leverage than equities (or $\theta > \gamma$ in the language of EOS)[5].

A critical new ingredient in our model is a short-sale cost paid by the trader to a third party who lends them the shares. The market maker posts a bid price for the stock b_s, and corresponding quantity q_{ss}. To sell to the market maker, traders must borrow shares from an unmodelled third party who charges a lending fee equal to a fraction $\rho > 0$ of the total amount shorted $q_{ss}b_s$. The lender is able to charge such a fee because they have some market power, or because there is some counterparty risk. No such fee exists when writing options because there is no need to find a contract to borrow – the market maker can simply create a new contract. The parameter ρ can also be thought of as a reduced form of any cost to shorting stock, for example recall risk or the indirect costs described in Nagel (2005). The result is that the market maker pays $q_{ss}b_s$ in exchange for q_{ss} shares, but the trader only nets $q_{ss}b_s(1-\rho)$ from the transaction.

It is important for our argument that option market makers do not pay the short-sale cost ρ in the course of hedging their position, and therefore embed the short-sale cost in option prices. If option prices fully reflected the short-sale cost, option markets would not be an effective alternative to shorting for traders with negative information and our results would no longer hold. In the model, option prices are set by the equilibrium between informed

[5]We could use the exogenous order sizes as they do in EOS, and all our results would carry though as long as options had more leverage than stocks ($\theta > \gamma$). Using order sizes set endogenously by the budget constraints allows us to argue that options are more levered than stocks regardless of model parametrization.

trader demand and bid-ask spreads, and ρ only impacts option prices through informed trader demand. In reality, market makers hedge their positions using equities and take into account their cost of shorting when setting prices. However, option market makers have access to cheaper shorting than ordinary investors, and therefore the option-embedded short-sale cost is smaller than the actual short-sale cost in equity markets. Moreover, the evidence in Bakshi, Cao, and Chen (2000) suggests options are not completely redundant securities, which means the equilibrium between informed trader demand and marker maker pricing described by our model plays an important role in determining option prices even when option market makers face short-sale costs.

3.1. Equilibrium

An equilibrium in our model consists of an optimal trading strategy for informed traders as a function of their signal, and bid-ask prices and quantities that yield zero expected profit for the market maker. In equilibrium, informed traders use the following cutoff strategy $f(\epsilon)$ that maps the range of possible signals to the space of possible trades:[6]

$$f(\epsilon) = \begin{cases} \text{buy puts} & \text{for } \epsilon \leq k_1 \\ \text{sell stock} & \text{for } \epsilon \in (k_1, k_2] \\ \text{sell calls} & \text{for } \epsilon \in (k_2, k_3] \\ \text{make no trade} & \text{for } \epsilon \in (k_3, k_4] \\ \text{sell puts} & \text{for } \epsilon \in (k_4, k_5] \\ \text{buy stock} & \text{for } \epsilon \in (k_5, k_6] \\ \text{buy calls} & \text{for } \epsilon > k_6 \end{cases} \tag{8}$$

For extremely good or bad signals, informed traders buy options despite the large information-based transaction costs in these markets because of the greater leverage they provide. The large bid-ask spread makes options unattractive for weaker signals, and so informed traders trade equities instead. For even weaker good or bad signals, however, informed traders value the stock near its unconditional mean and therefore cannot profitably trade stock; however,

[6]When choosing to trade, the informed trader always borrows the maximum allowed, either directly or by shorting a security. The uninformed traders choose among the same set of possible trades for simplicity.

they value the options well below their unconditional mean because extreme outcomes are unlikely, and therefore sell options. If bid prices are below informed trader's valuation of both a put and a call for a given signal, the informed traders choose not to trade. The cutoff points k_i arise endogenously in equilibrium and are chosen so that informed traders strictly prefer writing puts for all $\epsilon < k_1$, selling stock for all $k_1 < \epsilon < k_2$, etcetera. Some regions may be empty in equilibrium, meaning $k_i = k_{i+1}$ for some i. The addition of short-sale costs has the effect of shrinking the region of signals for which informed traders short stock, making $k_2 - k_1$ smaller than it would be otherwise.

The bid and ask prices for each asset (a_s, b_s, a_c, b_c, a_p, and b_p), the associated quantities (q_{bs}, q_{ss}, q_{bc}, q_{sc}, q_{bp} and q_{sp}), and the informed traders' cutoff points k_i are the 18 equilibrium parameters. Together they satisfy 18 equations, presented fully in Appendix B, which assure that the market maker's expected profit is zero for each trade, that the quantities and prices match the budget constraint, and that informed traders are indifferent between the two relevant trades at each cutoff point.

3.2. Results and Empirical Predictions

Due to the non-linearity of the simultaneous equations, no closed form solution for the equilibrium parameters is available. We proceed by first deriving results and empirical predictions which hold generally from the simultaneous equations themselves, and second by examining a numeric example in Appendix A. The focus of this paper is on the information content of option volume when there are short-sale costs. To this end, we assume throughout that $\rho > 0$. Proofs are in Appendix D.

Result 1. *When each asset is equally likely to be bought or sold by an uninformed trader, the stock is worth less conditional on an option trade than it is conditional on a stock trade.*

Empirical Prediction 1. *Option volume, scaled by volume in the underlying equity, is negatively related to future stock returns for the underlying.*

The main result is that an option trade is bad news for the value of the stock and a stock trade is good news. This result differs from EOS in that the conditioning variable is the location of trade rather than the direction of trade. Option volume corresponds to bad news because informed traders use stocks more frequently to trade on good news than they do for bad news due to the extra cost of short selling stock. Therefore, the expected equity value is lower conditional on an option trade than the unconditional mean, which is in turn lower than the expectation conditional on a stock trade. Result 1 requires that uninformed traders buy and sell each asset with equal probability, but holds regardless of how uninformed traders distribute their demand across the different assets; for example, uninformed traders may trade stock much more frequently than options or trade calls more than puts.

In order to translate Result 1 into an empirical prediction, we consider the implications of our static model in a multiperiod setting. If equity markets fully incorporate the information revealed through options trading into their valuations, stock prices will immediately reflect the new conditional expectation of $\tilde{V}$ after each option trade. Otherwise, stock prices do not fully reflect the information content of options trades for the time between when the informed option market trading occurs and when the information becomes public through another channel. In this case, there will be a negative relation between option volume and subsequent returns until the public release of the information. Empirical Prediction 1 is, therefore, a joint hypothesis that (a) short-sale costs make O/S a negative cross-sectional predictor of future prices, and (b) some of the information in option volume reaches equities through other channels, such earnings announcements, that occur after the option is traded.

A common intuition is that call volume reflects good news and put volume reflects bad news. Therefore, a natural question is why we do not examine the call to put volume ratio, or separate O/S into the call to stock and put to stock volume ratios. Equation (8) demonstrates that this common intuition does not hold in our setting because informed traders buy calls and sell puts for good news, and buy puts and sell calls for bad news. Unless trade direction is observable, it is unclear whether call (put) volume reflects good or bad news. Depending on the fraction of informed traders θ, the behavior of uninformed traders

γ_i, and the distribution of the underlying asset value, both put and call volume can be good or bad news about future equity value. Unlike separated call and put volumes, Result 1 illustrates that total option volume is a negative signal of private information regardless of model parameterization.

Our model makes no prediction about the overall volume in options and stocks together, only that option trades are bad news relative to stock trades. Our goal is to focus on informed traders' choice between equities and options, conditional on having a signal about the future value of a firm. Therefore, our predictive measure is the ratio of option volume to equity volume, rather than unscaled option volume.

Result 2. *The disparity in conditional mean equity values between option and stock trades, as a fraction of the mean stock price μ, is weakly increasing in the short-sale cost ρ.*[7]

Empirical Prediction 2. *The predictive power of the option volume ratio for future stock returns is increasing in the cost of shorting equity.*

Although Empirical Prediction 1 does not rely on cross-sectional differences in short-sale costs, if such differences exist the model predicts that option volume is a worse signal for high short-sale cost equities than low short-sale cost equities, but is still a valuable signal as long as short-sale costs exist. When short-sale costs are higher, option volume is worse news about future equity value because informed traders deviate from shorting stock to trading options for a larger range of signals. We discuss the empirical proxies for short-sale costs used to test Empirical Prediction 2 in Section 4.

Result 3. *The disparity in conditional mean equity values between option and stock trades, as a fraction of the mean stock price μ, is weakly decreasing in the option's leverage $\lambda \equiv \frac{\partial C}{\partial S}\frac{S}{C} = \frac{\mu}{2\phi(0)\sqrt{\sigma_\epsilon^2+\sigma_\eta^2}}$.*

[7] We scale the difference in conditional means by μ because we want the conditional mean difference to be relative to the unconditional mean. In the language of investment, we consider returns rather than raw profits.

Empirical Prediction 3. *The predictive value of relative option volume for future stock returns is decreasing in the average Black-Scholes λ of options traded.*

Result 3 may be surprising at first because leverage is usually an attractive feature of options. Indeed, in our model leverage allows an informed trader's investment to be more sensitive to their private information on a per dollar basis, and therefore the overall use of options by informed traders increases with leverage. However, this very attractiveness creates large information-based transaction costs in options markets. Larger bid-ask spreads make it more expensive for informed traders to divert their demand from stocks to options to avoid the short-sale cost, meaning that when option leverage is high, informed traders use options to avoid short-sale costs for a smaller range of signals. This weakens the O/S-return relation.

Empirically, Result 3 suggests that volume in options markets with higher leverage provides a weaker signal than volume in options markets with lower leverage. For a measure of leverage, we use $\lambda = \frac{\partial C}{\partial S}\frac{S}{C}$, the elasticity of C with respect to S, reflecting the "bang-for-the-buck" notion of leverage. We show in Appendix C that $\frac{\partial C}{\partial S}\frac{S}{C} = \frac{\mu}{2\phi(0)\sqrt{\sigma_\epsilon^2+\sigma_\eta^2}}$ in our model. Empirically, we use the Black-Scholes λ because some of the parameters in the model λ are unobservable. Result 3 indicates there exists a spread between conditional means of $\tilde{V}$ regardless of the leverage λ but that the spread is larger for smaller values of λ. Therefore, our empirical prediction is that O/S predicts returns for all levels of λ, but most strongly for low λ.

Result 4. *Equity value has higher skewness conditional on a call trade than conditional on a put trade.*[8]

Empirical Prediction 4. *The ratio of call volume to put volume varies positively with the future skewness of stock returns.*

[8] Our proof of this result requires that the uninformed trader demand for each asset γ_i does not approach zero. If it did, markets would begin to fail and the skewness result may reverse.

Result 4 follows from the equilibrium trading strategy described in Section 3.1. Following the notation used to describe the informed trader's strategy in equation (8), skewness conditional on a put trade is low because, if informed, it reflects either moderately good news (i.e. $\epsilon \in (k_4, k_5]$) or extremely bad news (i.e. $\epsilon \leq k_1$). Similarly, skewness conditional on a call trade is high because, if informed, it reflects either moderately bad (i.e. $\epsilon \in (k_2, k_3]$) or extremely good news (i.e. $\epsilon > k_6$). Empirical Prediction 4 restates Result 4 as a relation between the ratio of call volume to put volume and future return skewness.

To summarize the results up to this point, our model provides three principle predictions regarding the price discovery process across option and equity markets. First, we predict that the concentration of option volume, relative to equity volume, is a negative cross-sectional signal of future equity returns. Second, relative option volume carries greater predictive power when short-sale costs are high or leverage is low. Third, call-put volume differences predict the skewness of future returns. In Section 4, we explore these predictions empirically.

4. Empirical tests

The option data for this study comes from the Ivy OptionMetrics database, which provides end-of-day summary statistics on all exchanged-listed options on U.S. equities. The summary statistics include option volume, quoted closing prices, and option Greeks. The OptionMetrics database, and hence the sample for this study, spans from 1996 through 2008. The final sample for this study is dictated by the intersection of OptionMetrics, Compustat Industrial Quarterly and Center for Research in Security Prices (CRSP) Monthly data. We restrict the sample to firm-months with at least 50 call and 50 put contracts traded to reduce measurement problems associated with illiquid option markets. We also eliminate closed-end funds, real estate investment trusts, American Depository receipts, and firms with a stock price below \$1. The intersection of these databases and data restrictions results in 175,654 firm-months corresponding to approximately 150 months and 1,700 total unique firms.

For each firm i in month m, we sum the total option and equity volumes, denoted by

$\text{OPVOL}_{i,m}$ and $\text{EQVOL}_{i,m}$, respectively. Specifically, $\text{OPVOL}_{i,m}$ equals the total volume in option contracts across all strikes for options expiring in the 30 trading days beginning 5 days after the trade date.[9,10] We report $\text{EQVOL}_{i,m}$ in round lots of 100 to make it more comparable to the quantity of option contracts that each pertain to 100 shares. We define the option to stock volume ratio, or $\text{O/S}_{i,m}$, as:

$$\text{O/S}_{i,m} = \frac{\text{OPVOL}_{i,m}}{\text{EQVOL}_{i,m}} \tag{9}$$

Panel A of Table 1 contains descriptive statistics of $\text{O/S}_{i,m}$ (hereafter O/S for notational simplicity) for each year in our sample. The sample size increases substantially over the 1996-2008 window. The number of firm-months increases from 7,501 in 1996 to 18,363 in 2008. The remainder of Panel A presents descriptive statistics of O/S for each year of the sample. The mean value of O/S for the entire sample is 4.34%, which indicates that there are roughly 23 times more equity round lots traded than option contracts with times to expiration between 5 and 35 trading days. Average O/S has increased in recent years due to growth in options volume outpacing growth in equity volume. O/S is positively skewed throughout the sample period due to a high concentration of relative option volume among a small subset of firms.

[Insert Table 1 about here]

Panel B of Table 1 presents firm characteristics by deciles of O/S. SIZE (LBM) equals the log of market capitalization (book-to-market) corresponding to the firm's most recent quarterly earnings announcement. High O/S firms tend to be larger and have lower book-

[9] We exclude options expiring within 5 trading days to avoid measuring mechanical trading volume associated with option traders rolling forward to the next expiration date. The results are qualitatively unchanged if we include options with longer expirations.

[10] As a robustness check, we separate option volume into moneyness categories and find that at-the-money, in-the-money, and out-of-the-money option volumes all predict future returns once scaled by equity volume. This is consistent with the our model's intuition that options, regardless of their moneyness, serve as an alternative trading venue with symmetric costs for both positive and negative news trades. The consistency of the O/S-return relation across moneyness categories provides comfort that our model is not omitting critical determinants of the O/S-return relation by focusing on a single strike price.

to-market ratios. Although low O/S firms tend to be smaller, our initial data requirement of 50 calls and 50 puts traded in a month tilts our sample toward larger and more liquid firms, which mitigates, but fails to eliminate, concerns that the O/S-return relation is attributable to transaction costs. The average market capitalization of firms exceeds $2 billion in each O/S decile. VLC and VLP indicate the number of call and put contracts traded in a given month, respectively. Across all deciles of O/S, the number of call contracts traded exceeds the number of put contracts, which is consistent with calls being more liquid than put contracts. High O/S firms also tend to have higher levels of both option and equity volume. In our model, high O/S reflects negative private information, and hence our univariate trading strategy based on O/S consists of taking a short position in higher equity volume stocks (i.e. high O/S stocks) and a long position in lower equity volume stocks (i.e. low O/S stocks). This raises concerns that the predictive power of O/S may reflect compensation for taking positions in low liquidity firms. We attempt to mitigate these concerns in several ways, which are discussed in greater detail below.

[Insert Table 2 about here]

Panel A of Table 2 presents time-series equal-weighted average returns for each O/S decile. MOMEN equals the cumulative market-adjusted return measured over the 6 months leading up to portfolio formation. Market-adjusted returns equal the return of firm i minus the CRSP value-weighted index inclusive of dividends. RET(m) equals the portfolio's market-adjusted monthly return m months following portfolio formation, where portfolios are formed on the last trading day of month 0. To ensure that portfolios are formed on the basis of publicly available information, we exclude the last trading day of month $m = 0$ when calculating O/S. For example, we measure total volume from October 1st to October 30th and match the observed volume totals with returns accumulated from the close of trading on October 31st. Excluding volume on the last trading day of each month enforces a one-day separation between observing O/S and the formation of portfolios.[11]

[11]This restriction is important because of non-synchronous closing times across option and equity markets.

Table 2 demonstrates that high O/S firms tend to have high momentum, suggesting that a strategy that takes a long position in low O/S firms and short position in high O/S firms results in a momentum reversal bet.[12] The RET(1) column contains market-adjusted returns in the month following portfolio formation, where the bottom row contains one-month returns resulting from an equal-weighted short position in the highest decile of O/S and an equal-weighted long position in low O/S firms. Low O/S firms outperform high O/S firms by 114 basis points per month (14.54% annualized) prior to risk adjustments. The corresponding t-statistic, based on the monthly time-series, is -2.46 indicating that O/S is a negative cross-sectional signal of future equity returns. The difference in RET(2) across the extreme deciles of O/S is also negative but statistically insignificant, indicating that univariate return prediction fails in the second month following portfolio formation.

Panel B of Table 2 presents an analogous test where we sort firms by changes in O/S rather than levels of O/S. ΔO/S is defined as the difference between O/S in the portfolio formation month and its average over the prior six months, all scaled by this average. ΔO/S captures the extent to which O/S deviates from its historical average. Paralleling the construction of O/S, all of the information needed to calculate ΔO/S is available at the portfolio formation date. Return patterns for ΔO/S portfolios are similar to the Panel A results that rely on O/S. Low ΔO/S firms outperform high ΔO/S firms by an average of 1.15% in the month following portfolio formation with a corresponding t-statistic of -3.07. Our finding that O/S and ΔO/S contain comparable predictive power mitigates concerns that the O/S-return relation reflects compensation for a static form of risk.

[Insert Figure 1 about here]

Our main analyses focus on the relation between O/S and monthly returns. We chose

Removing this restriction does not materially affect our results.

[12]The negative correlation between pre- and post-formation portfolio returns raises concerns that the ability of O/S to predict returns is attributable to patterns of short-term momentum reversal shown in Lehmann (1990) and Jegadeesh (1990). We mitigate this concern by including a short-term reversal factor in our factor regressions (see footnote 14) and by explicitly controlling for portfolio formation month returns in our multivariate tests (see discussion of Table 4).

a monthly horizon, rather than daily or weekly, to mitigate concerns that the pattern of predictable returns is attributable to portfolio rebalancing costs. Although our model does not formally define the length of a given period, the endogenous determination of bid-ask spreads is intuitively linked to short horizons. Consistent with this intuition, EOS focus their analyses on intraday volumes and RSS focus on O/S at the daily level. We also find that the O/S-return relation holds when both are measured at the daily level. Figure (1) plots equal-weighted future daily O/S strategy returns corresponding to a long position in the lowest $\mathrm{O/S}_{i,d}$ decile firms and a short position in the highest $\mathrm{O/S}_{i,d}$ decile firms on day d. We use two return metrics: four-factor-adjusted and raw returns. To calculate expected returns, we estimate historical firm-specific factor loadings over the prior year using daily returns, and apply the estimated coefficients to contemporaneous daily factors. Factor-adjusted returns equal raw returns minus historically estimated expected returns.

Figure (1) captures two important patterns. First, $\mathrm{O/S}_{i,d}$ possesses strong predictive power for daily future returns over the 20 trading days following portfolio formation. The sum of the daily long-short O/S strategy returns is 100 (91) basis points when including (excluding) returns accrued on the day immediately following portfolio formation. Risk adjustment increases the returns associated with the O/S strategy, consistent with traders having private information about the idiosyncratic component of returns. The sum of daily factor-adjusted returns is 133 (123) basis point when including (excluding) returns accrued on the day immediately following portfolio formation. Second, the predictive power of $\mathrm{O/S}_{i,d}$ dissipates over time, with most of the predictive power coming in the first few days following portfolio formation. Both of these patterns are consistent with the monthly return results documented in Table 2.[13] Given the similar predictive power of daily and monthly O/S, the remainder of the analysis is conducted at the monthly level.

[Insert Table 3 about here]

Panel A of Table 3 presents portfolio alphas for each O/S decile using CAPM, four-

[13] In untabulated results, we find that weekly O/S provides similar predictive power for future returns.

factor, and five-factor risk adjustments. To calculate five-factor portfolio alphas, we regress the monthly excess return corresponding to each O/S decile on the three Fama-French, momentum, and Pastor-Stambaugh liquidity factors. Specifically, we estimate three variants of following regression for each O/S decile:

$$r_m^p - r_m^f = \alpha + \beta_1(r_m^{mkt} - r_m^f) + \beta_2\text{HML}_m + \beta_3\text{SMB}_m + \beta_4\text{UMD}_m + \beta_5\text{LIQ}_m + \epsilon_m \tag{10}$$

where r_m^p is the return on an equal-weighted portfolio of stocks in a given O/S decile, r_m^f is the risk-free rate, and r_m^{mkt} is the market return in month m. HML and SMB correspond to the monthly returns associated with high-minus-low market-to-book and small-minus-big strategies. Similarly, UMD and LIQ equal the monthly returns associated with high-minus-low momentum and Pastor-Stambaugh liquidity strategies, respectively. The CAPM risk-adjustment omits all factors except for $r_m^{mkt} - r_m^f$ and the four factor model omits LIQ. All factor returns are obtained from Ken French's data library via WRDS.

Panel A contains portfolio alphas corresponding to lagged decile ranks of O/S. A one-month lag indicates that the portfolio is formed at the conclusion of month $m-1$ and the returns accumulated in month m. The one-month lag portfolio alphas provide a risk-adjusted return comparable to the univariate RET(1) results of Table 2. Notice that risk-adjustment increases the returns associated with the O/S strategy, consistent with the results in Figure 1. Low (High) O/S firms earn a five-factor-adjusted alpha of 87.34 (-59.77) basis points in the month following portfolio formation with a t-statistic of 3.45 (-3.04). The long-short strategy results in an average risk-adjusted return of 1.47% per month or, equivalently, 19.15% annualized.[14] Return prediction extends into the second month following portfolio formation, where the long-short strategy results in additional average risk-adjusted return of 0.69% per month. The long-short strategy fails to accumulate statistically significant abnormal returns in the third month following the observation of O/S. This short-term predictive power of

[14] To mitigate concerns that the O/S-return relation is due to short term momentum reversal, we obtain the short term reversal factor from Ken French's website. In untabulated results, we find that the inclusion of short-term reversal factors in Eq. (10) result in portfolio alphas of similar magnitude and significance.

O/S is consistent with relative option volume reflecting private information about near term changes in price, rather than serving as a proxy for a static risk characteristic that is priced in the cross-section.

As predicted by our model, in addition to high O/S indicating bad news, low O/S indicates good news: a portfolio of firms with low O/S has significantly positive alphas in the month after portfolio formation. In the context of our model, low option volume indicates good news because informed traders use equity more (and options less) frequently for positive signals than negative ones due to the equity short-sale costs.

Panel B of Table 3 presents an analogous test where we sort firms by ΔO/S. Firms in the lowest decile of ΔO/S earn a factor-adjusted alpha of 1.13% per month with a corresponding t-statistic of 4.28. Similarly, firms in the highest decile of ΔO/S earn -0.52% per month with a t-statistic of -3.16. The alpha associated with a long-short strategy is 1.66% per month (t-statistic = 5.54), which corresponds to 21.77% on an annualized basis. Similar to the Panel A results, we find that ΔO/S retains predictive power for future returns in the second month following portfolio formation. The lowest ΔO/S decile continues to outperform the highest decile in the second month, earning an additional monthly alpha of 1% (t-statistic = 4.59). This result does not extend to the third month following the portfolio formation, suggesting that ΔO/S also provides a transitory signal of future returns.

Panels C and D contain the estimated factor loadings corresponding to the O/S and ΔO/S strategies, respectively. The results demonstrate that both strategies have negative loadings on the market factor. The O/S strategy returns load positively on the HML and negatively on UMD, indicating that the strategy involves selling glamour firms with strong recent equity performance. The ΔO/S strategy results in similar factor loadings except that it displays a slighty negative but statistically insignificant relation with the HML factor.

[Insert Table 4 about here]

Table 4 presents pooled regression results where RET(1) is the dependent variable and year fixed effects are used throughout. Columns (1) through (4) contain the results of regressing RET(1) on deciles of O/S. For example, in column (1) the coefficient on O/S is -0.106, indicating that firms in the highest O/S decile outperform firms in the lowest decile by an average of 0.95% ($= -0.106 \times 9$) per month. The coefficient on the O/S decile has a corresponding t-statistic of -8.16, where standard errors are clustered at the monthly level. Columns (2) and (3) demonstrate that the relation between O/S and RET(1) is robust to controlling for MOMEN, log market capitalization (SIZE), and log book-to-market (LBM). Columns (3) through (7) also control for the Amihud illiquidity ratio, AMIHUD, defined as the ratio of absolute returns to total dollar volume where higher values indicate lower liquidity, and vice versa. AMIHUD is measured on a daily basis and then averaged over the month prior to portfolio formation. In Column (4) we explicitly control for returns in the portfolio formation month, RET(0). We include RET(0) as a control because Table 2 shows a negative association between O/S and returns in the portfolio formation month suggesting O/S may reflect expected monthly return reversals. Consistent with Jegadeesh (1990) and Lehmann (1990), the coefficient on RET(0) is significantly negative, indicating a negative relation between returns in months $m-1$ and m. Across Columns (1) through (4), the coefficient on O/S is significantly negative, with the coefficients and t-statistics remaining stable across specifications.

Columns (5) through (7) of Table 4 contain regression results where O/S is decoupled into separate measures of option and equity volume, OPVOL and EQVOL. Column (5) demonstrates that the coefficient corresponding to deciles of OPVOL is -0.132 (t-statistic = -8.38), consistent with high option volume reflecting negative private information. In column (6) we find that equity volume has a negative relation with future returns, which parallels the findings in Brennan, Chordia, and Subrahmanyam (1998). Column (7) includes both equity volume and option volume deciles. While the coefficient on OPVOL remains negative, there is a marginally significant and *positive* relation between deciles of EQVOL and RET(1). Column (8) demonstrates that O/S and OPVOL are both negatively related to future returns after controlling for the other, though both coefficients attenuate in size and significance. Finally, Column (9) documents a similar result when both OPVOL and ΔO/S are included. Taken together, the results of Table 4 demonstrate a robust negative association between option market volume and future equity returns, distinct from monthly return reversals, the pricing of liquidity, and the relation between equity market volume and future returns.

[Insert Table 5 about here]

Panel A of Table 5 presents a monthly transition matrix for deciles of O/S, where firms are sorted by deciles of O/S and a one-month lag of O/S. Hence, the diagonal terms indicate the percentage of firms that remain within their respective portfolio in consecutive months. Approximately 56% of low O/S firms change portfolios from month to month. Similarly, over 40% of high O/S firms change portfolios. The average of the diagonals is 26.91% indicating that on average approximately 75% of all firms change O/S deciles in adjacent months. That approximately half of the extreme portfolios turn over each month reduces concerns that the O/S-return relation reflects compensation for a time-invariant risk characteristic.

Panel B of Table 5 examines portfolio alphas associated with O/S when sorting firm-months into deciles based on the time-series for a given firm, as opposed to sorting in the cross-section for a given month. For each firm, we observe the entire time-series of O/S during our 1996-2008 sample window and sort firm-months into portfolios using the realized O/S distribution. The sample for this analysis is limited to firms with at least 20 firm-months during our sample window, though the results do not appear sensitive to this restriction. This approach is not implementable as an investment strategy because it requires observing the entire firm-specific time-series of O/S for a given firm. However, this approach mitigates concerns that the extreme O/S deciles possess firms with inherently different levels of option volume that are relatively stable over time but that vary negatively with expected returns. Panel B demonstrates that firm-specific O/S deciles possess a strong negative relation with future returns, mirroring the results in Tables 2 through 4. The five-factor alpha associated with this strategy is over 3% per month, with a corresponding t-statistic of 12.35. These results confirm and extend our earlier findings by demonstrating that firm-specific ΔO/S possesses strong predictive power for future returns.

[Insert Table 6 about here]

Consistent with Empirical Prediction 2, Panel A of Table 6 demonstrates that the predictive power of O/S for future returns is increasing in short-sale costs. Our first measure of firm-specific short-sale costs, following Nagel (2005), is the level of residual institutional ownership $\text{RI}_{i,t}$. We define $\text{RI}_{i,t}$ as the percentage of shares held by institutions for firm i in quarter t, adjusted for size in cross-sectional regressions. Specifically, $\text{RI}_{i,t}$ equals the residual $\epsilon_{i,t}$ from the following regression:

$$\text{logit}(INST_{i,t}) = \log(\frac{INST_{i,t}}{1 - INST_{i,t}}) = \alpha_t + \beta_{1,t} SIZE_{i,t} + \beta_{2,t}(SIZE^2_{i,t}) + \epsilon_{i,t}, \tag{11}$$

where $INST_{i,t}$ equals the fraction of shares outstanding held by institutions as reflected in the Thomson Financial Institutional Holdings (13F) database. We calculate quarterly holdings as the sum of stock holdings of all reporting institutions for each firm and quarter. Values of $INST_{i,t}$ are winsorized at 0.0001 and 0.9999. Low levels of $\text{RI}_{i,t}$ (hereafter referred to as RI) correspond to high short-sale costs because stock loans tend to be scarce and, hence, short selling is more expensive when institutional ownership is low. We match RI to a given firm-month by requiring a three-month lag between the Thompson Financial report date and the first trading day of a given month. Panel A of Table 6 contains O/S portfolio alphas across quintiles of RI. In each month, firms are independently sorted into deciles of O/S and quintiles of RI. Within each RI quintile, we compute the equal-weighted return of a long-short position in the extreme deciles of O/S. We repeat this process each month yielding a time-series of long-short returns for each month during our 1996-2008 sample window. Portfolio alphas equal the intercept from regressing the monthly long-short returns on the five factors used in Eq. 10. Panel A demonstrates that the O/S portfolio alphas are monotonically decreasing across the RI quintiles, consistent with our model's prediction that the O/S-return relation is most pronounced when short-sale costs are high. Portfolios alphas are significantly positive across all RI quintiles, where the alpha for the low RI quintile is 2.49% (t-statistic = 3.64) while the alpha for the highest RI quintile is 0.91% (t-statistic = 2.21). The difference in alphas across the extreme RI quintiles is 1.58% per month with a t-statistic of 2.29.

Panels B and C of Table 6 retest Empirical Prediction 2 using more direct measures of short-sale costs. These tests rely on a proprietary dataset of institutional lending provided to us by Data Explorers. Data Explorers aggregates information on institutional lending from several market participants including hedge funds, investment banks, and prime brokers.[15] Similar to the datasets used in D'Avolio (2002) and Geczy, Musto, and Reed (2002), this dataset contains monthly institutional lending data on transacted loan fees and available loan supply. The sample period is June of 2002 through December of 2008, covering approximately half of our main sample period. Using these measures, we again find that the O/S-return relation is most pronounced when short-sale costs are high. Panel B's measure, LF_m, is the value-weighted average loan fee for institutional loans occurring in month m. Higher values of LF_m reflect higher short-sale costs because investors must pay the lending fee in order to obtain the shares necessary for shorting. In Panel B, O/S alphas are positive in each LF quintile and the difference in alphas across the extreme LF quintiles is 3.157% per month with a t-statistic of 2.91. Panel C's measure, LS_m, is the total quantity of shares available for lending, as a fraction of firms' total shares outstanding, at the conclusion of month m. Lower values of LS_m correspond to higher short-sale costs because investors must first locate lendable shares before implementing a short position. In Panel C, the difference in alphas across extreme LS quintiles is 1.799% per month with a t-statistic of 1.98. Across all three measures of short-sale costs, the results in Table 6 are consistent with informed traders using option markets more frequently when short-sale costs are high.

[Insert Table 7 about here]

[15] See www.dataexplorers.com for more details regarding the data.

Consistent with Empirical Prediction 3, Table 7 demonstrates that the predictive power of relative option volume for future stock returns is strongest when option leverage is low. For each firm-month, leverage is defined as the open-interest-weighted average of $\frac{\partial C}{\partial S}\frac{S}{C}$, as provided by OptionMetrics, which we refer to as LM.[16] Panel C contains the long-short O/S portfolio alphas across quintiles of LM, where firms are independently sorted on LM and O/S. The O/S alphas are monotonically decreasing across quintiles of LM, where the difference in portfolio alphas across the extreme LM quintiles is significant at the 1% level (t-statistic = 4.40). The results are consistent with informed traders moving a larger portion of their bets from shorting stock to trading options when leverage is lowest. Although we find that O/S alphas are decreasing in leverage, we do not find that the O/S strategy has positive alpha in each leverage quintile. Specifically, the alpha in the highest leverage quintile is negative but not statistically significant. One possible explanation for this finding is that certain levels of option leverage attract more volume than others for institutional reasons, a feature not described by our model.

[Insert Figure 2 about here]

Figure 2 presents annual cumulative returns to three long-short strategies assuming monthly portfolio rebalancing for each year in the sample. The first strategy consists of an equal-weighted long-short position in the extreme O/S deciles. The long-short strategy is implemented each month and the monthly returns are accumulated within each calendar year. The unconditional long-short strategy (shown in grey) results in positive returns in 10 out of 13 years, with a mean return of 14.72% and a standard deviation of 20.24%. The second takes long-short positions in extreme O/S deciles among firms in the bottom quintile of residual institutional ownership (RI), corresponding to firms with the highest short-sale costs. The strategy (shown in black) produces positive returns in every year of the sample, while significantly increasing the mean and decreasing the standard deviation of the annual

[16] The results are qualitatively similar when using volume-weighted average option leverage.

cumulative returns.[17] The increased performance of the long-short strategy among firms with high short-sale costs is consistent with the model prediction that informed traders with negative news shift their capital allocation toward option markets in response to short-sale costs. The third strategy corresponds to analogous long-short returns for firms in the lowest leverage (LM) quintile. Conditional upon being in the lowest LM quintile, the long-short O/S strategy (shown in white) results in positive hedge returns in all but one year while again increasing the mean return and decreasing the variance relative to the unconditional O/S strategy. Together, the results of Figure 2 demonstrate a robust association between O/S and future returns throughout our sample period, and that this association is stronger when short-sale costs are high or option leverage is low.

[Insert Table 8 about here]

In addition to the above results pertaining to O/S, we also examine what the call to put volume ratio, C/P, tells us about future equity returns. Empirical Prediction 4 states that C/P is a positive predictor of future return skewness. The results of Table 8 confirm this prediction. For each firm-month, we compute C/P as:

$$\mathrm{C/P}_{i,m} = \frac{\mathrm{VLC}_{i,m}}{\mathrm{VLP}_{i,m}}, \tag{12}$$

where $\mathrm{VLC}_{i,m}$ is the total call volume for firm i in month m and $\mathrm{VLP}_{i,m}$ is defined analogously for puts. Firms are sorted into deciles based on C/P, where the tenth (first) decile corresponds to high (low) levels of call volume relative to put volume. For each calendar month, we calculate the cross-sectional skewness of monthly returns within each decile portfolio of C/P, which results in a panel dataset of approximately 1,500 observations.

Table 8 contains the results of regressing portfolio skewness on the C/P decile rank. In column (1), the coefficient on the C/P decile rank is positive with a t-statistic of 2.31, indicating that C/P is positively associated with future return skewness. Column (2) demonstrates

[17]Because the long-short strategy results rely upon taking positions among equities with high short-sale costs, the reported results are not intended to reflect the actual returns achieved through implementation.

that the relation between C/P and return skewness remains significant after controlling for the lagged skewness of a given portfolio. The evidence in Table 8 is consistent with our model's equilibrium prediction that informed traders buy puts for extremely bad news, sell calls for moderate bad news, sell puts for moderate good news, and buy calls for extremely good news.

5. Additional analyses

Several existing studies examine the link between option market activity and earnings announcements. Skinner (1990) finds that the information content of earnings announcements declines following options listing, consistent with options facilitating informed trade prior to announcements. Amin and Lee (1997) finds that open interest increases prior to announcements and possesses some predictive power for the sign of earnings news. RSS finds that O/S significantly increases immediately prior to earnings announcement, suggesting that O/S reflects private information regarding earnings news. Consistent with this interpretation, they find that O/S positively predicts the absolute magnitude of earnings news and that the effect is more pronounced when the earnings news is negative. Both findings are consistent with our prediction that option markets serve as alternative venue for traders with negative private information seeking to avoid short-sale costs. Additionally, RSS finds that the relation between O/S and absolute announcement returns is less pronounced when there is a significant movement in equity prices prior to the announcement date, consistent with informed traders impounding private information into prices ahead of the announcement. In this section, we provide additional evidence that relative option volume reflects private information by examining whether prior month's O/S provides predictive power for the sign and magnitude of quarterly earnings surprises. Our tests build upon RSS by examining the relation between O/S and signed earnings news and returns.

We assemble a new dataset from four sources. The OptionMetrics, Compustat industrial quarterly file, CRSP daily stock file, and IBES consensus file provide information on option volume, quarterly firm attributes, equity prices, and earnings surprises, respectively. We apply the same sample restrictions outlined in Section 4. The intersection of these four databases results in a final sample consisting of 46,670 firm/quarter observations.

To the extent that informed traders gravitate toward options ahead of negative news, we predict that O/S is negatively associated with the resulting earnings surprise. For each earnings announcement, we measure O/S in the calendar month that directly precedes it. For example, we match earnings announcements that occur in July with O/S measured in June of the same year. This empirical design directly mimics the main analyses used in Section 4 by using O/S in month $m-1$ to predict returns in month m, except here we focus the analysis on the prediction of earnings news and earnings-announcement window returns revealed in month m. We use three variables to capture the news released at earnings announcements. The first, SURPRISE, is the earnings surprise as measured by the firm's actual EPS minus the analyst consensus EPS forecast immediately prior to the announcement, scaled by the beginning of quarter stock price. The second, standardized unexplained earnings (SUE), is an alternative measure of earnings surprise defined as the realized EPS minus EPS from four quarters prior, divided by the standard deviation of this difference over the prior eight quarters. The final, CAR$(-1,+1)$, equals three-day cumulative market-adjusted returns during the earnings announcement window from $t-1$ to $t+1$, where day t is the earnings announcement date.

[Insert Table 9 about here]

Column (1) of Table 9 demonstrates that the prior calendar month's O/S decile carries predictive power for future earnings surprises. The negative relation between relative option volume and earnings surprises is consistent with the negative O/S-return relation reflecting informed trade. Column (2) produces analogous results where SUE is the dependent variable. The coefficient on O/S is negative with a t-statistic of -4.50, indicating that O/S is negatively associated with earnings innovations. Column (3) presents the regression results when the announcement window abnormal returns, CAR$(-1,+1)$, is the dependent variable. The coefficient on O/S remains negative and statistically significant incremental to the firm's momentum, size, and book-to-market, which is consistent with relative option volume reflecting private information about future asset values revealed in part by the earnings announcement. As an example of the economic significance, the lowest O/S decile outperforms the highest by 0.79% ($= -0.088 \times 9$) in the 3 day announcement window (all else equal).

Additionally, we examine the predictive power of O/S for returns following the announcement. To the extent that O/S reflects private information about impending earnings news, we expect that the predictive power of O/S attenuates following the earnings announcement. Column (4) presents regression results where the dependent variable, CAR$(+2,+60)$, equals the cumulative buy-and-hold market-adjusted return from t+2 to t+60. We find that O/S coefficient is negative but insignificant at the 10% level (t-statistic=-1.64). Collectively, the results in Table 9 are consistent with O/S reflecting private information which is gradually impounded into prices via earnings news.

6. Conclusion

The central contribution of this paper is a mapping between observed transactions and the sign and magnitude of private information that does not require estimating order flow imbalances. Specifically, we examine the information content of option and equity volumes when agents are privately informed but trade direction is unobserved. We provide theoretical and empirical evidence that O/S, the amount of trading volume in option markets relative to equity markets, is a negative cross-sectional signal of private information. Stocks in the lowest decile of O/S outperform the highest decile by 1.47% per month on a factor-adjusted basis in the month following portfolio formation. We offer a simple explanation for this finding, specifically that it results from how informed traders choose between trading in equity and option markets in the presence of short-sale costs.

We model the capital allocation and price-setting processes in a multimarket setting and develop novel predictions regarding information transmission across markets. In equilibrium, short-sale costs and capital constraints cause informed traders to trade more frequently in option markets when in possession of a negative signal than when in possession of a positive signal, thus predicting that volume in options markets, relative to equity markets, is indicative of negative private information. By empirically documenting that O/S is a negative cross-sectional signal for future equity returns, our results are consistent with market frictions preventing equity prices from immediately reflecting the information content of O/S.

Our model also predicts that O/S is a stronger signal when short-sale costs are high or option leverage is low, and that volume differences across calls and puts predict future return skewness, all of which we confirm in the data. In particular, using proprietary firm-specific data on institutional loan fees and loan supply from 2002-2008, we find that O/S alphas are a positive and increasing function of short-sale costs. Similarly, conditional on low average leverage traded in options, sorting stocks by deciles of O/S results in an average annual hedge return of 21.27% and produces positive hedge returns in 12 out of 13 years in our sample. Finally, we show that O/S predicts the sign and magnitude of earnings surprises and abnormal returns at quarterly earnings announcements, consistent with O/S reflecting trader's private information.

Appendix A: A Numeric Example

One of the primary goals of our analysis is to model the equilibrium decision rule of informed traders in a setting where the range of possible private signals and terminal asset values are continuous. While these continuity assumptions allow for a richer set of predictions not available in the discrete case, they make the model sufficiently complex and non-linear that it can only be solved by a computer given numeric values for exogenous parameters. To illustrate the features of the model, we present the solution for exogenous parameters chosen to match the characteristics of a typical publicly traded US stock.

The stock has mean value $\mu = 30$, reflecting the average price of a publicly traded share of equity, and standard deviation $\sqrt{\sigma_\epsilon^2 + \sigma_\eta^2} = 3$, so that volatility is 10% of value, approximately the average monthly return volatility. We examine the case of a minority of traders $\theta = 10\%$ having fairly precise information, meaning the known shock's volatility $\sigma_\epsilon^2 = 5$ is slightly larger than the unknown shock's volatility $\sigma_\eta^2 = 4$. Equity volume is, as we show in this paper, about 20 times option volume on average. For this reason, while uninformed traders are equally likely to buy or sell each asset, they are twenty times more likely to use stocks than to trade calls or puts, meaning $\gamma_1 = \gamma_2 = \frac{40}{84}$ and $\gamma_3 = \gamma_4 = \gamma_5 = \gamma_6 = \frac{1}{84}$. The capital constraint κ is 100, which in equilibrium acts as a numeraire and impacts the quantities traded but not equilibrium prices or strategies. Regulation T requires that margin accounts for shorting equity contain 50% of the trade's value. Contrasting this with the CBOE margin requirement when writing at-the-money options of 20% of the underling equity's value, we find that by posting the same margin, traders can write $\varphi = 2.5$ options for each share they can short. Finally, we use a short-sale cost of $\rho = 1\%$.

Equilibrium bid-ask spreads for a numeric example			
	Stock	Call	Put
Ask price	30.04	2.19	2.19
Mean asset value	30	1.20	1.20
Bid price	29.97	0.78	0.83

Informed Traders' Equilibrium Strategy		
Trade	Range of signals	Fraction of signals
Buy $q_{bp} = 45.7$ put options	$\epsilon \leq -2.19$	16.4 %
Sell $q_{ss} = 3.3$ shares of stock	$-2.19 < \epsilon \leq -1.74$	5.4%
Sell $q_{sc} = 8.3$ call options	$-1.74 < \epsilon \leq -0.05$	27.2%
Sell $q_{sp} = 8.3$ put options	$-0.05 < \epsilon \leq 1.42$	24.7%
Buy $q_{bs} = 3.3$ shares of stock	$1.42 < \epsilon \leq 2.21$	10.2%
Buy $q_{bc} = 45.7$ call options	$2.21 < \epsilon$	16.1 %

We solve for the equilibrium prices and cutoff points numerically given the above parameters. The exact simultaneous equations used to find this solution are in Appendix B. The process converges quickly and it does not appear that there are multiple solutions. The equilibrium bid and ask prices, presented above, show that spreads are bigger in the options markets than the stock market (1.41 and 1.36 vs. 0.07). The above table also summarizes informed traders' equilibrium strategy, in which they use options for 84% of signals despite the enormous bid-ask spread in those markets. The primary reason is the leverage options afford: given their budget constraint $\kappa = 100$, traders can buy 45.7 options contracts but only 3.3 shares of stock, and due to the margin restriction $\varphi = 2.5$ they can write 8.3 options contracts but short only 3.3 shares of stock. Note that equity market bid-ask spreads are not symmetric around the mean because stock buys are more likely to come from informed traders than stock sells. Option spreads are also asymmetric around their mean values because informed traders buy options for extreme signals and sell them for moderate ones.

Informed traders use stocks less frequently, and options more frequently, for bad signals than they do for good signals because of the short-sale cost ρ. In this example, informed traders buy stock for 10.2% of signals (or 20.4% of positive signals) but only short stock for 5.4% of signals (10.8% of negative signals). Since they use options for the remainder of signals, there is a spread in conditional expectations as discussed in Result 1, namely that conditional on a stock trade the mean of $\tilde{V}$ is 30.01, while conditional on an option trade it is 29.95. The asymmetry in informed traders' behavior leads to a larger transaction cost for buying stock (0.04), than for selling (0.03).[18] Short-sale costs result in informed traders writing calls and buying puts for part of the range of signals that they would otherwise short stock. Hence, there is a higher concentration of informed trade and higher transaction costs in put buys than call buys, and similarly a higher concentration in call sells than put sells.

We confirm Result 2 in this example by increasing ρ to 2%. In untabulated results, we find that increasing ρ also increases the scaled difference in conditional means from 0.18% to 0.36%. We illustrate Result 3 by varying the mean equity value μ while keeping the other exogenous parameters fixed. The leverage in options increases with μ, which reduces the scaled difference in conditional means for stock and option trades from 0.20% for $\mu = 20$ back to 0.18% for $\mu = 30$. Finally, we confirm Result 4 in this setting; because informed traders write options for moderate signals and buy options for extreme signals, the skewness conditional on a call trade is higher (0.15) than the skewness conditional on a put trade (-0.10).

[18] Transaction costs are defined here as the difference between the quoted price and the mean asset value. For example, the transaction cost for buying equity is the ask prices less the mean stock value, or $30.04 - 30 = 0.04$.

Appendix B: Simultaneous Equations

The full set of simultaneous equations that characterize the equilibrium are:

$$q_{bs}a_s = \kappa \tag{B.1}$$
$$q_{ss} = q_{bs} \tag{B.2}$$
$$q_{bc}a_c = \kappa \tag{B.3}$$
$$q_{sc} = \varphi q_{ss} \tag{B.4}$$
$$q_{bp}a_p = \kappa \tag{B.5}$$
$$q_{sc} = \varphi q_{ss} \tag{B.6}$$

$$a_s = \mu + \frac{\theta(\phi(\frac{k_5}{\sigma_\epsilon}) - \phi(\frac{k_6}{\sigma_\epsilon}))}{\theta(\Phi(\frac{k_5}{\sigma_\epsilon}) - \Phi(\frac{k_6}{\sigma_\epsilon})) + (1-\theta)\gamma_1}\sigma_\epsilon \tag{B.7}$$
$$b_s = \mu - \frac{\theta(\phi(\frac{k_2}{\sigma_\epsilon}) - \phi(\frac{k_1}{\sigma_\epsilon}))}{\theta(\Phi(\frac{k_2}{\sigma_\epsilon}) - \Phi(\frac{k_1}{\sigma_\epsilon})) + (1-\theta)\gamma_2}\sigma_\epsilon \tag{B.8}$$
$$a_c = \frac{\theta\int_{k_6}^{\infty}\phi(\epsilon)C(\epsilon,\sigma_\eta)d\epsilon + (1-\theta)\gamma_3\phi(0)\sqrt{\sigma_\epsilon^2+\sigma_\eta^2}}{\theta(1-\Phi(\frac{k_6}{\sigma_\epsilon})) + (1-\theta)\gamma_3} \tag{B.9}$$
$$b_c = \frac{\theta\int_{k_2}^{k_3}\phi(\epsilon)C(\epsilon,\sigma_\eta)d\epsilon + (1-\theta)\gamma_4\phi(0)\sqrt{\sigma_\epsilon^2+\sigma_\eta^2}}{\theta(\Phi(\frac{k_3}{\sigma_\epsilon}) - \Phi(\frac{k_2}{\sigma_\epsilon})) + (1-\theta)\gamma_4} \tag{B.10}$$
$$a_p = \frac{\theta\int_{-\infty}^{k_1}\phi(\epsilon)P(\epsilon,\sigma_\eta)d\epsilon + (1-\theta)\gamma_5\phi(0)\sqrt{\sigma_\epsilon^2+\sigma_\eta^2}}{\theta(\Phi(\frac{k_1}{\sigma_\epsilon})) + (1-\theta)\gamma_5} \tag{B.11}$$
$$b_p = \frac{\theta\int_{k_4}^{k_5}\phi(\epsilon)P(\epsilon,\sigma_\eta)d\epsilon + (1-\theta)\gamma_6\phi(0)\sqrt{\sigma_\epsilon^2+\sigma_\eta^2}}{\theta(\Phi(\frac{k_5}{\sigma_\epsilon}) - \Phi(\frac{k_4}{\sigma_\epsilon})) + (1-\theta)\gamma_6} \tag{B.12}$$

$$q_{bp}(P(k_1,\sigma_\eta) - a_p) = q_{ss}(b_s(1-\rho) - \mu - k_1)) \tag{B.13}$$
$$q_{ss}(b_s(1-\rho) - \mu - k_2) = q_{sc}(b_c - C(k_2,\sigma_\eta)) \tag{B.14}$$
$$q_{sc}(b_c - C(k_3,\sigma_\eta)) = 0 \tag{B.15}$$
$$0 = q_{sp}(b_p - P(k_4,\sigma_\eta)) \tag{B.16}$$
$$q_{sp}(b_p - P(k_5,\sigma_\eta)) = q_{bs}(a_s - \mu - k_5) \tag{B.17}$$
$$q_{bs}(a_s - \mu - k_6) = q_{bc}(a_c - C(k_6,\sigma_\eta)) \tag{B.18}$$

Equations (B.1) – (B.6) ensure that the trade quantities satisfy the budget constraint. For example, (B.1) ensures that the amount of a stock transaction $a_s q_{bs}$ equals the budget κ. Equations (B.7) – (B.12) are the zero profit conditions for the market-maker. Equation (B.7), for example, ensures that the ask price for a stock is exactly the expectation of $\tilde{V}$ conditional on a stock trade. Computing this conditional expectation in the case of options trades (equations (B.9) – (B.12)) requires integrating the value function for options C and P.

These functions are the mean value of a call and put, respectively, conditional on the signal ϵ and the standard deviation of η. Specifically,

$$C(\epsilon, \sigma_\eta) \equiv E(\tilde{C}|\tilde{\epsilon} = \epsilon) = \Phi\left(\frac{\epsilon}{\sigma_\eta}\right)\epsilon + \phi\left(\frac{\epsilon}{\sigma_\eta}\right)\sigma_\eta \tag{B.19}$$

$$P(\epsilon, \sigma_\eta) \equiv E(\tilde{P}|\tilde{\epsilon} = \epsilon) = -\Phi\left(\frac{-\epsilon}{\sigma_\eta}\right)\epsilon + \phi\left(\frac{\epsilon}{\sigma_\eta}\right)\sigma_\eta \tag{B.20}$$

When solving these equations numerically, we estimate the integrals using a Reimann sum. Finally, equations (B.13) through (B.18) ensure informed traders are indifferent between the two neighboring portfolios at the cutoff points. For example, (B.13) ensures they are indifferent between buying puts and shorting stock given the signal k_1.

These equations cannot be solved in closed form due to the nonlinearity of C and P, however we can prove some general results directly from the simultaneous equations without needing a closed form solution. Throughout, we assume the exogenous parameters are chosen so that there exists a set of equilibrium parameters satisfying (B.1) through (B.18) as well as $k_1 < k_2 < k_3 \leq k_4 < k_5 < k_6$. For some parameters no such equilibrium exists, typically because the informed trader never finds it optimal to trade stock so $k_2 = k_3$; we focus on the case when informed traders use stock because our goal is to model the impact of short sale costs, which are only relevant when informed traders use equity. We do consider parametrizations where the informed trader chooses to trade every signal, meaning $k_3 = k_4$. In this case, we have one fewer free parameter and we need to replace equations (B.15) and (B.16) with a the single equation:

$$q_{sc}(b_c - C(k_3, \sigma_\eta)) = q_{sp}(b_p - P(k_3, \sigma_\eta)) \tag{B.21}$$

Appendix C: Measure of Leverage

Leverage λ in options markets is measured by the elasticity of the option pricing function $C(S)$ with respect to S. For options priced according to Black-Scholes, we have:

$$\lambda = \Phi(\frac{\log(\frac{S}{K}) + (r + \frac{\sigma^2}{2}T}{\sigma\sqrt{T}})\frac{S}{C}$$

In our model, we compute:

$$\begin{aligned}\lambda = \frac{\partial C}{\partial S}\frac{S}{C} &= \frac{\frac{\partial C(\epsilon)}{\partial \epsilon}}{\frac{\partial V}{\partial \epsilon}}\frac{E(\tilde{V})}{E(\tilde{C})} \\ &= \frac{\phi(\frac{\epsilon}{\sigma_\eta})\frac{\epsilon}{\sigma_\eta} + \Phi(\frac{\epsilon}{\sigma_\eta}) + 2\sigma_\eta\phi(\frac{\epsilon}{\sigma_\eta})}{1}\frac{\mu}{\phi(0)\sqrt{\sigma_\eta^2 + \sigma_\epsilon^2}}\end{aligned}$$

Since the options in our model are struck at μ, we measure λ when $\epsilon = 0$, giving us:

$$\lambda_{JS} = \frac{\mu}{2\phi(0)\sqrt{\sigma_\epsilon^2 + \sigma_\eta^2}} \tag{C.1}$$

For comparison's sake, the λ of an at-the-money call option in Black-Scholes with no dividends, $r = 0$, volatility σ and time-to-expiration T is equal to:

$$\lambda_{BS} = \Phi(\frac{1}{2}\sigma T)\frac{S}{S(\Phi(\frac{1}{2}\sigma T) - \Phi(-\frac{1}{2}\sigma T)} = \frac{\Phi(\frac{1}{2}\sigma T)}{\Phi(\frac{1}{2}\sigma T) - \Phi(-\frac{1}{2}\sigma T)} \tag{C.2}$$

Since Black-Scholes σT is the standard deviation of returns, the equivalent in our model is $\sqrt{\sigma_\epsilon^2 + \sigma_\eta^2}$. Rewriting equation C.1 as a function of the volatility of returns σT, we get:

$$\lambda_{JS} = \frac{1}{2\phi(0)\sigma T} \tag{C.3}$$

Written this way, it is clear that both λ_{JS} and λ_{BS} are decreasing functions σT and no other variable. The two are very close for small σT and farther apart for larger σT because the normal distibution use in our model is closer to the log-normal distribution used in Black-Scholes when volatility is small.

Appendix D: Proofs

Result 1. *When uninformed demand satisfies $\gamma_1 = \gamma_2$ and $\gamma_3 = \gamma_4 = \gamma_5 = \gamma_6$, in equilibrium $E(\tilde{V}|option\ trade) \leq E(\tilde{V}|equity\ trade)$. We obtain a strict inequality when $\rho > 0$.*

Proof. Denote k_1^* as the equilibrium cutoff points and a_i and b_i as the prices in the case that $\rho = 0$. Here, the problem is completely symmetric and we have that $k_1 = -k_6$, and $k_2 = -k_5$. Therefore, we get $E(\epsilon|\epsilon \in (k_5, k_6)) = -E(\epsilon|\epsilon \in (k_1, k_2))$ which yields $E(\tilde{V}|\text{option trade}) = E(\tilde{V}|\text{equity trade})$. When $\rho > 0$, we show that $k_1 > -k_6$, and $k_2 < -k_5$. Assuming informed traders can trade at the quantities and prices available when $\rho = 0$, but that they consider short sale cost $\rho > 0$ when choosing their cutoff points, the informed trader prefers buying puts to shorting stock at k_1^* since

$$q_{bp}^*(p(k_1^*, \sigma_n) = a_p^*) = q_{ss}^*(b_s^* - \mu - k_1^*) < q_s^* s(b_s^*(1-\rho) - \mu - k_1^*) \tag{D.1}$$

Similarly, informed traders prefers writing calls to shorting stock at k_2^* because

$$q_{sc}^*(b_c^* - C(k_2^*, \sigma_n) = a_p^*) = q_{ss}^*(b_s^* - \mu - k_1^*) > q_s^* s(b_s^*(1-\rho) - \mu - k_2^*) \tag{D.2}$$

However, because short sale costs do not appear in equations (B.15) – (B.18), they remain satisfied by $k_3^* \ldots k_6^*$. The best response to $\rho = 0$ prices is therefore $k_1 > k_1^*$, $k_2 < k_2^*$, $k_i = k_i^* \,\forall\, i \geq 3$.

If the informed trader uses the cutoff strategy k_i, the market-maker needs to adjust its bid-ask prices in order to satisfy the zero profit condition. In particular, the informed trader less frequently shorts stock and more frequently buys puts and writes calls, meaning the market maker should increase b_s, increase a_p, and decrease b_c. Continuing this sequential thinking, the informed trader would respond to the price changes by decreasing k_1 and increasing k_2, partially reversing their initial change from the $\rho = 0$ case. The process continues ad infinitum, market-makers adjusting spreads and informed traders adjusting their cutoff points, until equilibrium is reached. But because there are uninformed traders, each adjustment in this process is smaller than the previous, and therefore the dominant effect is the first response of the informed traders, namely a reduced range of signals for which informed traders short stock.

Informed traders' new equilibrium strategy k_i^ρ therefore satisfies $-k_2^\rho > k_4^\rho$ and $-k_1^\rho > k_5^\rho$, which implies that $E(\epsilon|\epsilon \in (k_4, k_5)) > -E(\epsilon|\epsilon \in (k_1, k_2))$, yielding the desired result that $E(\tilde{V}|\text{option trade}) \leq E(\tilde{V}|\text{equity trade})$. □

Result 2. *Given the same assumptions as Result 1, the scaled difference in conditional means $D \equiv \frac{E(\tilde{V}|\text{stock trade})-E(\tilde{V}|\text{option trade})}{\mu}$ is weakly increasing in the short sale cost ρ.*

Proof. Since the short sale cost ρ has no impact on μ, an equivalent statement is that

$$d \equiv E(\tilde{V}|\text{stock trade}) - E(\tilde{V}|\text{option trade})$$

is weakly increasing in ρ. Say we have $0 < \rho_1 < \rho_2$, and that $k_i^{(1)}$ are the equilibrium cutoff points in the informed traders strategy when the short sale cost is ρ_1 while $k_i^{(2)}$ are the cutoffs when short sale costs are ρ_2. As long as informed traders still short equity for some non-empty range of signals (k_2^*, k_3^*), by the same line of reasoning used in the proof of Result 1 we have that when $\rho = \rho_2$, the range of signals for which the informed trader shorts equity shrinks and d strictly increases. We state that d is only weakly increasing in ρ because when ρ_1 is sufficiently large that the informed trader never shorts equity, increasing short sale costs to ρ_2 has no impact on the informed trader's strategy and therefore no impact d. □

Result 3. *Holding the signal strength $r = \frac{\sigma_\epsilon}{\sigma_\eta}$ constant, the scaled difference in conditional means $D \equiv \frac{E(\tilde{V}|\text{option trade}) \leq E(\tilde{V}|\text{equity trade})}{\mu}$ is decreasing in the leverage in options as measured by $\lambda = \frac{\mu}{2\sqrt{\sigma_\epsilon^2+\sigma_\eta^2}}$.*

Proof. First we show that D is invariant to changes in μ as long as σ_ϵ^2 and σ_η^2 change proportionally so that λ remains the same.

Suppose that parameters k_i, a_i, and b_i constitute an equilibrium for the case $\mu = \mu_0$, $\sigma_\epsilon = \sigma r$, and $\sigma_\eta = \sigma$, where r is the signal strength ratio. We show that $k_i' = \omega k_i$, $a_i' = \omega a_i$, and $b_i' = \omega b_i$ are an equilibrium for the case $\mu' = \mu_0$, $\sigma_\epsilon' = \omega\sigma r$, and $\sigma_\eta' = \sigma\omega$, holding all other parameters fixed. We call the first the "unprimed" case and the second the "primed" case. In order to verify we have an equilibrium, we need to assure all 18 simultaneous equations are satisfied.

The first six equations are met provided that we set $q_b's = \frac{\kappa}{a_s'} = \frac{\kappa}{\omega a_s} = \frac{q_b s}{\omega}$, and similarly $q_{ss}' = \frac{q_{ss}}{\omega}$, $q_b'c = \frac{q_{bc}}{\omega}$, $q_s'c = \frac{q_{sc}}{\omega}$, $q_b'p = \frac{q_{bp}}{\omega}$, and $q_s'p = \frac{q_{sp}}{\omega}$. Noting that $C(\omega\epsilon, \omega\sigma) = \Phi\frac{\omega\epsilon}{\omega\sigma}\omega\sigma + \phi\frac{\omega\epsilon}{\omega\sigma}\omega\sigma = \omega C(\epsilon, \sigma)$, and similarly $P(\omega\epsilon, \omega\sigma) = \omega P(\epsilon, \sigma)$, the market-maker's zero profit conditions (B.7) through (B.12) are trivially satisfied since the cutoff points are always scaled by $\sigma_\epsilon = \sigma r$. The informed traders' indifference equations (B.13) through (B.18) are satisfied as well, for example we calculate for k_1':

$$
\begin{aligned}
q_b'p(P(k_1',\sigma_\eta')-a_p') =& \frac{q_{bp}}{\omega}/(P(\omega k_1,\omega\sigma_\eta)-\omega a_p) = q_{bp}(P(k_1,\sigma_\eta)-a_p)\\
=& q_s s(b_s(1-\rho)-\mu-k_1) = \frac{q_{ss}}{\omega}(\omega b_s(1-\rho)-\omega\mu-\omega k_1)\\
=& q_s' s(b_s'(1-\rho)-\mu'-k_1')
\end{aligned}
\tag{D.3}
$$

Now consider the expectation of $\tilde{V}$ conditional on a specific trade. Denoting expectations and probabilities using the primed parameters E' and P' respectively, we have that:

$$
\begin{aligned}
E'(\tilde{V}|\text{trade i}) =& \mu' + P'(\text{informed}|\text{trade i})E'(\tilde{\epsilon}|k_{i-1}'<\tilde{\epsilon}<k_i')\\
=& \omega\mu + P(\text{informed}|\text{trade i})E'(\tilde{\epsilon}|\omega k_{i-1}<\tilde{\epsilon}<\omega k_i)\\
=& \omega\mu + P(\text{informed}|\text{trade i})E(\omega\tilde{\epsilon}|k_{i-1}<\tilde{\epsilon}<k_i)\\
=& \omega E(\tilde{V}|\text{trade i})
\end{aligned}
\tag{D.4}
$$

Therefore, we have that D is the same in the primed and unprimed case:

$$
\begin{aligned}
D' =& \frac{(E'(\tilde{V}|\text{option trade}) - E'(\tilde{V}|\text{stock trade}))}{\mu'}\\
=& \frac{\omega E(\tilde{V}|\text{option trade}) - \omega E(\tilde{V}|\text{stock trade})}{\omega\mu}\\
=& D
\end{aligned}
\tag{D.5}
$$

We can therefore focus on the change in D as σ changes, since a change from μ to $c\mu$ is equivalent to a change from σ to $\frac{\sigma}{c}$ by the above. We show that as σ increases (and λ decreases), D increases. To achieve this, we show that for when $\rho > 0$, the following inequality holds for all exogenous parameters permitting an equilibrium:

$$
\frac{\partial}{\partial\sigma}\left(\frac{k_5(\sigma)}{\sigma} - \frac{k_4(\sigma)}{\sigma}\right) > \frac{\partial}{\partial\sigma}\left(\frac{k_2(\sigma)}{\sigma} - \frac{k_1(\sigma)}{\sigma}\right)
\tag{D.6}
$$

where $k_i(\sigma)$ is the equilibrium cutoff point k_i when the volatility parameter is σ. This inequality implies that, holding other parameters fixed, the range of signals (scaled to the standard normal) for which informed traders buy stock increases with σ faster than the range for which they sell stock. Because D is increasing in the difference between these two signal ranges, we can conclude from (a) that D itself increases in σ.

All that remains is to show (a). To simplify notation, we call the normalized cutoff point $j_i = \frac{k_i}{\sigma_\epsilon}$ and continue to use $\sigma = \sigma_\eta$ and $\sigma r = \sigma_\epsilon$. Consider a set of exogenous parameters and corresponding equilibrium. Together they must satisfy equation (13) above, and therefore:

$$\begin{aligned} q_{ss}(b_s - \mu - j_1\sigma r) =& q_{bp}(P(j_1\sigma r, \sigma) - a_p) \\ \Rightarrow j_1\sigma r =& b_s - \mu - \frac{q_{bp}}{q_{ss}}(P(j_1\sigma r, \sigma) - a_p) \end{aligned} \tag{D.7}$$

Following the reasoning in the proof of Result 1, we must consider which of the terms in the above change with σ even if the other parameters do not change. The informed trader will want to adjust their cutoff points in response to a new σ because the leverage in options has changed, so the cutoff points j_i experience a first order impact. Changes in σ also impact the mean value of the option, so the quote prices for options markets must change even if the informed traders do not change their strategy. The zero-profit bid and ask prices are $a_i = \sigma\underline{\text{a}}_i$ and $b_i = \sigma\underline{\text{b}}_i$, where $\underline{\text{a}}_i$ and $\underline{\text{b}}_i$ do not vary with σ. The value functions for options, $P(j_1\sigma r, \sigma)$ and $C(j_1\sigma r, \sigma)$ are also directly proportional to σ and therefore are directly impacted by changed in σ. The buy quantity for puts q_{bp} is set according to $q_{bp} = \frac{\kappa}{a_p} = \frac{\kappa}{\sigma\underline{\text{a}}_p}$. The stock prices and quantities, b_s and q_{ss} do not change directly with σ because the mean value of the asset does not depend on σ.

Denoting those terms that change directly with σ as functions of σ, we have:

$$\begin{aligned} j_1(\sigma)\sigma r =& \frac{b_s - \mu - (q_{bp}(\sigma))}{q_{ss}(P(j_1(\sigma)\sigma r, \sigma) - a_p(\sigma))} \\ \Rightarrow \frac{\partial}{\partial\sigma}(j_1(\sigma)\sigma r) =& -\frac{\partial}{\partial\sigma}\frac{q_{bp}(\sigma)\sigma}{q_{ss}}(\frac{P(j_1(\sigma)\sigma r, \sigma)}{\sigma})) - \underline{\text{a}}_p) \end{aligned} \tag{D.8}$$

We compute that $\frac{\partial}{\partial\sigma}\frac{q_{bp}(\sigma)\sigma}{q_{ss}} = 0$, $\frac{P(j_1(\sigma)\sigma r,\sigma)}{\sigma} = \Phi(-\epsilon r)j_i + \phi(-\epsilon)$, and $\frac{\partial}{\partial j_1}P(j_1\sigma r, \sigma) = \Phi(-\epsilon r)\sigma r$. Therefore, we get:

$$\begin{aligned} \frac{\partial}{\partial\sigma}(j_1(\sigma)\sigma r) =& \frac{q_{bp}(\sigma)\sigma}{q_{ss}}\Phi(-j_1 r)\sigma r\frac{\partial j_1}{\partial\sigma} \\ \Rightarrow \sigma r\frac{\partial j_1}{\partial\sigma} + j_1 r =& \frac{q_{bp}(\sigma)\sigma}{q_{ss}}\Phi(-j_1 r)\sigma r\frac{\partial j_1}{\partial\sigma} \\ \Rightarrow \frac{\partial j_1}{\partial\sigma} =& \frac{-j_1}{\sigma(1 - \Phi(-j_1 r)\frac{q_{bp}}{q_{ss}})} \end{aligned} \tag{D.9}$$

Similar calculations yield:

$$\frac{\partial j_2}{\partial\sigma} = \frac{j_2 r + \psi\sigma(b_c - C(j_2, \sigma))}{\sigma r(\psi\Phi(-j_2 r))} \tag{D.10}$$

$$\frac{\partial j_5}{\partial\sigma} = \frac{-\psi\Phi(b_p - P(j_5, \sigma)) - k_5 r}{\sigma r(1 - \psi\Phi(-j_5 r))} \tag{D.11}$$

$$\frac{\partial j_6}{\partial\sigma} = \frac{j_6}{\sigma}(\Phi(j_6 r)\frac{q_{bc}}{q_{ss}} - 1) \tag{D.12}$$

Noting that $\frac{\partial j_2}{\partial\sigma} > 0$, that $\frac{\partial j_2}{\partial\sigma}$ increases with j_2, and that $\frac{\partial j_5}{\partial\sigma} = -(\frac{\partial j_2}{\partial\sigma})_{j_2=-j_4}$, we have that

$\frac{\partial j_5}{\sigma} > \frac{\partial j_2}{\sigma}$.

Similarly, $\frac{\partial j_6}{\partial \sigma} > 0$, $\frac{\partial j_6}{\partial \sigma}$ increases with j_6, and $\frac{\partial j_6}{\partial \sigma} = -(\frac{\partial j_1}{\partial \sigma})_{j_1=-j_6}$ so we get that $\frac{\partial j_6}{\sigma} > \frac{\partial j_1}{\sigma}$. Adding these together, we get exactly inequality (a), which completes the proof. □

Result 4. *Equity value has a higher skewness conditional on a call trade than conditional on a put trade when* $\gamma_i > \frac{\theta}{(1-\theta)139.2}$.

Proof. We show that the third centralized moments conditional on call and put trades satisfy:

$$E((\tilde{V} - \hat{V}_{\text{call}})^3|\text{call trade}) > 0 > E((\tilde{V} - \hat{V}_{\text{put}})^3|\text{put trade}), \tag{D.13}$$

where $\hat{V}_i$ is the expected value of $\tilde{V}$ conditional on trade type i. Inequality D.13 implies Result 4 because skewness is the third centralized moment scaled by a positive number.

We show here that $E((\tilde{V} - \hat{V}_{\text{call}})^3|\text{call trade}) > 0$. The other half of inequality D.13 follows from same the derivation applied to the put option.

To simplify notation, we write $E^C(\cdot)$ as short hand for $E(\cdot|\text{call trade})$, and cm_3^C for the third centralized moment conditional on call trade.

$$\begin{aligned} cm_3^C &= E^C((\tilde{V} - \hat{V}_{\text{call}})^3) \\ &= E^C((\tilde{\epsilon} - E^C(\tilde{\epsilon}) + \tilde{\eta})^3) \end{aligned}$$

Since $\tilde{\epsilon} - E^C(\tilde{\epsilon})$ and $\tilde{\eta}$ are independent and both have zero mean conditional on a call trade, we have:

$$\begin{aligned} cm_3^C &= E^C((\tilde{\epsilon} - E^C(\tilde{\epsilon}))^3) \\ \Rightarrow cm_3^C &\propto E^C((\tilde{\delta} - E^C(\tilde{\delta}))^3) \end{aligned} \tag{D.14}$$

where $\tilde{\delta} = \frac{\tilde{\epsilon}}{\sigma_\epsilon}$ and $\propto$ indicates that the two expressions have the same sign.

Next we break up the expectation in D.14 into two exhaustive cases: the trade was initiated by an informed trader and the trade was initiated by an uninformed trader. In each case, we expand $(\tilde{\delta} - E^C(\tilde{\delta}))^3$, and in order to keep the expression as brief as possible we write:

$$\begin{aligned} m_i^I &\equiv E(\tilde{\epsilon}^i|\text{informed call trade}) \\ m_i^U &\equiv E(\tilde{\epsilon}^i|\text{uninformed call trade}) \\ \hat{\delta} &\equiv E^C(\tilde{\delta}) \\ \theta_C &\equiv P(\text{informed}|\text{call trade}) \end{aligned}$$

After breaking up and expanding the expectation, we find:

$$\begin{aligned} cm_3^C &\propto \theta_C(m_3^I - 3m_2^I\hat{\delta} + 3m_1^I\hat{\delta}^2 - \hat{\delta}^3) + (1-\theta_C)(m_3^U - 3m_2^U\hat{\delta} + 3m_1^U\hat{\delta}^2 - \hat{\delta}^3) \\ &= \theta_C(m_3^I - 3m_2^I\hat{\delta} + 3m_1^I\hat{\delta}^2 - \hat{\delta}^3) + (1-\theta_C)(-3\hat{\delta} - \hat{\delta}^3) \\ &= m_3^I\theta_C + 2\hat{\delta}^3 - 3\hat{\delta}(1 + \theta_C(m_2^I - 1)) \end{aligned} \tag{D.15}$$

To arrive at equation D.15 we use the fact that $\hat{\delta} = \theta_C m_1^I + (1-\theta_C) m_1^U = \theta_C m_1^I$.

From here, we prove three lemmas that together complete the proof under the following condition:

$$\gamma_i > \frac{\theta}{(1-\theta)139.2} \tag{D.16}$$

This condition ensures that the number of uninformed traders in options markets does not approach zero, in which case markets begin to fail and the skewness result can reverse. It is a condition easily satisfied for any normal parametrizations. If $\theta > \frac{1}{10}$ we only require $\gamma_i > \frac{1}{1250}$ and if $\gamma_i > \frac{1}{84}$ we only require $\theta < 63\%$.

Lemma 1 shows that $m_3^I > 0$ when (D.16) holds. Lemma 2 shows that $2\hat{\delta}^3 - 3\hat{\delta}(1 + \theta_C(m_2^I - 1)) > 0$ when $\delta < 0$. Lemma 3 shows that $m_3^I \theta_C > -2\hat{\delta}^3 + 3\hat{\delta}(1+\theta_C(m_2^I - 1))$ when $\delta > 0$ and (D.16) holds. Put together with (D.15), these lemmas complete the proof. □

Lemma 1. *The third moment of $\tilde{\delta}$ conditional on an informed call trade, m_3^I, is positive whenever $\gamma_i > \frac{\theta}{(1-\theta)139.2}$.*

Proof. The lemma follows from informed trader's equilibrium cutoff strategy, which assures that a call trade is either weakly bad news or extremely good news. We only need to rule out the possibility that uninformed traders are so scarce the informed trader almost never buys calls, which would make the the distribution of $\tilde{\delta}$ conditional on an informed trade similar to the distribution of $\tilde{\delta}$ conditional on a call *sell*, which has a negative third moment.

From the moments of the truncated normal distribution given in Jawitz (2004), we have:

$$m_3^I = \frac{(j_2^2+2)\phi(j_2) - (j_3^2+2)\phi(j_3) + (j_6^2+2)\phi(j_6)}{\Phi(j_3) - \Phi(j_2) + 1 - \Phi(j_6)} \tag{D.17}$$

where j_i are the equilibrium cutoff points scaled down by σ_ϵ so they are $\tilde{\delta}$ cutoffs rather than $\tilde{\epsilon}$ cutoffs. The function $f(x) = (x^2+2)\phi(x)$ is positive, symmetric about $x = 0$, decreasing for $x > 0$, increasing for $x < 0$, and satisfies $f(-\bar{j}) + f(\bar{j}) = f(0)$ for $\bar{j} = 1.832$. In equilibrium, we know that $j_2 \le j_3 \le 0 \le j_6$ and $|j_3| < |j_2| < |j_6|$, so D.17 tells us that $m_3^I > 0$ whenever $j_6 < \bar{j}$.

Next we show that $j_6 < \bar{j}$ whenever (D.16) holds. Assume the contrary, that $j_6 > \bar{j}$. We consider only equilibria where the informed trader buys equity for some signals, so we know that at $\tilde{\epsilon} = \bar{j}\sigma_\epsilon$ the informed trader prefers equity to calls. Writing $C(x)$ for $E(\tilde{C}|\tilde{\epsilon} = x)$, We have that:

$$\begin{aligned} q_{bs}(\mu + \sigma_\epsilon \bar{j} - a_s) &> q_{bc}(C(\bar{j}\sigma_\epsilon) - a_c) \\ \Leftrightarrow q_{bs}(\mu + \sigma_\epsilon \bar{j}) &> q_{bc}(C(\bar{j}\sigma_\epsilon)) \\ \Leftrightarrow \frac{q_{bs}}{q_{bc}} &> \frac{\Phi(\frac{\bar{j}\sigma_\epsilon}{\sigma_\eta})\bar{j}\sigma_\epsilon + \phi(\frac{\bar{j}\sigma_\epsilon}{\sigma_\eta})\sigma_\eta}{\mu + \sigma_\epsilon \bar{j}} \end{aligned} \tag{D.18}$$

The right hand side of (D.18) is increasing in σ_η, so if (D.18) holds when $\sigma_\eta = 0$ it holds for all σ_η.

When $\sigma_\eta = 0$ we can solve for the equilibrium k_6 directly from the simultaneous equations in Appendix B. In particular, we find that:

$$k_6 = \frac{a_c}{a_s + a_c}\mu$$

So if $k_6 > \bar{j}\sigma_\epsilon$ we have:

$$\begin{aligned}
&\frac{a_c}{a_s + a_c}\mu > \bar{j}\sigma_\epsilon \\
&\Leftrightarrow \frac{a_c}{\mu + a_c}\mu > \bar{j}\sigma_\epsilon \\
&\Rightarrow a_c > \bar{j}\sigma_\epsilon \\
&\Rightarrow \frac{\theta\phi(\bar{j})}{\theta(1-\Phi(\bar{j})) + (1-\theta)\gamma_6}\sigma_\epsilon > \bar{j}\sigma_\epsilon
\end{aligned} \tag{D.19}$$

Solving D.19 for γ_6, we find exactly the opposite of the condition (D.16), so we know that (D.16) implies $k_6 < \bar{j}\sigma_\epsilon$ and $m_3^I > 0$. □

Lemma 2. *When $\delta < 0$, we have that $2\hat{\delta}^3 - 3\hat{\delta}(1 + \theta_C(m_2^I - 1)) > 0$.*

Proof. This lemma holds because the quantity in question measures the difference between non-centralized moments and centralized moments due to the change in mean. The lemma shows that when the mean of a variable is negative, the centralized third moment is greater than the un-centralized third moment. To see this technically, first note that:

$$\begin{aligned}
\mathrm{var}(\tilde{\delta}|\text{call trade}) &= E^C(\tilde{\delta}^2) - \hat{\delta}^2 \\
&= \theta_C m_2^I + (1-\theta_C) - \hat{\delta}^2 \\
&= 1 + \theta_C(m_2^I - 1) - \hat{\delta}^2
\end{aligned}$$

And since variances are positive, we have:

$$\begin{aligned}
&1 + \theta_C(m_2^I - 1) - \hat{\delta}^2 > 0 \\
&\Rightarrow \hat{\delta}(1 + \theta_C(m_2^I - 1)) < \hat{\delta}^3 \\
&\Rightarrow 2\hat{\delta}^3 - 3\hat{\delta}(1 + \theta_C(m_2^I - 1)) > 0
\end{aligned}$$

□

Lemma 3. *When $\delta > 0$ and (D.16) holds, we have that $m_3^I\theta_C > -2\hat{\delta}^3 + 3\hat{\delta}(1+\theta_C(m_2^I - 1))$.*

Proof. The intuition for Lemma 3 is that when $\hat{\delta} > 0$ the centralized third moment is less than the un-centralized third moment, but the positive mean makes the third moment so large it is positive even after centralization. More rigorously, we have:

$$\begin{aligned}
&m_3^I\theta_C + 2\hat{\delta}^3 - 3\hat{\delta}(1 + \theta_C(m_2^I - 1)) \\
&\propto m_3^I + 2(m_1^I)^3(\theta_C)^2 - 3m_1^I(1 + \theta_C(m_2^I - 1)) \\
&> m_3^I - 3m_1^I(1 + \theta_C(m_2^I - 1))
\end{aligned} \tag{D.20}$$

From Jawitz (2004) we have:

$$m_3^I = \frac{(j_2^2+2)\phi(j_2) - (j_3^2+2)\phi(j_3) + (j_6^2+2)\phi(j_6)}{\Phi(j_3) - \Phi(j_2) + 1 - \Phi(j_6)}$$
$$m_2^I = \frac{(j_2)\phi(j_2) - (j_3)\phi(j_3) + (j_6)\phi(j_6)}{\Phi(j_3) - \Phi(j_2) + 1 - \Phi(j_6)}$$
$$m_1^I = \frac{\phi(j_2) - \phi(j_3) + \phi(j_6)}{\Phi(j_3) - \Phi(j_2) + 1 - \Phi(j_6)}$$

Noting that any equilibrium satisfying (D.16) and $\hat{\delta} > 0$ in which the informed trader uses each asset with positive probability satisfies:

1. $-\bar{j} < j_2 < j_3 < 0 < j_6 < \bar{j}$.
2. $|j_3| < |j_2| < |j_6|$.
3. $\phi(j_2) - \phi(j_3) + \phi(j_6) > 0$.

We can substitute these conditions into D.20 and find that $m_3^I - 3m_1^I(1 + \theta_C(m_2^I - 1)) > 0$, which in turn implies Lemma 3. □

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Figure 1: Daily O/S Ratios and Future Returns

Figure 1 presents equal-weighted future daily O/S strategy returns corresponding to a long position in the lowest $\mathrm{O/S}_{i,d}$ decile firms and a short position in the highest $\mathrm{O/S}_{i,d}$ decile firms, where $\mathrm{O/S}_{i,d}$ equals the ratio of option volume to equity volume of firm i on day d. We use two return metrics: four-factor-adjusted (shown in grey) and raw returns (shown in black). To adjust for the four risk factors, we estimate historical firm-specific factor loadings over the prior year using daily returns, and apply the estimated coefficients to contemporaneous daily factors. The sample consists of 2,969,570 firm-days spanning 1996 through 2008.

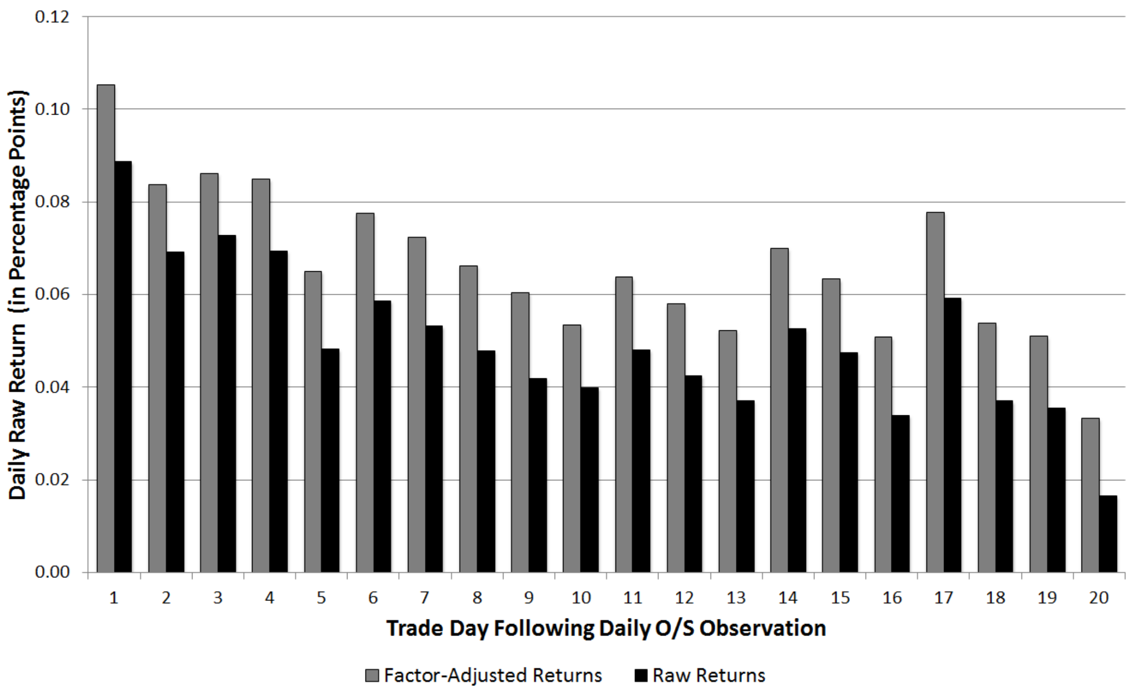

Figure 2: Cumulative Hedge Returns by Year

Figure 2 presents cumulative annual unadjusted returns to three strategies assuming monthly portfolio rebalancing for each year in the sample. The first strategy (shown in grey) consists of an equal-weighted long position in the lowest $\mathrm{O/S}_{i,m}$ decile together with an equal-weighted short position in the highest $\mathrm{O/S}i_{m}$ decile. $\mathrm{O/S}_{i,m}$ equals the ratio of option volume to equity volume of firm i in month m. Decile portfolios are formed at the conclusion of each month. In addition to $\mathrm{O/S}_{i,m}$ deciles, firms are independently sorted into quintiles of residual institutional ownership and option leverage. The second strategy (shown in black) consists of a long-short $\mathrm{O/S}_{i,m}$ position for firms in the lowest quintile of residual institutional ownership (RI). RI is obtained from cross-sectional regressions as detailed in Nagel (2005), where lower values of RI correspond to higher short-sale costs, and vice versa. The third strategy (shown in white) consists of a long-short $\mathrm{O/S}_{i,m}$ position for firms in the lowest leverage (LM) quintile, where $\mathrm{LM}_{i,m}$ is the open-interest-weighted average λ of firm i in month m. The sample consists of 175,654 firm-months spanning 1996 through 2008. All returns are shown as percentages.

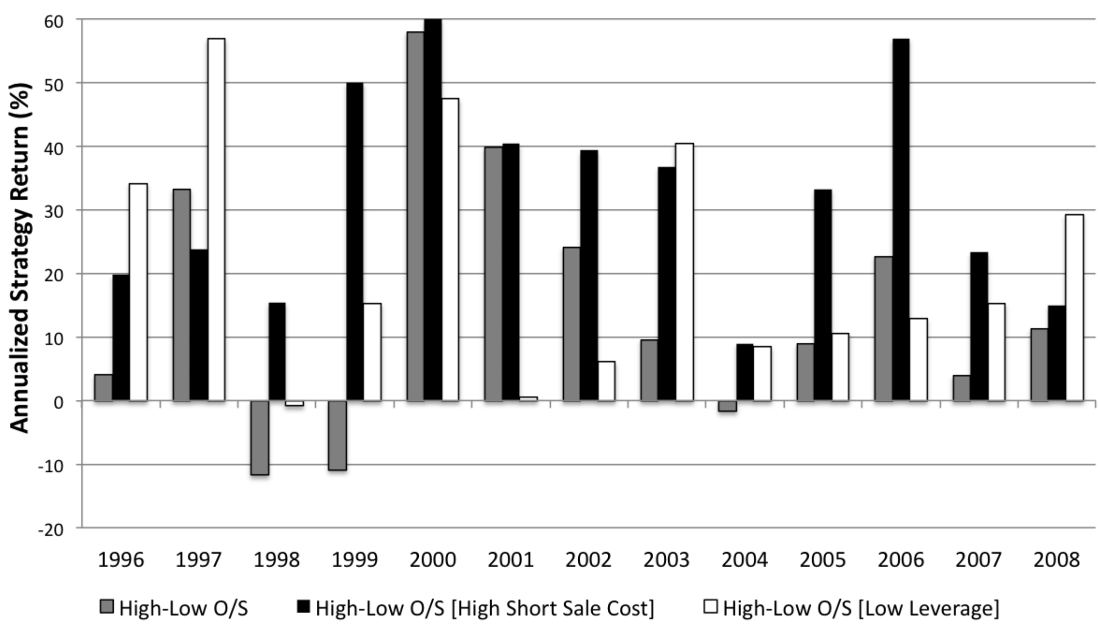

Table 1: Descriptive Statistics By Year

Panel A provides sample size information and descriptive statistics of $O/S_{i,m}$ (shown as a percentage), where $O/S_{i,m}$ equals the ratio of option volume to equity volume of firm i in month m as outlined in Section 4. Panel B gives average firm characteristics by decile of O/S. The sample consists of 175,654 firm-months spanning 1996 through 2008. SIZE is the log of market capitalization of the firm and LBM is the log of the firm's book-to-market ratio measured at the firm's last quarterly announcement date. VLC (VLP) equals the total call (put) contract volume traded in a given month; each contract represents 100 shares. OPVOL equals the sum of VLC and VLP. EQVOL equals the total equity volume traded, in units of 100 shares.

Panel A: Sample Characteristics and O/S Descriptive Statistics by Year

	Firms	Firm-Months	MEAN	P25	MEDIAN	P75	SKEW
1996	1,094	7,501	4.725	1.555	2.992	5.662	2.887
1997	1,425	10,292	4.495	1.534	2.917	5.510	2.497
1998	1,609	11,600	3.873	1.281	2.520	4.856	2.774
1999	1,719	12,255	3.833	1.248	2.559	4.986	2.133
2000	1,800	14,027	3.747	1.330	2.662	5.010	1.897
2001	1,662	12,883	3.093	1.035	2.146	4.090	2.130
2002	1,564	12,383	2.770	0.832	1.803	3.684	2.127
2003	1,497	12,146	3.425	0.886	2.034	4.497	2.543
2004	1,676	13,961	4.246	1.023	2.422	5.423	2.514
2005	1,791	15,031	4.972	1.073	2.615	6.110	2.779
2006	1,917	16,831	5.655	1.247	3.174	7.325	2.445
2007	2,062	18,381	5.658	1.231	3.078	7.173	2.651
2008	1,996	18,363	4.721	0.978	2.425	5.755	3.091
ALL		175,654	4.337	1.136	2.541	5.353	3.094

Panel B: Firm Characteristics by Deciles of O/S

	O/S	LMC	LBM	VLC	VLP	OPVOL	EQVOL
1 (Low)	0.363	7.492	0.384	522	259	782	272,478
2	0.753	7.278	0.360	1,016	487	1,503	210,911
3	1.166	7.248	0.344	1,677	814	2,491	221,327
4	1.648	7.306	0.336	2,674	1,380	4,054	250,668
5	2.244	7.383	0.322	4,124	2,240	6,364	286,555
6	3.015	7.509	0.312	6,806	3,659	10,465	347,914
7	4.049	7.676	0.301	11,318	6,227	17,544	431,857
8	5.554	7.862	0.285	18,018	10,637	28,655	507,843
9	8.107	8.051	0.272	29,881	18,692	48,573	603,144
10 (High)	16.488	8.220	0.250	71,186	45,074	116,260	669,223
High-Low	16.124	0.728	-0.134	70,664	44,814	115,478	396,745

Table 2: Cumulative Abnormal Returns By Deciles of Option Volume Ratio

The table below presents the time-series average equal-weighted returns by deciles of $O/S_{i,m}$ and $\Delta O/S_{i,m}$. As outlined in Section 4, $O/S_{i,m}$ equals the ratio of option volume to equity volume of firm i in month m, and $\Delta O/S_{i,m}$ is defined as the difference between $O/S_{i,m}$ and the average O/S over the prior six months, all scaled by this average . Decile portfolios are formed at the conclusion of each month. Deciles range from 1 to 10 with the highest (lowest) values located in the 10th (1st) decile. The sample consists of 175,654 firm-months spanning 1996 through 2008. MOMEN equals the cumulative market adjusted returns measured over the 6 months leading up to portfolio formation. RET(0) is the return in the portfolio formation month, RET(1) is the return in the first month following portfolio formation, and RET(2) is the return in the second month following portfolio formation, all market adjusted. All returns are shown as percentages. The t-statistics are calculated using the time-series difference in returns between the 1st and 10th deciles. ***, **, and * indicate the significance at the 1%, 5%, and 10% level, respectively.

Panel A: Abnormal Returns Across Deciles of O/S				
	MOMEN	RET(0)	RET(1)	RET(2)
1 (Low)	-4.836	-1.895	0.646	0.330
2	-1.075	-0.833	0.417	0.116
3	1.471	-0.073	0.109	0.271
4	3.431	0.297	0.182	-0.049
5	5.275	0.285	0.095	0.148
6	6.542	0.868	-0.166	0.057
7	7.367	0.628	-0.162	-0.188
8	9.960	0.873	-0.357	-0.065
9	12.589	0.956	-0.111	-0.187
10 (High)	17.944	1.107	-0.492	-0.169
High-Low	22.780***	3.003***	-1.138***	-0.498
t-stat High-Low	9.636	6.470	-2.464	-1.153

Panel B: Abnormal Returns Across Deciles of ΔO/S				
	MOMEN	RET(0)	RET(1)	RET(2)
1 (Low)	-5.648	-2.316	0.795	0.261
2	-2.585	-1.530	0.255	0.349
3	-0.618	-0.921	0.273	-0.041
4	0.835	-0.334	-0.023	-0.035
5	3.200	-0.160	-0.049	-0.021
6	6.426	0.497	0.009	0.035
7	10.216	1.250	-0.373	-0.033
8	12.542	1.648	-0.161	0.104
9	16.017	1.920	-0.217	-0.114
10 (High)	18.223	2.148	-0.351	-0.271
High-Low	23.871***	4.463***	-1.146***	-0.533*
t-stat High-Low	15.284	12.223	-3.073	-1.795

Table 3: One-Month Five-factor-adjusted Returns by Deciles of O/S and ΔO/S

Panel A presents portfolio alphas across lagged deciles of $\mathrm{O/S}_{i,m}$, where $\mathrm{O/S}_{i,m}$ equals the ratio of option volume to equity volume of firm i in month m. Decile portfolios are formed at the conclusion of each month, ranging from 1 to 10 with the highest (lowest) values located in the 10th (1st) decile. The sample consists of 175,654 firm-months spanning 1996 through 2008. A k-month lag indicates that the portfolio is formed at the conclusion of month $m-k$ and the returns are realized in month m. Portfolio alphas are calculated using the CAPM; three Fama-French and momentum factors; and the three Fama-French, momentum, and Pastor-Stambaugh liquidity factors. Panel B is defined analogously for ΔO/S, where ΔO/S equals the difference between O/S in the portfolio formation month and the average over prior six months, scaled by this average. Panels C and D contain estimated factor loadings. r_m^{mkt} is the market return in month m, r_m^f is the risk-free rate, HML ,SMB, UMD, and LIQ are the monthly returns associated with the high-minus-low market-to-book, small-minus-big strategies, momentum and Pastor-Stambaugh liquidity strategies, respectively. All returns are shown as percentages, t-statistics are shown in parentheses.

Panel A: Factor-Adjusted Alphas by Deciles of O/S

Lag:	One-Month Lag			Two-Month Lag			Three-Month Lag		
	CAPM	Four-Factor	Five-Factor	CAPM	Four-Factor	Five-Factor	CAPM	Four-Factor	Five-Factor
1 (Low)	0.607	0.974	0.873	0.302	0.513	0.422	0.076	0.130	0.058
	(1.75)	(3.92)	(3.45)	(1.14)	(2.67)	(2.17)	(0.31)	(0.72)	(0.32)
5	0.029	0.154	0.094	0.084	0.290	0.253	0.056	0.288	0.234
	(0.10)	(0.89)	(0.54)	(0.29)	(1.67)	(1.42)	(0.20)	(1.86)	(1.47)
10 (High)	-0.570	-0.573	-0.598	-0.246	-0.212	-0.269	-0.191	-0.087	-0.134
	-(1.86)	-(2.99)	-(3.04)	-(0.78)	-(1.09)	-(1.35)	-(0.61)	-(0.45)	-(0.68)
Low-High	1.177	1.547	1.471	0.548	0.725	0.690	0.268	0.217	0.193
	(2.61)	(4.90)	(4.55)	(1.33)	(2.70)	(2.52)	(0.68)	(0.85)	(0.74)
Annualized	15.081	20.231	19.154	6.784	9.053	8.608	3.262	2.635	2.338

Panel B: Factor-Adjusted Alphas by Deciles of ΔO/S

Lag:	One-Month Lag			Two-Month Lag			Three-Month Lag		
	CAPM	Four-Factor	Five-Factor	CAPM	Four-Factor	Five-Factor	CAPM	Four-Factor	Five-Factor
1 (Low)	0.737	1.220	1.132	0.213	0.586	0.490	-0.023	0.179	0.121
	(1.89)	(4.71)	(4.28)	(0.68)	(3.20)	(2.65)	-(0.09)	(1.18)	(0.78)
5	-0.118	0.083	0.045	-0.084	0.210	0.162	-0.124	0.108	0.060
	-(0.43)	(0.45)	(0.24)	-(0.30)	(1.23)	(0.93)	-(0.46)	(0.61)	(0.34)
10 (High)	-0.400	-0.492	-0.523	-0.316	-0.433	-0.506	0.020	-0.075	-0.151
	-(1.55)	-(3.03)	-(3.16)	-(1.20)	-(2.68)	-(3.09)	(0.06)	-(0.44)	-(0.88)
Low-High	1.137	1.711	1.655	0.529	1.019	0.996	-0.043	0.255	0.273
	(3.04)	(5.87)	(5.54)	(1.78)	(4.84)	(4.59)	-(0.17)	(1.30)	(1.35)
Annualized	14.534	22.581	21.770	6.535	12.936	12.633	-0.515	3.099	3.322

Table 3 [Continued]: One-Month Five-Factor-Adjusted Returns by Deciles of O/S and ΔO/S

Panel C: Factor Loadings Across Deciles of O/S

	Intercept	$r_m^{mkt} - r_m^f$	SMB_m	HML_m	UMD_m	LMI_m
1 (Low)	0.873	1.062	0.446	0.144	-0.550	0.092
	(3.45)	(16.08)	(6.49)	(1.68)	-(11.58)	(1.27)
5	-0.256	1.254	0.645	-0.253	-0.095	0.063
	-(1.59)	(29.78)	(14.73)	-(4.64)	-(3.12)	(1.37)
10 (High)	-0.598	1.324	0.561	-0.389	0.050	0.008
	-(3.04)	(25.74)	(10.49)	-(5.83)	(1.36)	(0.14)
Low-High Loading	1.47	-0.26	-0.12	0.53	-0.60	0.08
	(4.55)	(-3.10)	(-1.31)	(4.86)	(-9.88)	(0.91)

Panel D: Factor Loadings Across Deciles of ΔO/S

	Intercept	$r_m^{mkt} - r_m^f$	SMB_m	HML_m	UMD_m	LMI_m
1 (Low)	1.132	1.058	0.630	-0.132	-0.610	0.094
	(4.28)	(15.31)	(8.78)	-(1.47)	-(12.27)	(1.24)
5	0.057	1.232	0.550	-0.420	-0.104	0.048
	(0.29)	(23.67)	(10.18)	-(6.22)	-(2.77)	(0.85)
10 (High)	-0.523	1.212	0.659	-0.008	-0.023	-0.007
	-(3.16)	(27.97)	(14.65)	-(0.14)	-(0.72)	-(0.16)
Low-High Loading	1.65	-0.15	-0.03	-0.12	-0.59	0.10
	(5.54)	-(1.97)	-(0.36)	-(1.22)	-(10.46)	(1.18)

Table 4: Multivariate Regression

The table below presents pooled cross-sectional regression results from regressing RET(1) on deciles O/S, OPVOL, and EQVOL. The sample consists of 175,654 firm-months spanning 1996 through 2008. RET(1) is the return in the first month following the observation of O/S$_{i,m}$, the ratio of option volume to equity volume of firm i in month m. OPVOL$_{i,m}$ equals the total option volume of firm i in month m. EQVOL$_{i,m}$ is defined analogously for equity volume. Decile portfolios are formed at the conclusion of each month. Deciles range from 1 to 10 with the highest (lowest) values located in the 10th (1st) decile. RET(0) is the market-adjusted return in the portfolio formation month. MOMEN equals the cumulative market adjusted returns measured over the prior six months. All returns are calculated as percentages. SIZE is the log of market capitalization of the firm and LBM is the log of the firm's book-to-market ratio measured at the firm's last quarterly announcement date. AMIHUD is the Amihud illiquidity ratio of firm i in month m. ΔO/S equals the difference between O/S in the observation month and the average over prior six months, scaled by this average. Year fixed effects are included and standard errors are clustered at the monthly level. The resulting t-statistics are shown in parentheses. ***, **, and * indicate the coefficient is significant at the 1%, 5%, and 10% level, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Intercept	0.391***	0.416***	-1.592***	-1.676***	-2.194***	-2.255***	-2.018***	-1.910***	-1.862***
	-3.1	-3.31	(-5.28)	(-5.55)	(-7.13)	(-7.03)	(-6.31)	(-5.93)	(-5.83)
Decile(O/S)	-0.106***	-0.113***	-0.120***	-0.118***	–	–	–	-0.070***	–
	(-8.16)	(-8.74)	(-8.88)	(-8.69)	–	–	–	(-2.82)	–
Decile(OPVOL)	–	–	–	–	-0.132***	–	-0.159***	-0.068**	-0.098***
	–	–	–	–	(-8.38)	–	(-8.09)	(-2.36)	(-5.48)
Decile(EQVOL)	–	–	–	–	–	-0.062***	0.049**	–	–
	–	–	–	–	–	(-3.20)	-2.04	–	–
RET(0)	–	–	–	-0.022***	-0.022***	-0.023***	-0.022***	-0.022***	-0.022***
	–	–	–	(-5.82)	(-5.81)	(-5.92)	(-5.80)	(-5.80)	(-5.74)
MOMEN	–	0.003**	0.005***	0.008***	0.008***	0.007***	0.008***	0.008***	0.008***
	–	-(2.24)	-(3.21)	-(5.07)	-(4.84)	-(4.55)	-(4.93)	-(5.01)	-(5.16)
SIZE	–	–	0.209***	0.207***	0.280***	0.238***	0.247***	0.249***	0.254***
	–	–	-(6.97)	-(6.91)	-(8.36)	-(6.29)	-(6.53)	-(7.03)	-(7.41)
LBM	–	–	0.928***	1.141***	1.265***	1.461***	1.195***	1.175***	1.277***
	–	–	-(3.65)	-(4.46)	-(4.97)	-(5.76)	-(4.66)	-(4.59)	-(5.02)
AMIHUD	–	–	16.067**	15.620**	10.411	11.269	11.475	13.186*	10.903
	–	–	-(2.31)	-(2.25)	-(1.50)	-(1.61)	-(1.64)	-(1.84)	-(1.57)
Decile(ΔO/S)	–	–	–	–	–	–	–	–	-0.065***
	–	–	–	–	–	–	–	–	(-4.30)
Adj-R^2 (%)	0.26	0.268	0.305	0.35	0.349	0.314	0.351	0.354	0.359

Table 5: Time Series Properties of O/S

Panel A provides the pooled month-to-month transition probabilities by deciles of $O/S_{i,m}$, where $O/S_{i,m}$ equals the ratio of option volume to equity volume of firm i in month m. Decile portfolios are formed at the conclusion of each month. Deciles range from 1 to 10 with the highest (lowest) values located in the 10th (1st) decile. The sample consists of 175,654 firm-months spanning 1996 through 2008. Panel B re-examines the link between $O/S_{i,m}$ and future returns when sorting firm-months into deciles based on the time-series of O/S for a given firm. For each firm, we observe the entire time-series of $O/S_{i,m}$ during our sample window and sort firm-months into portfolios using the realized distribution of $O/S_{i,m}$. MOMEN equals the cumulative market adjusted returns measured over the 6 months leading up to portfolio formation. RET(0) is the return in the portfolio formation month, and RET(1) is the return in the first month following portfolio formation. Portfolio alphas are calculated using the three Fama-French, momentum, and Pastor-Stambaugh liquidity factors. All returns and transition probabilities are shown as percentages.

Panel A: Month-to-Month O/S Decile Transition Matrix

	1 (Low)	2	3	4	5	6	7	8	9	10 (High)
1 (Low)	**43.95**	22.34	13.12	8.46	5.03	3.19	1.77	1.14	0.71	0.31
2	23.10	**23.22**	18.14	13.30	9.03	5.80	3.66	1.98	1.14	0.62
3	13.90	18.70	**18.36**	16.16	12.65	8.77	5.55	3.26	1.75	0.89
4	7.97	13.30	17.45	**17.22**	15.10	11.95	8.51	4.66	2.65	1.18
5	4.80	9.27	12.98	15.25	**16.93**	15.48	11.89	7.46	4.20	1.74
6	2.83	5.78	8.78	12.69	15.25	**17.51**	16.09	11.90	6.76	2.41
7	1.68	3.42	5.71	8.41	11.92	16.33	**18.84**	17.82	11.85	4.03
8	0.98	1.98	3.30	4.80	7.86	12.14	18.07	**22.76**	19.94	8.17
9	0.58	1.15	1.62	2.40	4.42	6.71	11.47	20.35	**30.43**	20.88
10 (High)	0.40	0.63	0.73	1.07	1.57	2.20	4.09	8.57	20.88	**59.87**
Avg. Diag	26.91									

Panel B: Abnormal Returns Across Firm-Specific O/S Deciles

	Market-Adjusted Returns			Five-Factor Alpha	
	MOMEN	RET(0)	RET(1)	Alpha	t-Stat
1 (Low)	0.328	-0.537	2.030	2.189	10.663
2	2.923	-0.203	1.141	1.160	5.977
3	2.776	0.162	0.632	0.417	2.260
4	5.306	0.526	0.518	0.399	2.313
5	6.579	0.730	0.330	0.260	1.592
6	7.837	0.709	0.166	0.057	0.320
7	7.958	0.735	-0.110	-0.155	-0.907
8	11.203	0.951	-0.341	-0.430	-2.294
9	12.566	0.828	-0.667	-0.830	-4.313
10 (High)	15.046	1.273	-0.806	-0.914	-4.521
High-Low	14.718	1.810	-2.836	3.103	
t-stat High-Low	7.063	4.471	-8.304	12.349	

Table 6: Portfolio Alphas by Quintiles of Short-Sale Costs

Panel A presents portfolio alphas sorted by quintiles of residual institutional ownership (RI). RI is obtained from cross-sectional regressions as detailed in Nagel (2005), where lower values of RI correspond to higher short-sale costs, and vice versa. Within each RI quintile, portfolio alphas are obtained from initiating an equal-weighted long-short position in the extreme $O/S_{i,m}$ deciles. $O/S_{i,m}$ equals the ratio of option volume to equity volume of firm i in month m. Decile portfolios are formed at the conclusion of each month. The sample consists of 175,654 firm-months spanning 1996 through 2008. Portfolio alphas are calculated using the three Fama-French, momentum, and Pastor-Stambaugh liquidity factors. Panels B and C are defined analogously for quintiles of institutional lending loan fees (LF), defined as the value-weighted average loan fee for institutional loans occurring in month m and loan supply (LS), defined as the total quantity of shares available for lending scaled by total shares outstanding at the conclusion of month m. Loan fee and supply data are available on a monthly basis from June 2002 through 2008. All returns are shown as percentages.

Panel A: Five-Factor O/S Alphas by Quintiles of Residual Institutional Ownership

	Residual Institutional Ownership RI(1)					
	High Short Sale Costs RI(1)	RI(2)	RI(3)	RI(4)	Low Short Sale Costs RI(5)	High - Low Short Sale Costs
Alpha	2.489	1.645	1.503	1.146	0.914	1.575
t-Statistic	(3.64)	(3.00)	(3.44)	(2.52)	(2.21)	(2.29)
Annualized	34.318	21.630	19.600	14.649	11.535	20.630

Panel B: Five-Factor O/S Alphas by Quintiles of Institutional Lending Loan Fee

	Institutional Lending Loan Fee (LF)					
	Low Short Sale Costs LF(1)	LF(2)	LF(3)	LF(4)	High Short Sale Costs LF(5)	High - Low Short Sale Costs
Alpha	0.305	0.940	0.778	2.012	3.462	3.157
t-Statistic	(0.80)	(1.68)	(1.34)	(3.31)	(3.27)	(2.91)
Annualized	3.722	11.876	9.741	26.999	50.440	45.203

Panel C: Five-Factor O/S Alphas by Quintiles of Institutional Lending Loan Supply

	Institutional Lending Loan Supply (LS)					
	High Short Sale Costs LS(1)	LS(2)	LS(3)	LS(4)	Low Short Sale Costs LS(5)	High - Low Short Sale Costs
Alpha	2.996	1.444	1.179	0.567	1.197	1.799
t-Statistic	(3.40)	(2.82)	(1.82)	(0.95)	(2.08)	(1.98)
Annualized	42.510	18.778	15.103	7.018	15.346	23.861

Table 7: Portfolio Alphas by Quintiles of Option Leverage

This table presents portfolio alphas sorted by quintiles of open-interest-weighted average leverage (LM) of firm i in month m. Within each LM quintile, portfolio alphas are obtained from initiating an equal-weighted long-short position in the extreme $\mathrm{O/S}_{i,m}$ deciles. $\mathrm{O/S}_{i,m}$ equals the ratio of option volume to equity volume of firm i in month m. Decile portfolios are formed at the conclusion of each month. The sample consists of 175,654 firm-months spanning 1996 through 2008. Portfolio alphas are calculated using the three Fama-French, momentum, and Pastor-Stambaugh liquidity factors. All returns are shown as percentages.

	Option Leverage (LM)					
	Low Leverage LM(1)	LM(2)	LM(3)	LM(4)	High Leverage LM(5)	Low - High Leverage
Alpha	2.921	1.473	0.396	0.313	-0.120	3.041
t-Statistic	(4.53)	(2.47)	(0.90)	(0.82)	-(0.51)	(4.40)
Annualized	41.262	19.183	4.858	3.820	-1.435	43.259

Table 8: Future Return Characteristics by Deciles of Call-Put Volume Ratio

The dependent variable in the table below is SKEW, defined as the cross-sectional skewness of monthly returns within a given portfolio in the month following portfolio formation. SKEW is calculated each calendar month and for each decile of $C/P_{i,m}$, where $C/P_{i,m}$ equals the ratio of total call volume to total put volume of firm i in month m. Decile portfolios are formed at the conclusion of each month. Deciles range from 1 to 10 with the highest (lowest) values located in the 10th (1st) decile. Year fixed effects are included and standard errors are clustered at the monthly level. The resulting t-statistics are shown in parentheses. ***, **, and * indicates the coefficient is significant at the 1%, 5%, and 10% level, respectively.

Dependent Variable:	SKEW	
	(1)	(2)
Intercept	0.176*	0.129
	(1.91)	(1.41)
Decile(C/P)	0.022**	0.020**
	(2.31)	(2.16)
Lag(SKEW)	–	0.206***
	–	(7.41)
Adj-R^2 (%)	4.033	8.124

Table 9: Earnings Surprises and Earnings Announcement Returns

The sample for Table 9 consists of 46,670 quarterly earnings announcements during the 1996 through 2008 sample window. Each measure of earnings news is regressed on deciles of $O/S_{i,m}$ from the prior calendar month. $O/S_{i,m}$ equals the ratio of option volume to equity volume of firm i in month m. Decile portfolios are formed at the conclusion of each month. Deciles range from 1 to 10 with the highest (lowest) values located in the 10th (1st) decile. Columns 1, 2, and 3 contain the regression results where the dependent variables are SURPRISE, SUE, and CAR(-1,+1), respectively. SURPRISE equals the firm's actual EPS minus the consensus EPS forecasts immediately prior to the announcement, scaled by the beginning of quarter share price. SUE equals the standard unexplained earnings, calculated as realized EPS minus EPS from four-quarters prior, divided by its standard deviation over the prior eight quarters. $CAR(-1, +1)$ is the cumulative market-adjusted return during the three-day window surrounding the announcement date. $CAR(+2, +60)$ is the cumulative market-adjusted return from t+2 to t+60 where the t denotes the announcement date. SIZE is the log of the firm's market capitalization and LBM is the log of the firm's book-to-market ratio measured at the firm's last quarterly announcement date. MOMEN equals the cumulative market adjusted returns measured over the 6 months leading up to portfolio formation. All returns are calculated as percentages. Year fixed effects are included and standard errors are two-way clustered by firm and quarter. The resulting t-statistics are shown in parentheses. ***, **, * indicates the coefficient is significant at the 1%, 5%, 10% level, respectively.

	(1)	(2)	(3)	(4)
Dependent Variable:	SURPRISE	SUE	CAR(-1,+1)	CAR(+2,+60)
Intercept	-0.059*	0.340***	-0.910*	-1.537
	(-1.70)	(2.67)	(-1.88)	(-0.64)
Decile(O/S)	-0.006***	-0.025***	-0.088***	-0.107
	(-3.13)	(-4.50)	(-5.27)	(-1.64)
MOMEN	0.002***	0.010***	-0.001	0.016*
	(9.34)	(15.51)	(-0.53)	(1.66)
SIZE	-0.161***	-1.065***	0.187	0.221
	(-3.52)	(-6.23)	(0.52)	(0.94)
LBM	0.015***	-0.044***	0.164***	3.515**
	(5.42)	(-4.09)	(3.88)	(2.28)
Adj-R^2 (%)	2.33	3.551	0.198	1.463
Year Fixed Effects?	Yes	Yes	Yes	Yes