Which Index Options Should You Sell?[*]

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This paper explores historical return and risk properties of equity-hedged options across the S&P 500 option surface. We evaluate returns by estimating alpha to the S&P 500 index, and we quantify risk using three metrics: return volatility, losses under stress tests, and conditional value at risk. We show that analyzing option risk-adjusted alphas using different risk metrics leads to significantly different conclusions. We find that the most compensated options to sell on the S&P 500 surface per unit of stress-test loss are front-month options with strikes near-the-money and moderately below the index level. We apply these results to evaluate return expectations for short volatility strategies, potential added return from option selection, and implications for variance swaps.

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Many investors' portfolios can benefit from an allocation to the volatility risk premium, which may be captured by selling options. Options tend to be richly priced because of demand for portfolio protection.[1] Unfortunately for those who wish to earn the volatility risk premium, constructing option portfolios is not straightforward. For starters, there is no well-defined volatility risk premium harvesting benchmark.[2]

In 1996, an investor could trade about 250 different options on the S&P 500 Index across eight different maturities and a strike range from 60% to 120% of the S&P 500 Index value. Over the past 20 years, the option universe has grown significantly. As of early 2016, there are over 7,000 different S&P 500 options that an investor can trade across 27 different maturities and a strike range from 5% to 180% of the S&P 500 Index value.

Even when focusing on options on a single equity index, the degrees of freedom across option maturity and option strike price has clearly led to an expansive universe. Which of these options should you sell? In order to answer this question, it would be helpful to understand the risk and return properties of options across the volatility surface. Unfortunately, this critical piece of information has not been well documented. Our paper seeks to fill this void for options on the S&P 500 Index.

We begin by estimating the historical average returns for short delta-hedged daily-rebalanced S&P 500 Index options across different strikes and maturities. We find that out-of-the-money options had lower returns than near-the-money options, primarily due to lower exposure. We also find that short-dated options had higher average delta-hedged returns than their longer-dated counterparts.

Short delta-hedged option returns have beta to the S&P 500 Index because changes in option implied volatilities tend to be negatively correlated with equity returns. When evaluating the performance of option positions, we do not want to bias our conclusions due to returns attributable to equity beta. Therefore, we estimate the surface of option alphas as the intercept of the regression of their respective delta-hedged returns on S&P 500 Index returns. We find that the surface of option alphas is similar to that of delta-hedged returns, except that it is shifted lower because the short options' positive beta has been removed.

We then turn to quantifying the risk of short option positions across the surface. We evaluate three dimensions of risk: (1) traditional return volatility, (2) stress-test loss, and (3) conditional value at risk. Stress-test loss evaluates the expected portfolio return during extreme equity index return scenarios. Expected stress-test loss may be an important consideration for investors whose tail risk appetite may constrain leverage (and thereby limit expected return), more so than their aversion to return volatility.

Investors typically seek to maximize returns conditional on their risk budget. Assuming investors are willing to lever positions, the options with the highest return per unit of risk should help achieve this

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[1] Bakshi and Kapadia (2003) analyzed delta-hedged index option returns and found evidence in favor of a volatility risk premium. Hill et al. (2006) and Israelov and Nielsen (2015) observed that covered call returns are higher because of the spread between implied and realized volatility. Israelov, Nielsen, and Villalon (2016) showed that exposure to downside risk, through selling put options, captured both the equity risk premium and the volatility risk premium.

[2] Although the S&P 500 VIX short-term futures index (SPVXSTR) has become an often quoted benchmark, it does not directly capture the technical definition of the volatility risk premium in options: implied volatility minus coincident realized volatility.

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objective. Our analysis shows that short-dated, at-the-money and moderately below-the-money options have been the most compensated per unit of stress-test loss. On the other hand, short-dated, deep-out-of-the-money put options have realized the highest alpha per unit of return volatility. Which options you should sell ultimately depends on which risk you are concerned about: return volatility or tail risk.

We believe our findings help both portfolio managers who construct volatility risk premium harvesting strategies, as well as end investors who allocate to these portfolio managers. To portfolio managers, our analysis provides insights into the risk/return tradeoff across the option opportunity set. To end investors, our analysis can help set performance and risk expectations for volatility risk premium harvesting strategies. For example, our empirical investigation suggests that a pure volatility selling portfolio with double-digit expected returns likely has either (1) considerable stress-test loss exposure or (2) considerable alpha relative to a more passive methodology. The end investor must be comfortable with the level of tail risk exposure or confident in the manager's skill at providing alpha. And the transparent portfolio manager should attribute his expected performance accordingly.

Data Description

We analyze S&P 500 option performance from 1996 to 2015. The OptionMetrics IVY database provides daily closing bid and ask quotes, implied volatilities, dividends, and option deltas for the S&P 500 options analyzed in this paper. Underlying equity index values and USD LIBOR are from Bloomberg. Equity index futures prices and returns, which are used to calculate option hedge sizing and returns, are also derived from Bloomberg data.

To estimate option selling performance across different regions of the S&P 500 option surface, we bucket along two dimensions: moneyness and maturity.

With respect to option moneyness, we bucket option strikes by the number of standard deviations from the current index value:[3]

$$Option\ Strike\ Standard\ Deviation\ (STD)\ = \frac{\ln\frac{Option\ Strike}{Forward\ Price}}{Implied\ Volatility * \sqrt{Time\ Until\ Expiration}}$$

We bucket option strikes from -2.5 to +1 standard deviations, with 0.5 standard deviation increments. Historical option data is incomplete, and this strike range was selected based on data availability. Please see the Appendix for details on data availability and handling of missing data. For reference, a 0 standard deviation option has a strike equivalent to the index's forward price. If annualized implied volatility were 20%, a +1 standard deviation 1-month option would have a strike about 5.8% above the current index level, and a +1 standard deviation 2-month option strike would be about 8.2% above. Higher strike options are not symmetrically extended to +2.5 standard deviations due to sparse data for deep out-of-the-money call options.

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[3] Practitioners also refer to this definition as normalized strike. For implied volatility, we use a variance swap rate calculated by applying the VIX methodology to options of the same maturity. Implied volatility is annualized and time until expiration is expressed in years. The forward price reflects the implied index value at option expiration.

Within each bucket, we only include out-of-the-money options. This means that put options are included for negative implied standard deviation strikes and call options are included otherwise. At a given strike, the out-of-the-money option is generally more liquid than the in-the-money option. Excluding in-the-money options should lead to less noisy estimates, while not economically impacting our results.

We include S&P 500 options with standard monthly expiration dates (3rd Friday of the month). We bucket across maturities that range from front-month (monthly option maturity closest to expiration) to twelve months. Front-month options typically have less than one month to expiration, second-month options typically have between one and two months until their expiration, and so on.

Option Surface

The term structure and implied volatility skew of S&P 500 Index options have been well documented[4]. In order to provide context for the rest of our paper, which analyzes the properties of beta-adjusted options across the surface, we plot the average implied volatility surface in **Figure 1**.

As expected, implied volatility is higher for lower option strikes. Much of the existing literature, along the lines of Heston (1993), explains the implied skew with different assumptions about the dynamics of the underlying asset. Garleanu, Pedersen, and Poteshman (2009) offer another explanation for the shape of the implied volatility surface: option flows. Under this hypothesis, the implied volatility surface may exhibit a persistent skew due to protection buying on the put side and covered call selling on the call side.

An implied volatility surface is a price surface. By itself, it does not necessarily explain the profitability of selling options across the option surface — in the same way that a stock's price standalone does not explain the profitability of holding the stock. Carr and Wu (2016) estimate an option historical volatility surface, in which the surface reflects the localized volatility relevant for the specific option's strike and maturity. Combining their expected volatility surface with the average implied volatility surface yields a volatility risk premium surface (the spread between implied and realized volatility, conditional on strike and maturity). They find that implied volatility is typically higher than expected volatility (i.e. a volatility risk premium), which also suggests that the option surface may be affected by more than the dynamics of the underlying asset.

Due to the path dependence of option returns, the relationship between the volatility risk premium and option selling returns is imperfect. Investors who harvest the volatility risk premium are likely more interested in the properties of option P&L than in the difference between implied volatility and realized volatility. The remainder of this paper analyzes the return and risk characteristics of short delta-hedged option positions across the strike and maturity dimensions.

Option Returns

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[4] See Derman and Miller (2016) and Mixon (2007) for discussions of the implied volatility skew and term structure.

Similar to constructing portfolios in other asset classes, option selection balances expected returns against risk. We begin here with expected returns, or to be more precise, average historical returns over a 20-year period (1996 – 2015).[5]

We compute the annualized returns of short S&P 500 delta-hedged options, bucketed daily by moneyness and time until expiration. On a given day, we calculate the associated one-day, excess return of each available option within a given bucket after hedging its Black-Scholes delta exposure using S&P 500 Index futures:

$$R_{i,t} = \frac{-(P_{opt,i,t} - P_{opt,i,t-1}) + \Delta_{opt,i,t-1}(P_{fut,t} - P_{fut,t-1}) + R_f\, P_{opt,i,t-1}}{SPX_{t-1}}$$

The return for each bucket on a given day is then equally-weighted:[6]

$$R_{moneyness,maturity,t} = \frac{1}{N}\sum_{i}^{N} R_{i,t}$$

We repeat this exercise each day and report average annualized returns in **Figure 2**. This average return surface does not look like the implied volatility surface in **Figure 1**. Shorter-dated options of a given moneyness had, for the most part, higher average delta-hedged returns than their longer-dated counterparts. Deep out-of-the-money options had lower returns than near-the-money options, primarily due to lower volatility exposure. At-the-money options and out-of-the-money put options tended to have higher delta-hedged returns than out-of-the-money call options. This is likely related to high demand for protection as described in Bollen and Whaley (2004) and Constantinides and Lian (2015).

Alphas

The intent of delta-hedging is to neutralize an option's equity exposure. Hull and White (2015) show that delta-hedged S&P 500 Index options still have exposure (as measured by beta) to the S&P 500 Index. An option's beta exposure can be expressed as follows:

$$\underbrace{\frac{dP}{dS}}_{beta} = \underbrace{\frac{\partial P}{\partial S}}_{delta} + \underbrace{\frac{\partial P}{\partial IV}}_{vega}\frac{\partial IV}{\partial S}$$

If the last partial derivative in the above expression is zero, then delta equals beta and delta-hedging should result in an exposure that has no beta. But for S&P 500 Index options the last partial derivative in the above expression is not zero. Typically, implied volatility rises when the market sells off and falls when the market rallies. This negative relationship between changes in option implied volatility and equity returns leads to short option positions losing money, on average, when implied volatility increases.[7] Said another way, we expect that short delta-hedged options have positive beta.

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[5] Past performance is not a guarantee of future performance.

[6] We choose to equally-weight within each bucket for parsimony.

That's exactly what we find. **Figure 3** plots the beta of short delta-hedged option returns in each bucket. In all cases, short delta-hedged option returns had positive equity exposure. Equity beta was higher for near-the-money options, with a maximum value of 0.05 for options with a normalized strike of zero. The higher beta values for near-the-money and longer-dated options make sense because these options have higher exposure to changes in implied volatility.

We do not wish to attribute the performance of equity returns arising from an imperfect hedge to the short options positions. Thus, for the sake of performance attribution, we estimate *alphas*. Alpha should not be interpreted as the average return to an *ex ante* implementable strategy[8], but it does identify the excess return (relative to equity market exposure) earned within each bucket.

The top panel of **Figure 4** plots the alpha across the volatility surface. The alpha profile is similar to the return profile displayed in **Figure 2**, except that it is shifted lower because the options' positive equity exposure has been removed. The bottom panel of **Figure 4** shows this difference for reference. The difference is larger for near-the-money options because these options had higher beta.[9]

We believe the alphas in **Figure 4** are more appropriate for decision making than the returns in **Figure 2.** Option selection should be evaluated on the performance options provide that cannot be obtained elsewhere (i.e. alpha). Equity exposure is easily modifiable with instruments such as futures or ETFs.

Option Risk

Shorter-dated at-the-money and moderately lower strikes had the highest historical alphas, but that does not necessarily mean that they are the preferred options to sell. Investors have another lever to pull – leverage. Levering up other options on the surface would have led to higher earned alphas. Whether it makes sense to do so depends on the option risk profile and the investors' risk tolerance. Selling the options that provide the greatest alpha per unit of risk should lead to the highest alphas when constrained by a risk budget.

Return Volatility

Return volatility is often the default risk metric and our analysis of risk starts here. We compute the volatility of short delta-hedged options across each bucket. However, we do not wish to "contaminate" our risk estimate with volatility that is due to equity beta. We therefore construct beta-adjusted volatility estimates, which are estimated as the residual volatilities from the regression of option returns on the S&P 500 Index.

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[7] Short option positions have negative vega ($\frac{\partial P}{\partial IV}$) exposure.

[8] Our performance attribution uses *ex post* betas. Although it is beyond the scope of this paper, an investor could estimate an option's *ex ante* beta for use in a trading strategy. Hull and White (2015) describe potential approaches to option beta hedging.

[9] The higher beta of these options comes from higher exposure to implied volatility changes (higher vega).

Figure 5 reports annualized beta-adjusted volatility across the same buckets we analyzed for option returns. Out-of-the-money options had significantly lower return volatility than near-the-money options. This makes sense because option convexity exposures are highest at-the-money. For example, we see that the annualized beta-adjusted volatility of selling a 0.0 STD (at-the-money), front-month delta-hedged option is 2.9%. However, the annualized beta-adjusted volatility for selling a -2.0 STD (out-of-the money, lower strike) delta-hedged option with the same maturity is 0.9%.

With respect to the time dimension, beta-adjusted volatilities were similar across most maturities. However, shorter-dated (front-month) options were more volatile than their longer-dated counterparts. Our priors for the relationship between return volatility and option maturity are ambiguous. On the one hand, shorter-dated options have higher gamma exposure, meaning that they react more to large-sized equity moves than do longer-dated options. On the other hand, shorter-dated options have lower vega exposure, meaning they react less to changes in their implied volatility. Further complicating matters, short-dated implied volatilities tend to change more than longer-dated implied volatilities — potentially increasing the shorter-dated options' risk exposure attributable to changing implied volatility. Overall, the magnitude of gamma and vega exposures move in opposite directions with respect to maturity. The historical evidence appears to suggest that these two offsetting exposures result in a similar beta-adjusted volatility for options across most maturities. However, for short-dated options (front month), higher gamma exposure has been more impactful than lower vega exposure.

Stress exposure

One deficiency of return volatility as a risk metric is that it is agnostic to fat tails. Especially fat left tails, which is the tail that investors are more worried about. Short volatility certainly has fat tails, and for this reason we think that it is important to also consider a risk metric that accounts for this exposure. We turn to stress tests. What are the potential losses under realistic extreme scenarios?

On October 19, 1987, the S&P 500 Index was down approximately 20 percent, the worst daily loss for a broad-based US equity index going back to at least 1897.[10] Therefore, we use a 20 percent equity market move for our illustrative stress scenario parameters. Beta-adjusted, short option positions are exposed to losses during large, one-day market moves, regardless of direction. In the large, one-day market rally case (+20%), we increase 1-month implied volatility by 20 percentage points. Although implied volatilities tend to decrease in market rallies, we conservatively assume an implied volatility increase. In the large, one-day market crash case (-20%), we use a much larger 1-month implied volatility increase of 60 percentage points to reflect the possibility of an extreme volatility move in a market crash.[11]

All stress-test losses are beta-adjusted, thereby removing the option's unconditional equity exposure. An option's stress-test loss is defined as the maximum loss in the two shock scenarios. In our stress-test loss definition $BS_{opt,i,t}$ is the Black-Scholes option pricing equation, with underlier price as the first input and implied volatility as the second:

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[10] The S&P 500 Index was down -20.5% on this day, the largest percentage loss since its launch in 1957. From 1897 to 1957, the largest one-day percentage loss in the Dow Jones Industrial Average was -12.8% on October 28, 1929.

[11] Increase of 60 percentage points defined as 1-month implied volatility increasing from 20% to 80%. Implied volatility shocks for a given option are adjusted for time to expiration to reflect the fact that short-dated implied volatilities tend to react more than long-dated implied volatilities. Therefore, for an option with T calendar days to expiry, implied volatility is increased by 0.6/sqrt(T*12/365).

$$StressLoss_{i,t} = Max\left(Loss\ in\ Rally_{i,t}, Loss\ in\ Crash_{i,t}\right)$$

$$Loss\ in\ Rally_{i,t} = \frac{BS_{opt,i,t}\left(1.2SPX_t, IV_{opt,i,t} + \frac{0.2}{\sqrt{\frac{12}{365}DaysToExp_{opt,i,t}}}\right) - P_{opt,i,t}}{SPX_t} - 0.2\beta_{opt,i,t}$$

$$Loss\ in\ Crash_{i,t} = \frac{BS_{opt,i,t}\left(0.8SPX_t, IV_{opt,i,t} + \frac{0.6}{\sqrt{\frac{12}{365}DaysToExp_{opt,i,t}}}\right) - P_{opt,i,t}}{SPX_t} + 0.2\beta_{opt,i,t}$$

The option beta specified in these equations is the sum of two components: (1) Black-Scholes delta and (2) Adjustment for spot–implied volatility relationship. This beta is consistent with the calculations shown in **Figure 3**. The stress-test loss in each bucket is then defined as follows:

$$StressLoss_{moneyness,maturity,t} = \frac{1}{N}\sum_{i}^{N} StressLoss_{i,t}$$

As an example of this calculation, consider the S&P 500 put option maturing on January 16, 2015 with 1950 strike price on December 10, 2014, shown in **Table 1**. By construction, the S&P 500 Index can either lose 20% or gain 20% in our stress scenarios. On this date, a beta-adjusted portfolio that is short one option contract would have lost 11.7% in the hypothetical stress scenario that has the S&P 500 down 20% and the index's one-month implied volatility up by 60%.[12]

Figure 6 reports stress-test losses calculated across the S&P 500 options surface using the same defined stress scenarios. Stress-test losses shown in **Figure 6** look quite different than the return volatilities shown in **Figure 5**. Stress-test losses are *lower* for beta-adjusted at-the-money options than for the out-of-the-money options. For example, the 0.0 STD, front-month, beta-adjusted option had a stress-test loss of 10.3% versus the 14.0% stress-test loss for the -2.0 STD option. Whereas the return volatility of the -2.0 STD option was less than one-third of the 0.0 STD option, the stress-test loss exposure is actually over 35% higher. In other words, the lower strike option has more tail risk than suggested by return volatility. Lastly, stress-test losses tended to be higher for options nearer to expiration, a result consistent with our findings for the options' return volatility.

As risk measures, stress-test losses and return volatilities are different in that stress-test losses are computed *ex ante* and return volatilities are computed *ex post*. The distinction matters because a short options strategy may appear to be low risk when computing realized volatilities when it is in fact high risk in terms of its tail risk exposure. A lack of tail events over a strategy's life (backtested or live)[13] and path-dependent tail risk exposures can hide periods of potentially high tail risk. Evaluating exposure to losses in stress-tests helps to uncover the tail risk embedded in short option positions that may be hidden within positions that have low return volatility.

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[12] The sample option in Table 1 has 37 days to expiration. In the one-day crash scenario the implied volatility of this option is increased by 54.4% instead of 60% due to the time to expiration adjustment.

Conditional Value at Risk

Every day, investors experience the return volatility of their strategies. In their entire careers, they may never experience the type of stress-test scenario considered above, which resembles the worst day for US equities in the past 120 years. The conditional value at risk (CVaR) is an intermediate measure of tail risk between the two ends of this spectrum. CVaR measures the average of all returns below a specified percentile. Specifically, 0.1% CVaR can be interpreted as the average one-day loss during a once in approximately 4-year event.

Figure 7 reports 0.1% CVaR across the option surface. The shape of the CVaR surface is closer to that of return volatility (**Figure 5**) than stress-test loss (**Figure 6)**. The level of 0.1% CVaR was considerably lower than corresponding stress-test loss exposure, particularly for out-of-the-money options. For the front-month, 0.0 STD bucket the CVaR was 1.8%, which was around 1/5th of the corresponding average stress-test loss. For the front-month, -2.0 STD bucket the CVaR was 0.7%, which was around 1/20th of the corresponding average stress-test loss.

Appraisal Ratios

The Appraisal ratio is a leverage invariant measure of risk-adjusted alpha. It identifies the extent to which a strategy is additive to a portfolio. The appraisal ratio is computed by regressing a strategy's return in excess of cash on the existing portfolio's return in excess of cash and dividing the intercept (alpha) by the residual volatility. It bears resemblance to a Sharpe ratio. For the purpose of our analysis, we assume that the existing portfolio is a passive, unit holding of the S&P 500.

We compute three versions of the appraisal ratio. The first, **Volatility Appraisal Ratio (VAR)**, is the ratio of the strategy's alpha to its beta-adjusted volatility. The second, **Stress-Test Appraisal Ratio (STAR)**, is the average of the ratio of the strategy's beta-adjusted return[14] to its beta-adjusted stress-test loss. The final version, the **Conditional Value at Risk Appraisal Ratio (CVaRAR)**, is the ratio of the strategy's alpha to its 0.1% beta-adjusted conditional value at risk.

Volatility Appraisal Ratio

Figure 8 reports VARs across the S&P 500 options surface, dividing **Figure 4** by **Figure 5**. The average VAR across all buckets was 0.7. Front-month, lower-strike options have realized the highest VAR. For example, selling daily-rebalanced deep-out-of-the-money -2.5 STD front-month put options had a VAR of 2.5. This VAR is considerably higher than selling other front-month options, as seen from the 0.7 VAR realized by selling front-month at-the-money options.

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[13] As an example, over the period between 1996 and 2015, the S&P 500 Index's worst day was in October 2008 when it was down 9%. While this is a large, negative return, it is significantly less bad than Black Monday in October 1987 when the S&P 500 Index was down 20%.

[14] Both VAR and CVaRAR divide full-period average alpha by each respective full-period risk measure. However, STAR averages the point-in-time ratios of beta-adjusted return to beta-adjusted stress loss (calculated on each date). We chose to calculate STAR in this manner to maintain consistent beta-adjusted stress loss throughout time.

So it might seem that the answer to the question posed by our paper's title "Which Index Options Should You Sell?" is short-dated deep out-of-the-money put options. The -2.5 STD put option has realized 1.4% annualized alpha on average. In order to match the 2.1% annualized alpha of the at-the-money option, the out-of-the-money put option would need to have been levered 1.4x. In so doing, it would have realized 0.8% annualized volatility. This is significantly lower than the 2.9% annualized volatility realized by the at-the-money option, an apparent victory. However, this conclusion changes drastically when using stress-test loss, instead of return volatility, as the measure of risk.

The out-of-the-money put option's expected stress-test loss is 13.5%, about 30% higher than the at-the-money option's 10.3% expected stress-test loss. Levered up 1.4x to match the at-the-money option's average historical alpha, the out-of-the-money option's expected stress-test loss is 19.5%, considerably higher than the 10.3% stress-test loss for the at-the-money option. Which option looks more compensated (per unit of risk exposure) now?

Stress-Test Appraisal Ratio

Figure 9 plots the STARs across the option surface. The average STAR across the options surface was 8.3%. Over the period analyzed, the front-month put options with strikes between 0 and -1 standard-deviation away from the money realized the highest STARs, in the range of 19% to 27%. One explanation for higher STARs on this part of the options surface could be that protection buyers typically buy short-dated, moderately out-of-the-money put options.

It is interesting that the STAR of selling these options is comparable to the return-to-stress-test loss ratio of holding the S&P 500 index itself.[15] Levered to target the same long-term alpha, selling options on the most compensated part of the surface and holding equities both expose an investor to similar beta-adjusted tail risk. However, the levered short, option positions achieve this beta-adjusted return with significantly lower return volatility[16]. This means that the long equity position has a wider distribution of long-term outcomes than does the levered short options position, despite the two positions providing similar expected alphas per unit of tail risk.

Like the VAR measure, STAR was also higher for shorter-dated options. However, the profile across strikes was quite different. At-the-money and slightly lower strike options had relatively higher STARs. However, deep out-of-the-money options had lower STARs. The -2.5 STD, out-of-the-money front-month put option discussed above had 3.4x the VAR of the at-the-money option, but about 0.6x of its STAR.

Conditional Value at Risk Appraisal Ratio

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[15] Mechanically, the S&P 500 would lose 20% in a -20% move. Assuming the equity risk premium is 5%, the S&P 500 would have a return-to-stress-test loss ratio of 25%.

[16] The annualized volatility of the S&P 500 from 1996 – 2015 (measured using 21-day, overlapping returns) was 16.5%. The beta-adjusted volatility of front-month, 0.0 STD (ATM) options was 2.9%. Assuming the equity risk premium is 5%, the front-month, 0.0 STD option would need to be levered 2.4x to achieve an alpha of 5%. The corresponding beta-adjusted volatility for the levered short option position would be 6.9%, roughly 40% of the return volatility of equities.

Figure 10 reports CVaRAR across the option surface. The average CVaRAR across all buckets was 1.0. Front-month, lower-strike options had the highest CVaRAR, with a maximum of 2.9 for the front-month, -2.5 STD options. As expected from the comparison of risk characteristics, the shape of CVaRAR looks much closer to VAR (**Figure 8**) than STAR (**Figure 9**).

Statistical Significance

Our paper measures risk-adjusted alphas of S&P 500 options, exploring differences across the option surface. To test the statistical significance of our results, we bootstrap a distribution of cross-sectionally de-meaned Appraisal Ratios. **Figures 11, 12,** and **13** show these bootstrapped distributions for VAR, STAR, and CVaRAR, respectively. The cross-sectionally de-meaned VAR distribution in **Figure 11** supports our finding that short-dated, deep out-of-the-money put options had the highest VAR on the option surface. On average, the front-month, -2.0 STD bucket's VAR was 1.8 higher than the surface mean. This result is statistically significant with the 90% confidence interval ranging from 1.4 to 2.2. The cross-sectionally de-meaned CVaRAR distributions in **Figure 13** are similar[17] to those shown for VAR.

Our paper's primary finding is that the most compensated options to sell on the S&P 500 surface per unit of stress-test loss, as measured by STAR, are short-dated options with strikes near and moderately below the index level. We report the mean and 90% confidence interval of the cross-sectionally de-meaned STAR distribution in **Figure 12**. The front-month, -0.5 STD bucket's STAR was +18% higher than its peers on average, a statistically significant result with the 90% confidence interval ranging from +12% to +25%. Supporting the primary finding, near-dated, moderately-lower-strike options exhibited economically meaningful and statistically significant outperformance relative to the rest of the surface. Conversely, longer-dated options (particularly 12-month) and deep out-of-the-money options' underperformance was also statistically significant.

Subsample Analysis

We test whether our primary findings are consistent over time by splitting our sample into two 10-year periods: 1996 – 2005 and 2006 – 2015. **Figure 14** reports the bootstrapped distribution of the difference in the average cross-sectionally de-meaned STARs between the second and first half of the sample. Our primary finding is that few of the differences between the two periods were statistically significant.[18]

Implications for Sizing Option Selling Portfolios and Expected Alpha

How should investors evaluate risk when trading off something they experience every day (or every several years) against something they will likely never witness? That is not a simple question to answer. But we can ask a different question: suppose the unlikely stress scenario does occur? Do they want their strategy to live to see another day? If so, then the expected losses during that stress scenario are likely the binding risk constraint.

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[17] The confidence intervals for the CVaRAR distributions are wider than those for VAR because risk is defined as 0.1% CVaR, which may be less stable across bootstrap samples.

[18] Zero difference was within the 90% confidence interval for almost all buckets.

Thus, rather than define a risk budget constraint in terms of return volatility, portfolio managers can employ a stress-test loss framework and optimize their portfolios around a stress-test loss constraint.

For illustrative purposes, we assume a stress-test loss budget of 20%. **Figure 15** shows the leverage required for short beta-adjusted option positions to reach the 20% stress-test loss budget. For front-month, at-the-money options, roughly 2x leverage is needed on average, with longer maturities requiring additional leverage.[19]

Figure 16 then shows the average alpha across the options surface when options are sized to the 20% stress-test loss budget. This chart is simply calculated by scaling the STAR surface (**Figure 9**) by 20%. At this risk budget, an investor would have earned 1.7% per year on average across the option surface. Selling options on the most compensated part of the surface (0 to -1 standard deviation front-month strikes) would have earned an investor 4.5% per year on average.

These expected alphas scale linearly with the stress-test loss budget. For example, investors who set a more aggressive stress-test loss budget of 50% would have earned 11.3% per year on average selling options on the most compensated part of the surface. These numbers provide an important frame of reference to potential investors in short volatility strategies. If a passive short index option portfolio manager expects double-digit annualized alphas, what does that suggest about their strategies' potential stress-test exposures?

Implications for Alpha from Passive Option Selection

By comparing similarly-sized option portfolios across the option surface, we can see how much extra alpha could have been generated from passive option selection since 1996. By passive option selection, we mean always choosing to invest in options with the same maturity and moneyness (in contrast to point-in-time active selection of option strike and maturity).

Assuming the same illustrative stress-test loss risk budget of 20% for all options, **Figure 16** showed average annualized alpha across the surface. The average compensation across the option surface was 1.7% per year. The highest bucket on the option surface (front-month, -0.5 STD) earned 5.4% per year. This bucket earned 3.7% per year higher than the average, and is effectively an upper bound, with the benefit of hindsight, on the alpha from passive option selection (for the specified 20% stress-test loss budget).

A covered call is a well-known strategy that incorporates option selling. An S&P 500 covered call holds the S&P 500 index and sells an S&P 500 call option. Typical constructions of this strategy sell a near-dated, moderately out-of-the-money call option. Looking to **Figure 16**, selling a front-month 1.0 STD call[20] provided 0.8% of alpha on average, which is half of the surface's average alpha. This result illustrates that typical, passive covered call strategies may be selling options that are not as well compensated per unit of stress-test loss.[21]

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[19] This notional leverage measure, however, can be misleading as a measure of risk for a hedged portfolio. For one, the delta exposures of put and call options will partially offset each other, but this netting is not reflected in the notional leverage calculation. Further, consider a delta-hedged short options portfolio that is short two at-the-money put options and short one unit of the S&P 500. This portfolio would technically have a gross leverage of 3 (2 from the options, 1 from the short S&P 500 position), but its delta cannot fall below -1 or above +1.

Implications for Variance Swaps

Thus far, we have focused on the properties of options across the strike and maturity surface. However, selling variance swaps is an alternate approach to harvesting the volatility risk premium that seemingly does not require option strike selection. A variance swap is an over-the-counter instrument that swaps implied variance for realized variance. Variance swaps are favored by many investors because of their operational ease — no delta-hedging, explicit option selection, or option rebalancing is required.

As shown in Demeterfi et al (1999), a variance swap can be replicated with a static portfolio of options across an infinite range of strikes with 1/strike^2 weighting. **Figure 17** plots the relative weighting across the strikes in a variance swap replication portfolio, where the at-the-money strike has a baseline weight of 1. For reference, relative to at-the-money options the variance swaps holds approximately 1.2x as many 10% out-of-the-money puts, 2x as many 30% out-of-the-money puts, and 4x as many 50% out-of-the-money puts. On the call side, the variance swap holds 0.8x as many 10% out-of-the-money calls, 0.6x as many 30% out-of-the-money calls, and 0.4x as many 50% out-of-the-money calls. Although harvesting the volatility risk premium by selling variance does not explicitly require strike selection, it implicitly selects strikes, effectively weighting options at each strike as depicted by **Figure 17**.

We can compare the effective weights shown in **Figure 17** against the STAR surface shown in **Figure 9** to see if the variance swap replication portfolio holds strikes that are well compensated. The variance swap does hold relatively fewer out-of-the-money calls, which have lower risk-adjusted alphas, and it also holds relatively more out-of-the-money puts, which have higher risk-adjusted alphas. However, the variance swap holds an increasing quantity of deep out-of-the-money puts, which we have seen are not as well compensated per unit of stress-test loss (despite their higher Volatility Appraisal Ratios). Although variance swaps provide some operational conveniences, they may not be an optimal way of harvesting the volatility risk premium for those who are concerned about their strategy's stress-test loss exposure.

Conclusion

The volatility risk premium is a well-established risk premium backed by strong economic rationale and widely documented empirical evidence. Selling options is analogous to underwriting financial insurance, and it is therefore unsurprising that most parts of the S&P 500 option surface have historically compensated option sellers. However, implementing strategies to harvest the volatility risk premium is not straightforward. First and foremost, which of the 7,000+ available S&P 500 options should you actually sell to capture this risk premium?

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[20] On average, the moneyness of a front-month, 1.0 STD call was around 104%.

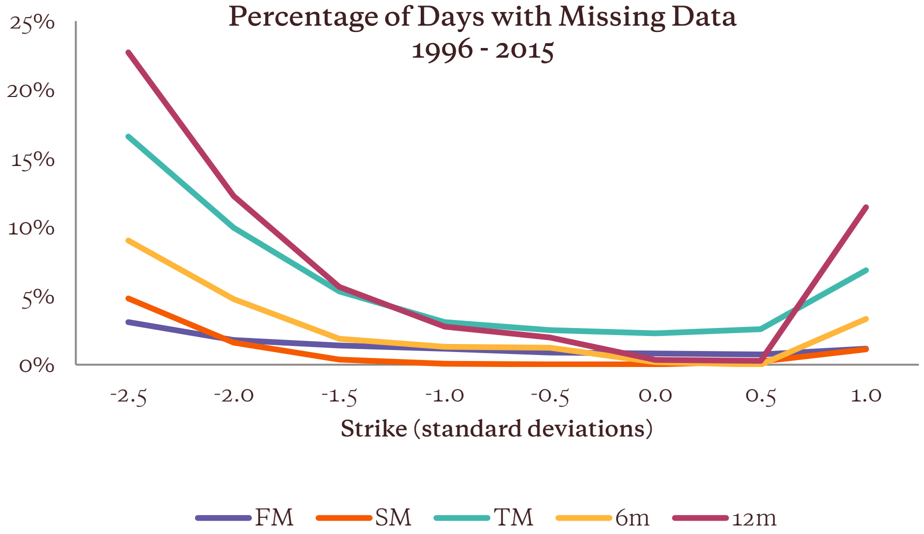
[21] Figure 10 shows annualized alpha for options that are selected within each strike/maturity bucket on a daily basis. For a covered call fund that sells a 1.0 STD call option and holds it to expiration, that option may span multiple buckets before expiration. The alpha-to-stress of that option over its lifespan is then an average of the various buckets' alpha-to-stress. An option sold in a more compensated bucket (i.e, front-month, -1.0 STD) and held to expiration is expected to have a higher alpha-to-stress because it averages over a more compensated part of the surface. However, this averaging may result in a smaller difference between the -1.0 and 1.0 STD strike options than shown in Figure 10.

We find that short-dated options with strikes moderately below the current index level are the most compensated per unit of stress-test loss exposure. Option buyers seek to purchase insurance for their portfolio, and are typically concerned about monthly or quarterly returns. They also typically purchase moderately out-of-the-money puts to cheapen the price of this insurance. It is intuitive that the options which most directly match these preferences are the most attractively compensated for option sellers.

Investors should look beyond typical risk metrics, such as return volatility, and beyond typical short option implementations, such as traditional covered calls and variance swaps, when seeking to efficiently harvest the volatility risk premium. Instead, prudent risk management of a short option portfolio requires evaluating tail risk, and that maximizing risk-adjusted alphas requires selecting and rebalancing options to maintain exposure to the most compensated part of the surface.

Appendix: Data Availability and Handling of Missing Data

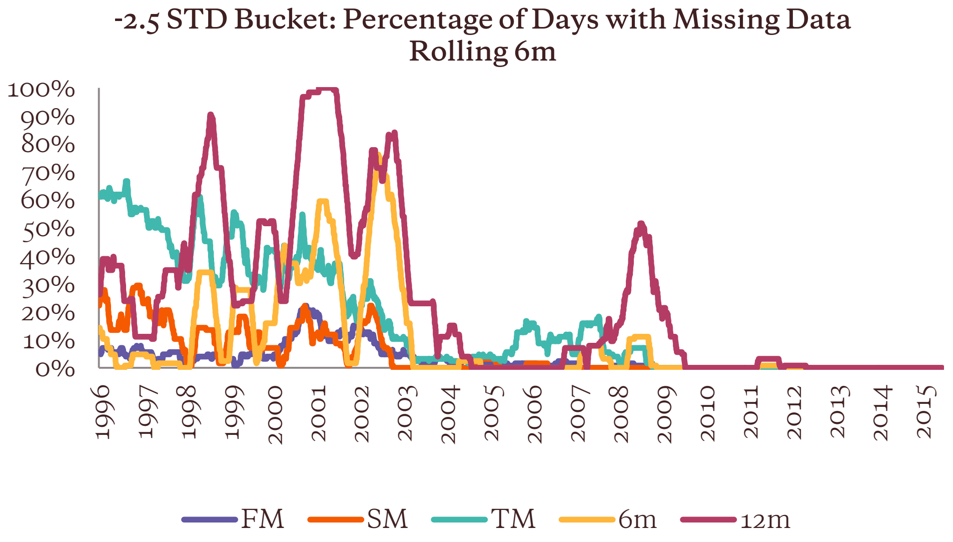
For our S&P 500 option surface analysis, we selected a strike range from -2.5 standard deviations to +1.0 standard deviations, bucketed by 0.5 standard deviation increments. We selected only out-of-the-money options because they are more liquid, and filtered options that had a zero bid price. In general, we found that shorter-dated, near-the-money buckets had better data availability. The below chart shows the percentage of days with no options available in each bucket:



We bucket S&P 500 options with non-zero bid prices on each date by moneyness (standard deviations) and time until expiration. We then calculated the percentage of days where there were no options in a given bucket to produce the chart above. In the legend, '6m' refers to the option maturity nearest 6 months to expiration, and '12m' refers to the option maturity nearest 12 months to expiration.

The data source is OptionMetrics. For illustrative purposes only.

Further out-of-the-money options had a higher percentage of missing data. Our selected strike range ensured that no bucket had more than 25% missing data. Furthermore, data availability improved over time. The chart below shows the rolling 6-month average of missing data for the most sparse strike bucket (-2.5 standard deviations):



We bucket S&P 500 options with non-zero bid prices on each date by moneyness (standard deviations) and time until expiration. For options in the -2.5 standard deviation strike buckets, we then calculated the percentage of days where there were no options in a given maturity over the trailing 6 months to produce the chart above. In the legend, '6m' refers to the option maturity nearest 6 months to expiration, and '12m' refers to the option maturity nearest 12 months to expiration.

The data source is OptionMetrics. For illustrative purposes only.

Before analyzing the option surface, within each bucket, we first filled in the following missing measures on dates with no available options: implied volatility, delta-hedged option returns, and stress-test loss. Specifically, on a date with missing data we identified all available buckets with the same maturity. We then ran a principal component analysis using all dates with complete data for these identified available buckets. On average, the first principal component explained 98% of the variation for implied volatility, 84% for delta-hedged returns, and 97% for stress-test loss. We then found all dates with complete data for both the 'missing bucket' and the available buckets used in the PCA. Using this complete data set, we regressed the 'missing bucket' on the first principal component. We subsequently used these regression coefficients to fill in the bucket's missing value on the date with no data available. We repeated this analysis across all buckets to ensure that we started with a complete data set before running our analysis.

For a small number of option buckets, we were unable to fill in missing data because were no options available within a specified maturity for some dates. Out of the 5,034 historical dates in the sample, this occurred on one day for second-month (SM) options and 96 days for third-month (TM) options.

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Table 1: Single Short Beta-Hedged Option Stress Test Example

As of December 10, 2014

	Dec. 10, 2014	Stress Test Example	Dec. 11, 2014 (Actual)
Strike	1950	1950	1950
Expiration	Jan. 2015	Jan. 2015	Jan. 2015
Option Type	Put	Put	Put
S&P 500 Index Level	2026.14	1620.91	2035.33
Implied Volatility	18.8%	73.2%	19.9%
Short Option Delta	0.26	0.75	0.24
Short Option Beta	0.31	0.80	0.29
Option Price ($)	19.75	382.35	19.05
Option P&L / NAV	-	-17.9%	0.0%
Beta-Hedge P&L / NAV	-	6.2%	-0.1%
Beta-Hedged Option P&L / NAV	-	-11.7%	-0.1%

The above table illustrates the stress-test loss calculation on December 10, 2014 for an S&P 500 put option maturing on January 16, 2015, with 1950 strike price.

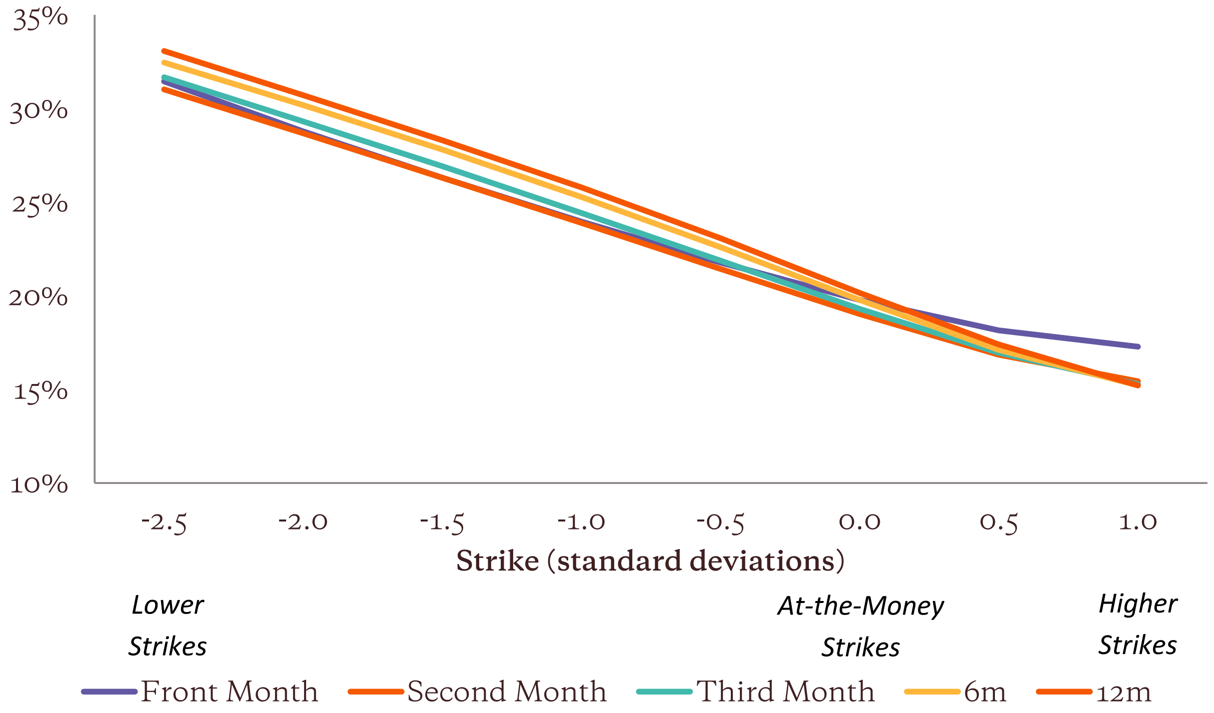
Stress-test loss is defined as the worst simulated one-day loss during 2 extreme one-day shock scenarios:
1. -20% S&P 500 crash with +60% IV spike
2. +20% S&P 500 crash with +20% IV spike

For this option on this date, the "Stress Test Example" column simulates a -20% one day S&P 500 index return and a +60% additive spike in one-month implied volatility. The "Actual" column shows the actual next-day statistics for this option, for comparison.

The data source is OptionMetrics. For illustrative purposes only.

Figure 1: Average Implied Volatility Across Surface

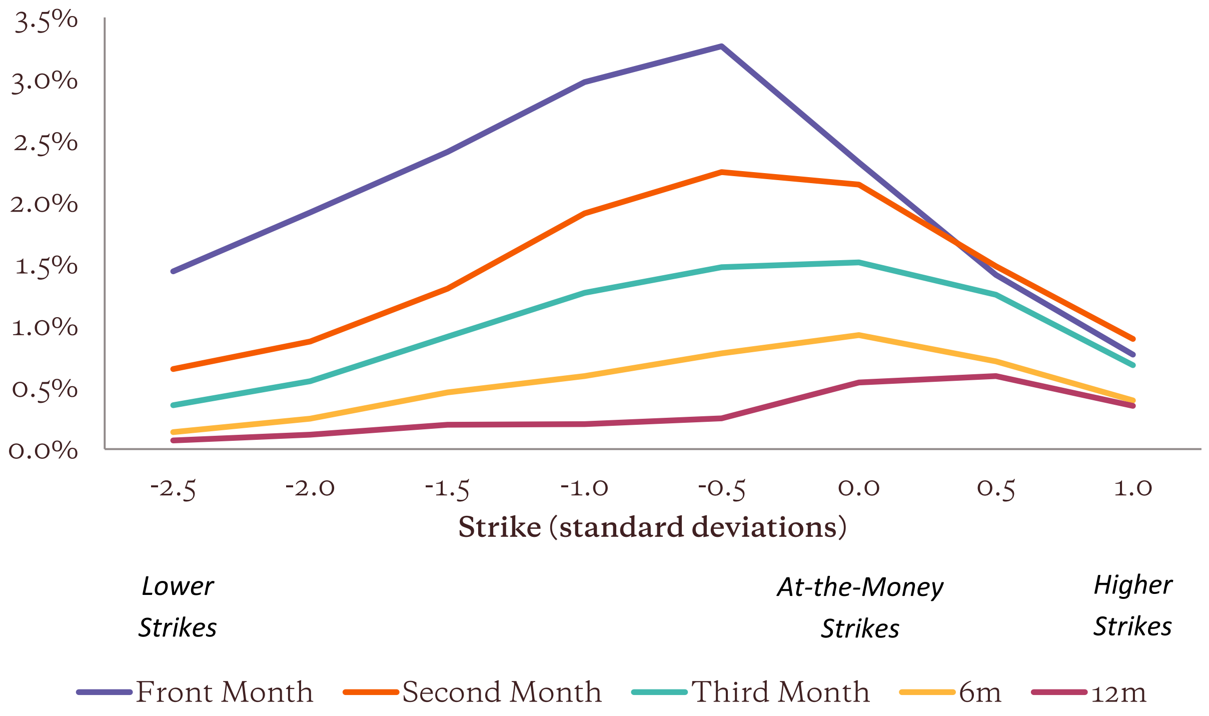
1996-2015



We bucket S&P 500 options on each date by moneyness (standard deviations) and time until expiration. Then for each option in a bucket, on each day we calculate the Black-Scholes implied volatility of that option, and we average across the options in a bucket to get one implied volatility value per bucket per date. For each bucket, this series is then averaged over time to produce the chart above. In the legend, '6m' refers to the option maturity nearest 6 months to expiration, and '12m' refers to the option maturity nearest 12 months to expiration.

The data source is OptionMetrics. For illustrative purposes only.

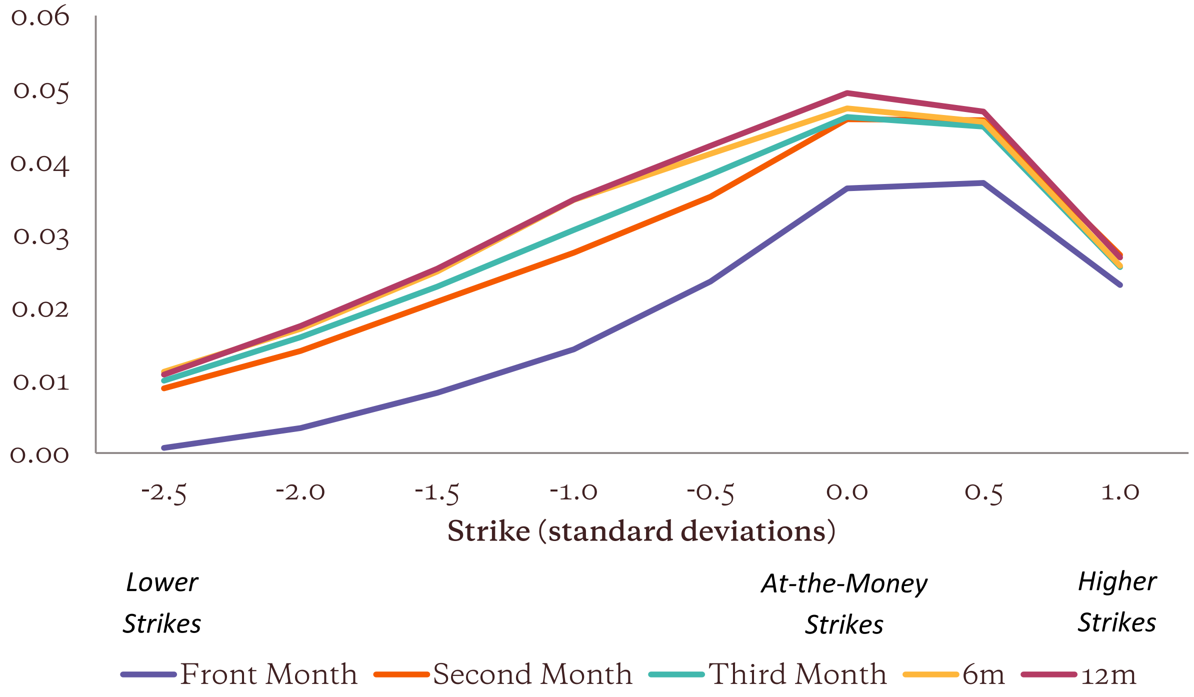
Figure 2: Average Annualized Short Delta-Hedged Option Return Across Surface
1996-2015



We bucket S&P 500 options on each date by moneyness (standard deviations) and time until expiration. Then for each option in a bucket, on each day we calculate the next day's annualized return of a delta-hedged portfolio that held just one contract of that option, and we average across the options in a bucket to get one "delta-hedged return" value per bucket per date. For each bucket, this series is then averaged over time to produce the chart above. In the legend, '6m' refers to the option maturity nearest 6 months to expiration, and '12m' refers to the option maturity nearest 12 months to expiration.

The data source is OptionMetrics. For illustrative purposes only.

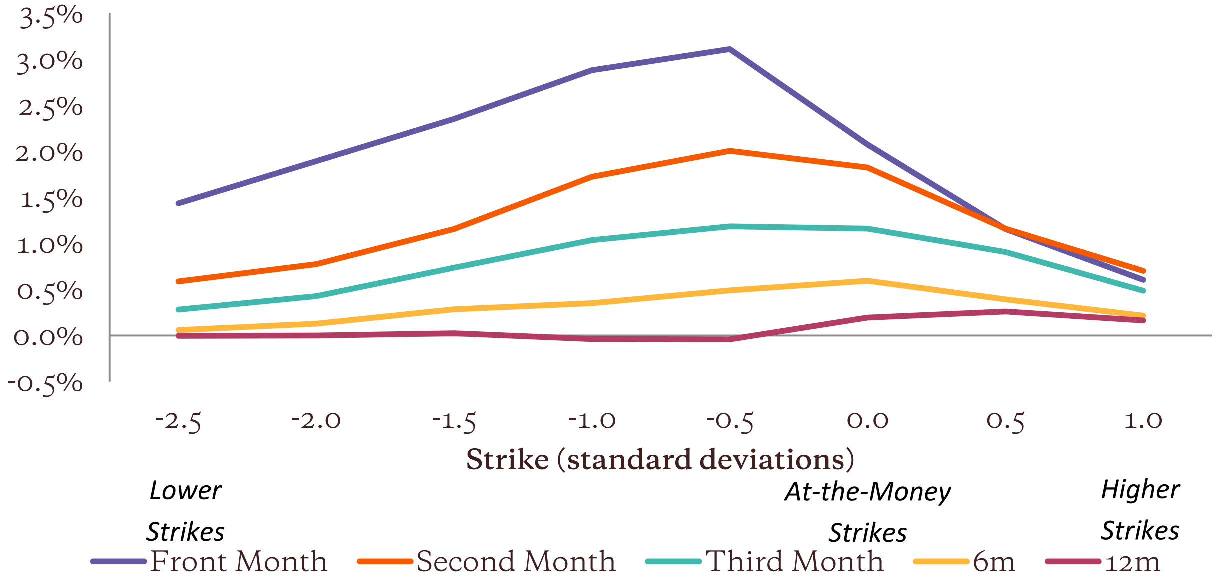
Figure 3: Beta of Short Delta-Hedged Option Returns Across Surface
1996-2015



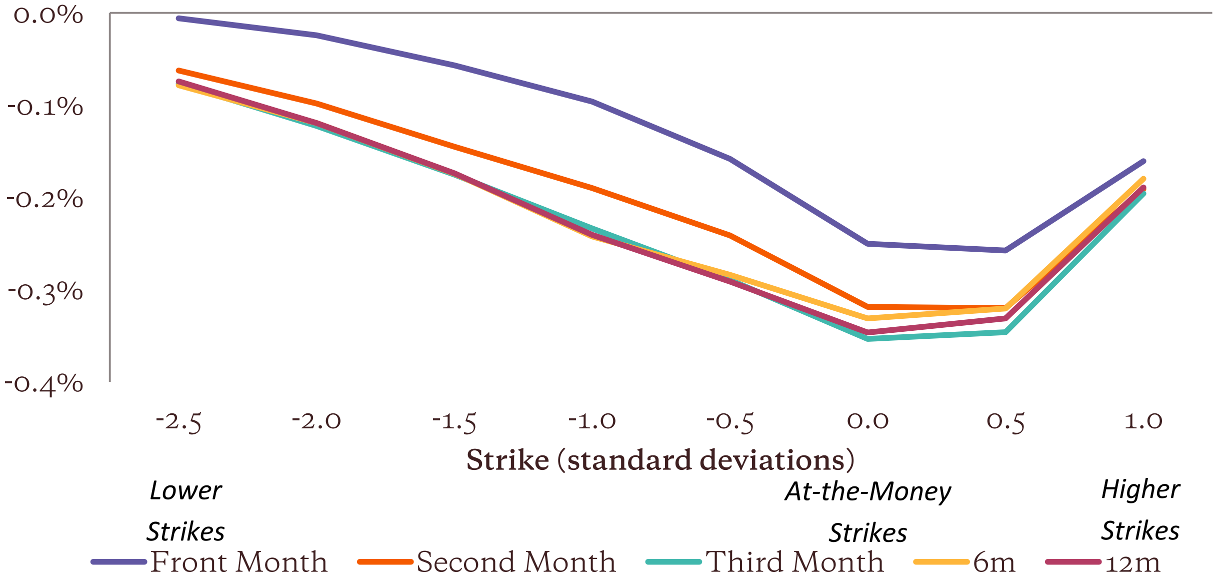
We bucket S&P 500 options on each date by moneyness (standard deviations) and time until expiration. Then for each option in a bucket, on each day we calculate the next day's annualized return of a delta-hedged portfolio that held just one contract of that option, and we average across the options in a bucket to get one "delta-hedged return" value per bucket per date. For each bucket's return series, we then compute the full-period beta to the S&P 500 to produce the chart above. In the legend, '6m' refers to the option maturity nearest 6 months to expiration, and '12m' refers to the option maturity nearest 12 months to expiration.

The data source is OptionMetrics. For illustrative purposes only.

Figure 4: Short Option Alphas Across Surface
1996-2015



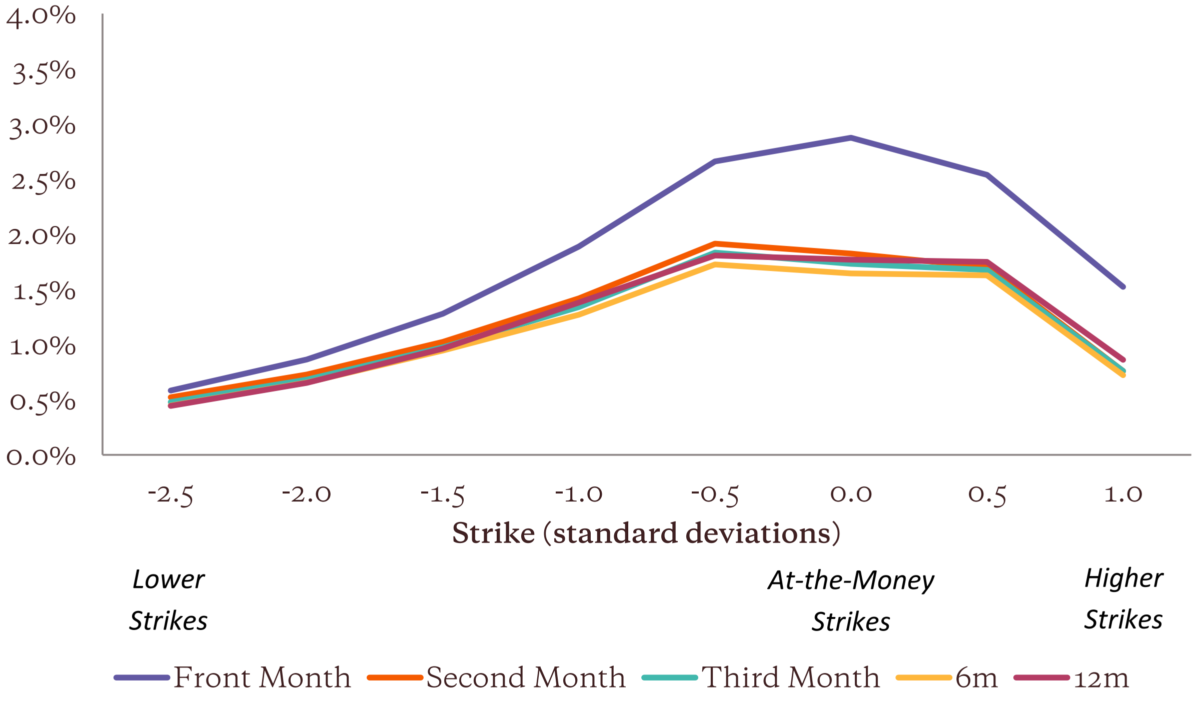
Difference between Short Option Alphas and Delta-Hedged Returns
1996-2015



We bucket S&P 500 options on each date by moneyness (standard deviations) and time until expiration. Then for each option in a bucket, on each day we calculate the next day's annualized return of a delta-hedged portfolio that held just one contract of that option, and we average across the options in a bucket to get one "delta-hedged return" value per bucket per date. For each bucket's return series, we then compute the full-period beta to the S&P 500. We then define "alpha" on each day as "delta-hedged return" minus full-period beta * S&P 500 return. We show the annualized alpha in the top panel of the chart above. The bottom panel shows the difference between option alphas and delta-hedged returns. In the legend, '6m' refers to the option maturity nearest 6 months to expiration, and '12m' refers to the option maturity nearest 12 months to expiration.

The data source is OptionMetrics. For illustrative purposes only.

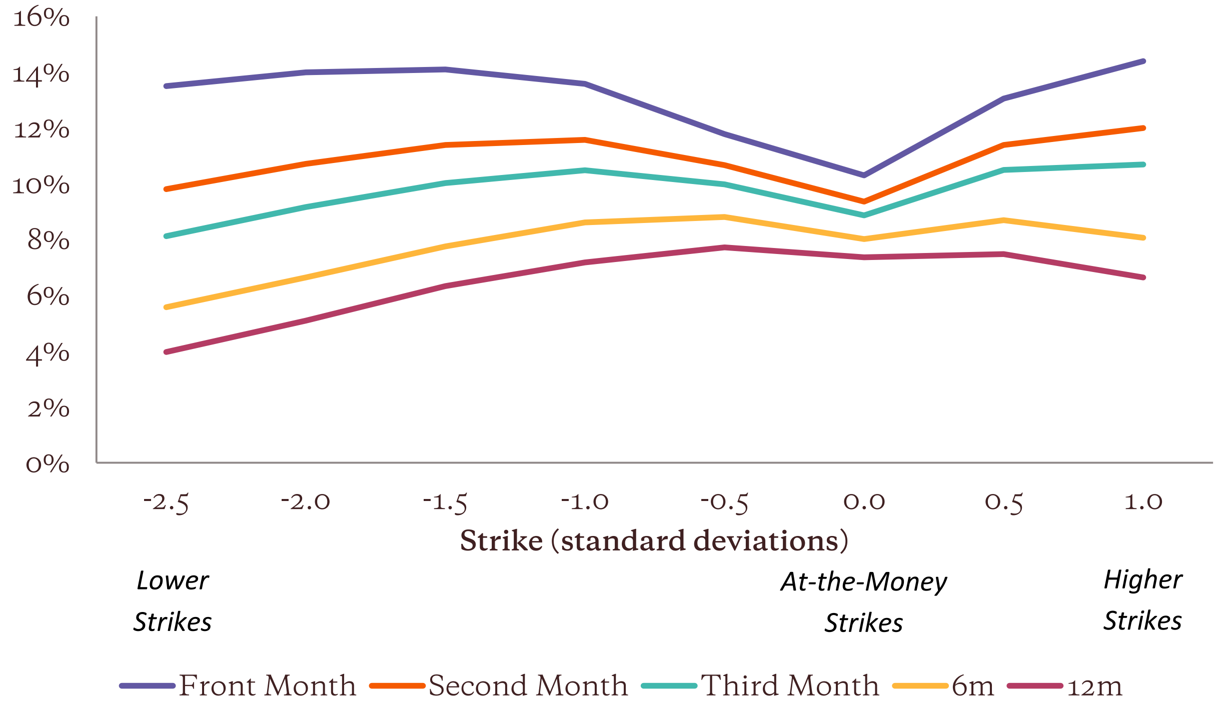
Figure 5: Beta-Adjusted Option Return Volatility Across Surface
1996-2015



We bucket S&P 500 options on each date by moneyness (standard deviations) and time until expiration. Then for each option in a bucket, on each day we calculate the next day's annualized return of a delta-hedged portfolio that held just one contract of that option, and we average across the options in a bucket to get one "delta-hedged return" value per bucket per date. For each bucket's return series, we then compute the full-period beta to the S&P 500. We then define "alpha" on each day as "delta-hedged return" minus full-period beta * S&P 500 return. For each bucket, we then compute the full-period annualized volatility of the alpha to produce the chart above. In the legend, '6m' refers to the option maturity nearest 6 months to expiration, and '12m' refers to the option maturity nearest 12 months to expiration.

The data source is OptionMetrics. For illustrative purposes only.

Figure 6: Average Stress-Test Loss Across Surface
1996-2015



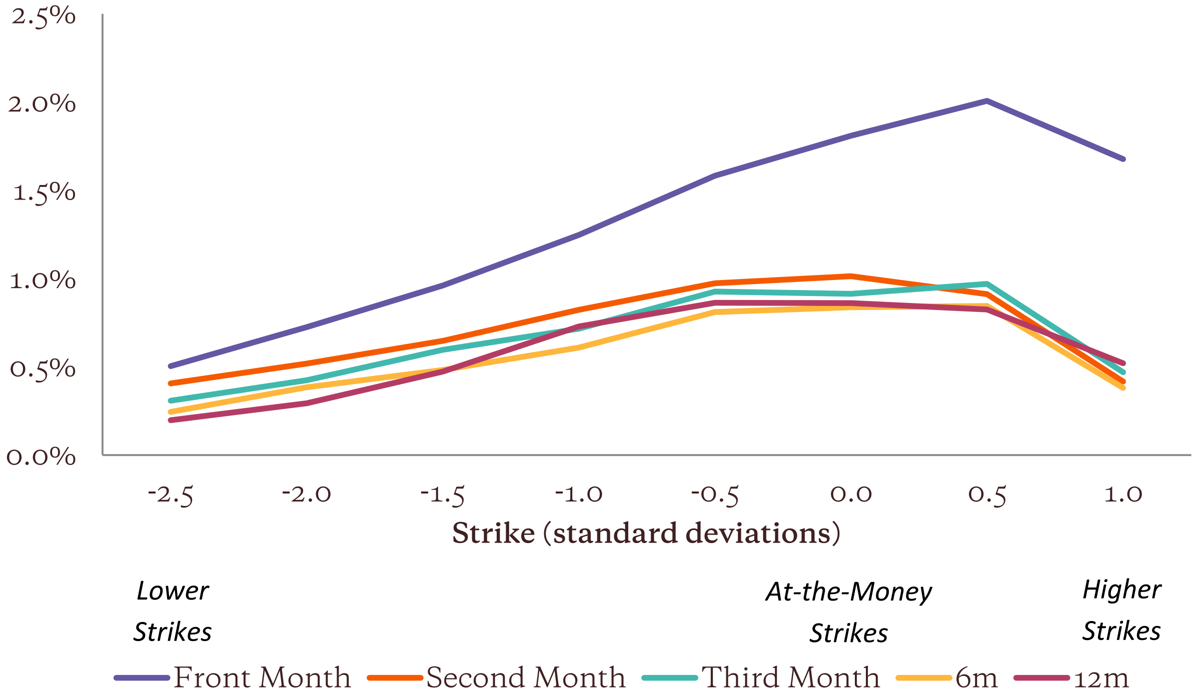
We bucket S&P 500 options on each date by moneyness (standard deviations) and time until expiration. Then for each option in a bucket, on each day we calculate the stress-test loss of a beta-adjusted portfolio that held just one contract of that option, and we average across the options in a bucket to get one "stress-test loss" value per bucket per date. For each bucket, this series is then averaged over time to produce the chart above. In the legend, '6m' refers to the option maturity nearest 6 months to expiration, and '12m' refers to the option maturity nearest 12 months to expiration.

Stress-test loss is defined as the worst simulated one-day loss during 2 extreme one-day shock scenarios:
1. -20% S&P 500 crash with +60% IV spike
2. +20% S&P 500 rally with +20% IV spike

The data source is OptionMetrics. For illustrative purposes only.

Figure 7: 0.1% Conditional Value at Risk Across Surface

1996-2015

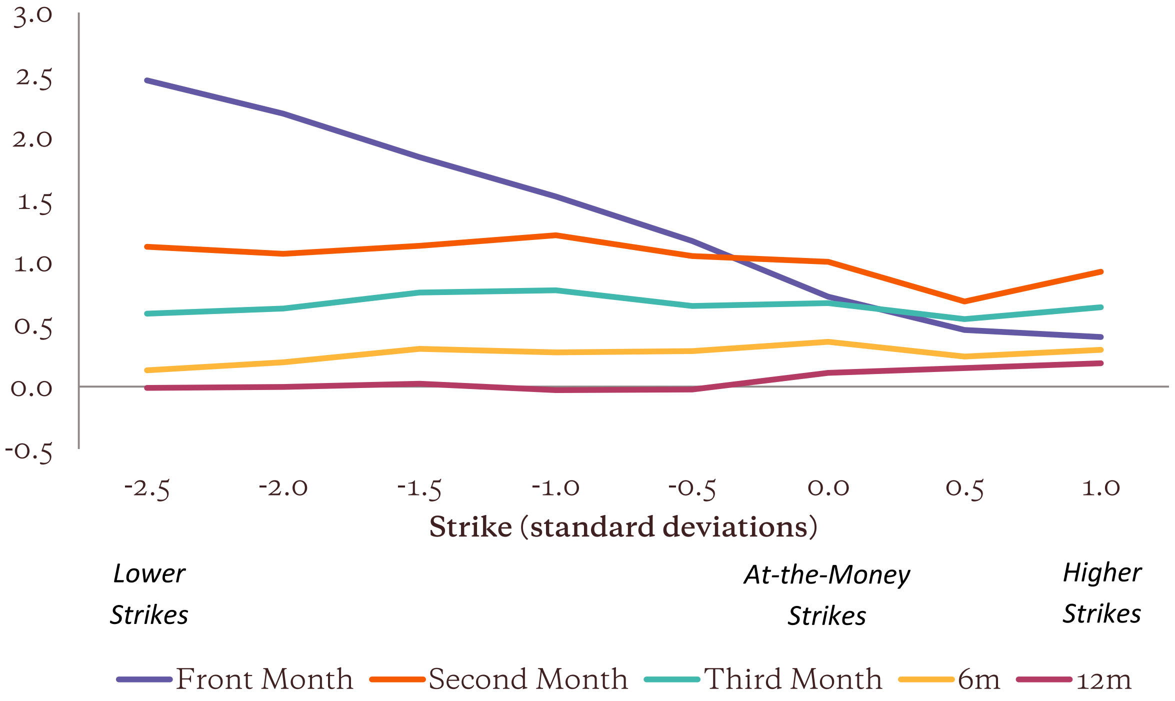


We bucket S&P 500 options on each date by moneyness (standard deviations) and time until expiration. Then for each option in a bucket, on each day we calculate the next day's return of a delta-hedged portfolio that held just one contract of that option, and we average across the options in a bucket to get one "delta-hedged return" value per bucket per date. For each bucket's return series, we then compute the full-period beta to the S&P 500. We then define "alpha" on each day as "delta-hedged return" minus full-period beta * S&P 500 return. We then report the 0.1% CVaR of the daily option alphas. In the legend, '6m' refers to the option maturity nearest 6 months to expiration, and '12m' refers to the option maturity nearest 12 months to expiration.

The data source is OptionMetrics. For illustrative purposes only.

Figure 8: Average Volatility Appraisal Ratio (VAR) Across Surface

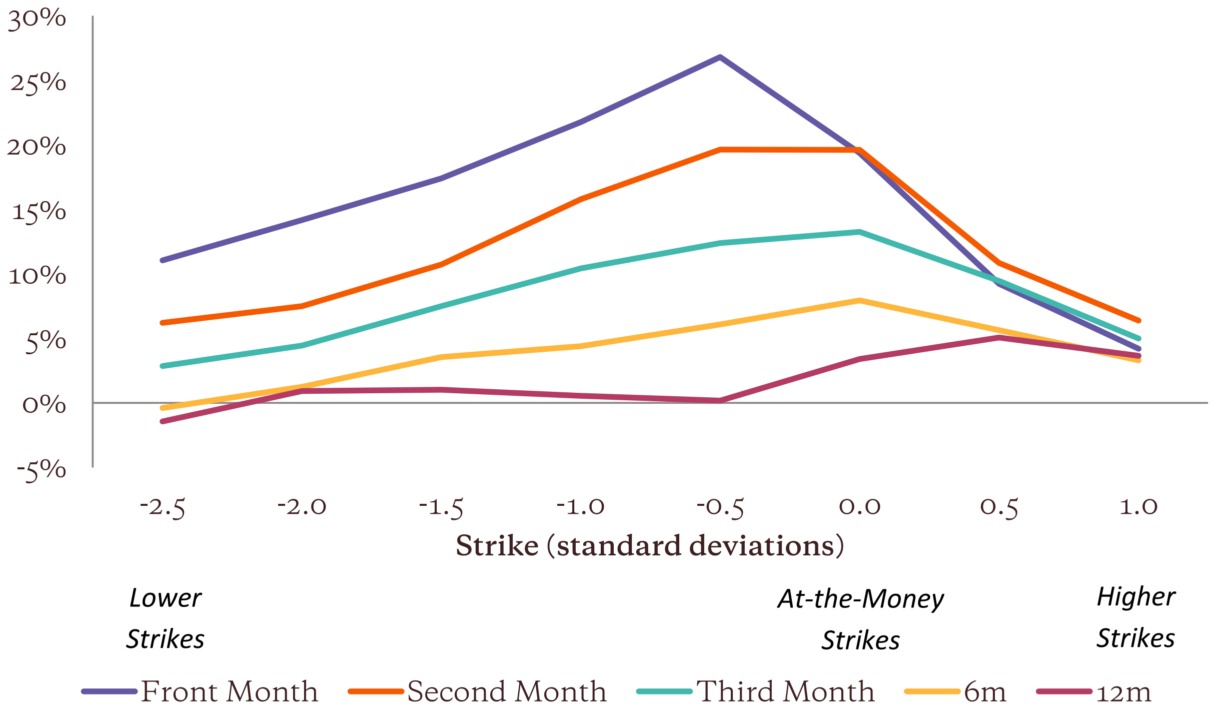
1996-2015



We bucket S&P 500 options on each date by moneyness (standard deviations) and time until expiration. Then for each option in a bucket, on each day we calculate the next day's return of a delta-hedged portfolio that held just one contract of that option, and we average across the options in a bucket to get one "delta-hedged return" value per bucket per date. For each bucket's return series, we then compute the full-period beta to the S&P 500. We then define "alpha" on each day as "delta-hedged return" minus full-period beta * S&P 500 return. For each bucket's series, we then divide the annualized average alpha by the volatility of alphas to generate a Volatility Appraisal Ratio (VAR), which is shown in the chart above. In the legend, '6m' refers to the option maturity nearest 6 months to expiration, and '12m' refers to the option maturity nearest 12 months to expiration.

The data source is OptionMetrics. For illustrative purposes only.

Figure 9: Average Stress-Test Appraisal Ratio (STAR) Across Surface
1996-2015

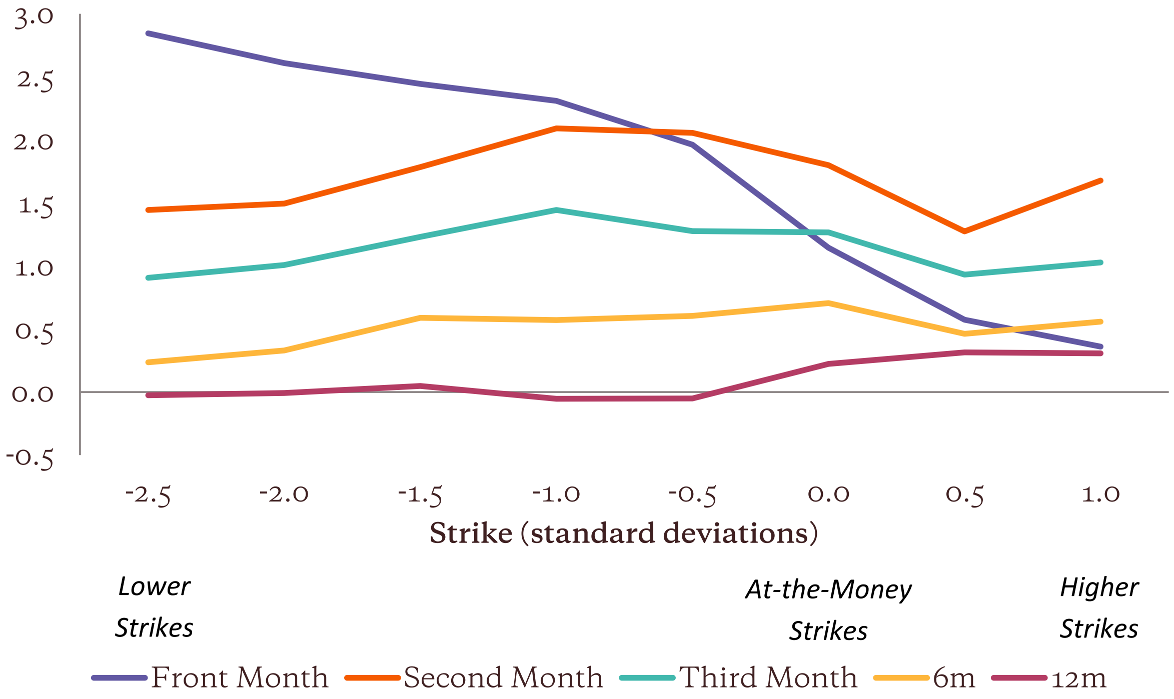


We bucket S&P 500 options on each date by moneyness (standard deviations) and time until expiration. Then for each option in a bucket, on each day we calculate the next day's return of a delta-hedged portfolio that held just one contract of that option, and we average across the options in a bucket to get one "delta-hedged return" value per bucket per date. For each bucket's return series, we then compute the full-period beta to the S&P 500. We then define "alpha" on each day as "delta-hedged return" minus full-period beta * S&P 500 return. Then for each option in a bucket, on each day we calculate the stress-test loss of a beta-adjusted portfolio that held just one contract of that option, and we average across the options in a bucket to get one "stress-test loss" value per bucket per date. For each bucket on each date we then divide the annualized alpha by the stress-test loss to generate a Stress-Test Appraisal Ratio (STAR), and report the full-period average of this ratio in the chart above. In the legend, '6m' refers to the option maturity nearest 6 months to expiration, and '12m' refers to the option maturity nearest 12 months to expiration.

Stress-test loss is defined as the worst simulated one-day loss during 2 extreme one-day shock scenarios:
1. -20% S&P 500 crash with +60% IV spike
2. +20% S&P 500 rally with +20% IV spike

The data source is OptionMetrics. For illustrative purposes only.

Figure 10: Average 0.1% Conditional Value at Risk Appraisal Ratio (CVaRAR) Across Surface
1996-2015

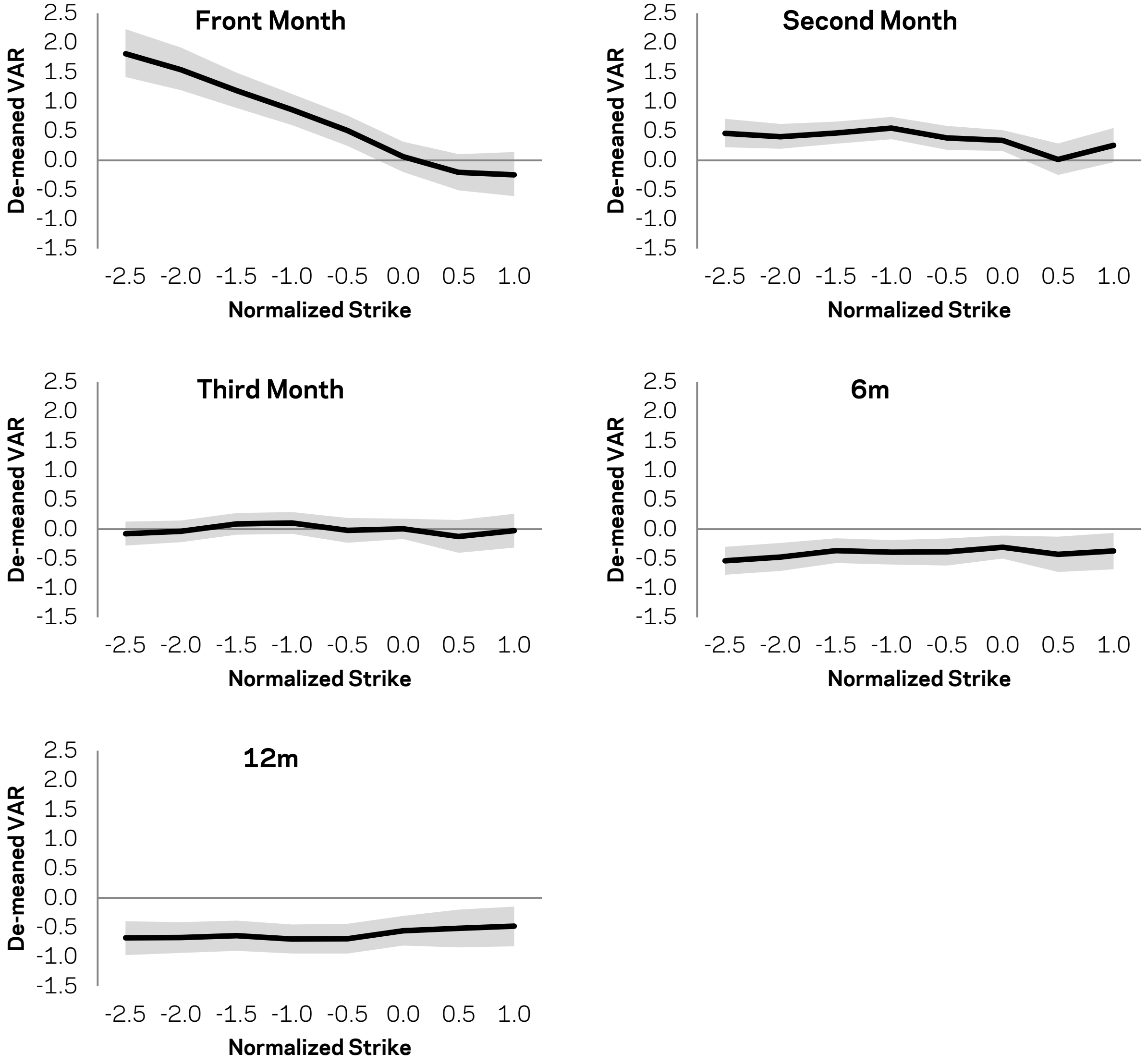


We bucket S&P 500 options on each date by moneyness (standard deviations) and time until expiration. Then for each option in a bucket, on each day we calculate the next day's return of a delta-hedged portfolio that held just one contract of that option, and we average across the options in a bucket to get one "delta-hedged return" value per bucket per date. For each bucket's return series, we then compute the full-period beta to the S&P 500. We then define "alpha" on each day as "delta-hedged return" minus full-period beta * S&P 500 return. We then divide the average annualized alpha by the 0.1% CVaR or alphas to generate a Conditional Value at Risk Appraisal Ratio (CVaRAR), and repot the full-period average of this ratio in the chart above. In the legend, '6m' refers to the option maturity nearest 6 months to expiration, and '12m' refers to the option maturity nearest 12 months to expiration.

The data source is OptionMetrics. For illustrative purposes only.

Figure 11: Surface De-meaned Volatility Appraisal Ratio (VAR) Distribution

1996-2015



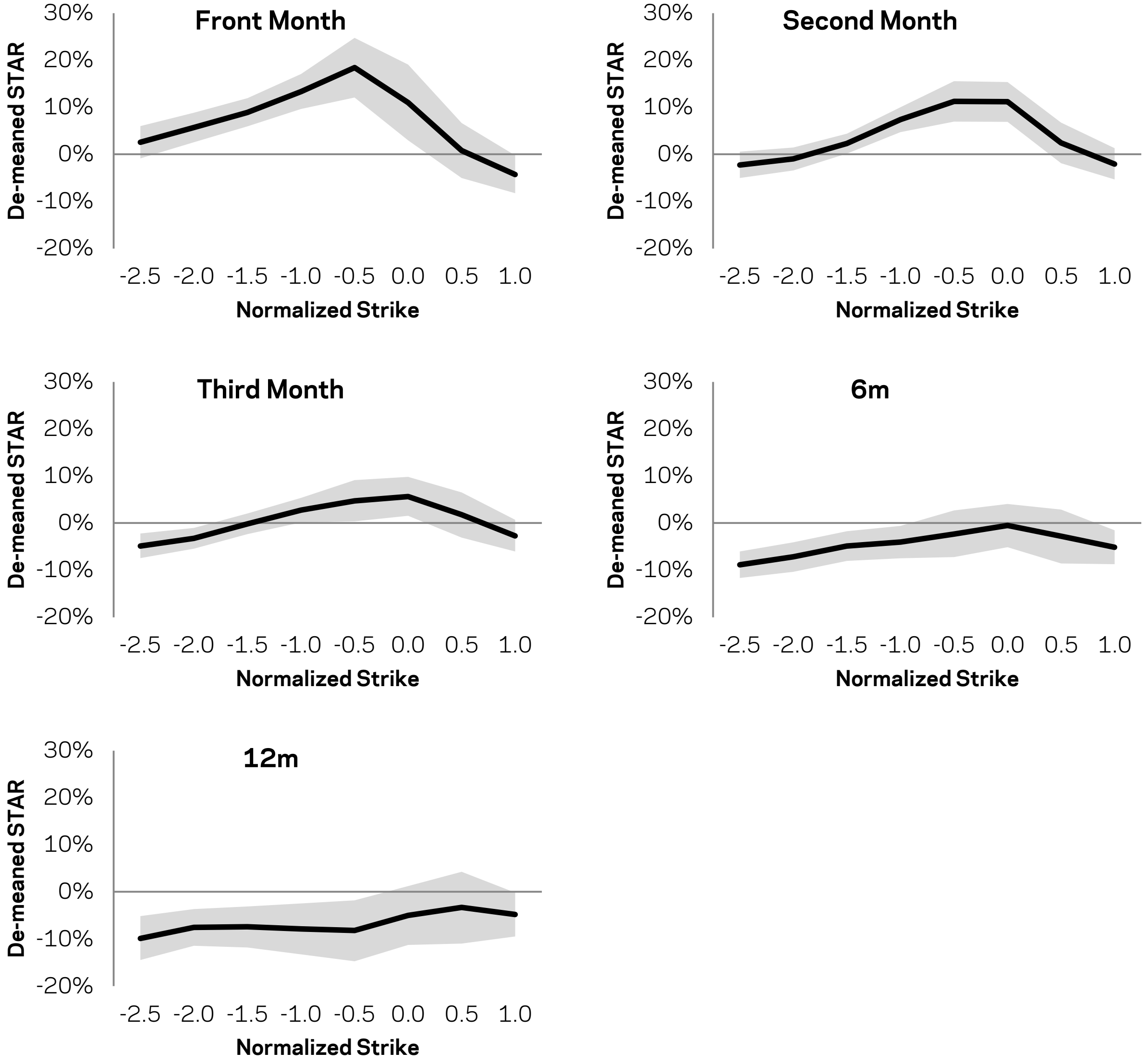
We bucket S&P 500 options on each date by moneyness (standard deviations) and time until expiration. Then for each option in a bucket, on each day we calculate the next day's return of a delta-hedged portfolio that held just one contract of that option, and we average across the options in a bucket to get one "delta-hedged return" value per bucket per date.

We then run 10,000 bootstraps. For each bootstrap we compute the beta of each bucket's return series to the S&P 500. We then define "alpha" on each day as "delta-hedged return" minus full-period beta * S&P 500 return. For each bootstrap, we divide the annualized alpha by the volatility of alphas to calculate a Volatility Appraisal Ratio (VAR) for each bucket. We then report the average cross-sectionally de-meaned VAR across bootstraps as a solid line and the 90% confidence interval as the shaded region.

In the chart titles, '6m' refers to the option maturity nearest 6 months to expiration, and '12m' refers to the option maturity nearest 12 months to expiration. The data source is OptionMetrics. For illustrative purposes only.

Figure 12: Surface De-meaned Stress-Test Appraisal Ratio (STAR) Distribution

1996-2015



Front Month

Second Month

Third Month

6m

12m

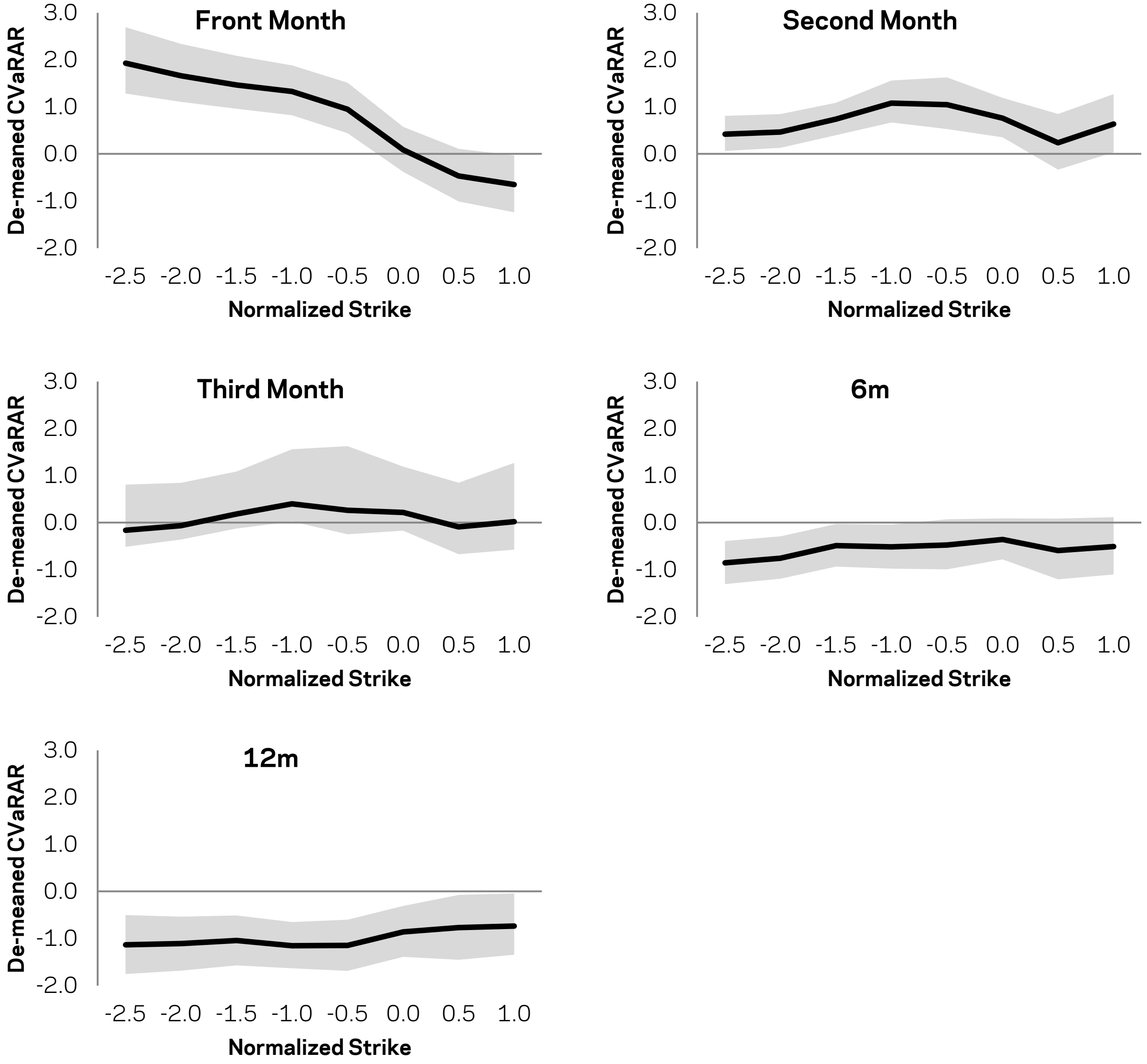
We bucket S&P 500 options on each date by moneyness (standard deviations) and time until expiration. Then for each option in a bucket, on each day we calculate the next day's return of a delta-hedged portfolio that held just one contract of that option, and we average across the options in a bucket to get one "delta-hedged return" value per bucket per date. For each option in a bucket, on each day we also calculate the stress-test loss of a beta-adjusted portfolio that held just one contract of that option, and we average across the options in a bucket to get one "stress-test loss" value per bucket per date.

We then run 10,000 bootstraps. For each bootstrap we compute the beta of each bucket's return series to the S&P 500. We then define "alpha" on each day as "delta-hedged return" minus full-period beta * S&P 500 return. On each date, we divide the annualized alpha by the stress-test loss to calculate a Stress-Test Appraisal Ratio (STAR) per bucket per date. We then calculate the time series average of cross-sectionally de-meaned STARs. We report the average cross-sectionally de-meaned STAR across bootstraps as a solid line and the 90% confidence interval as the shaded region.

In the chart titles, '6m' refers to the option maturity nearest 6 months to expiration, and '12m' refers to the option maturity nearest 12 months to expiration. The data source is OptionMetrics. For illustrative purposes only.

Figure 13: Surface De-meaned Conditional Value at Risk Appraisal Ratio (CVaRAR) Distribution

1996-2015

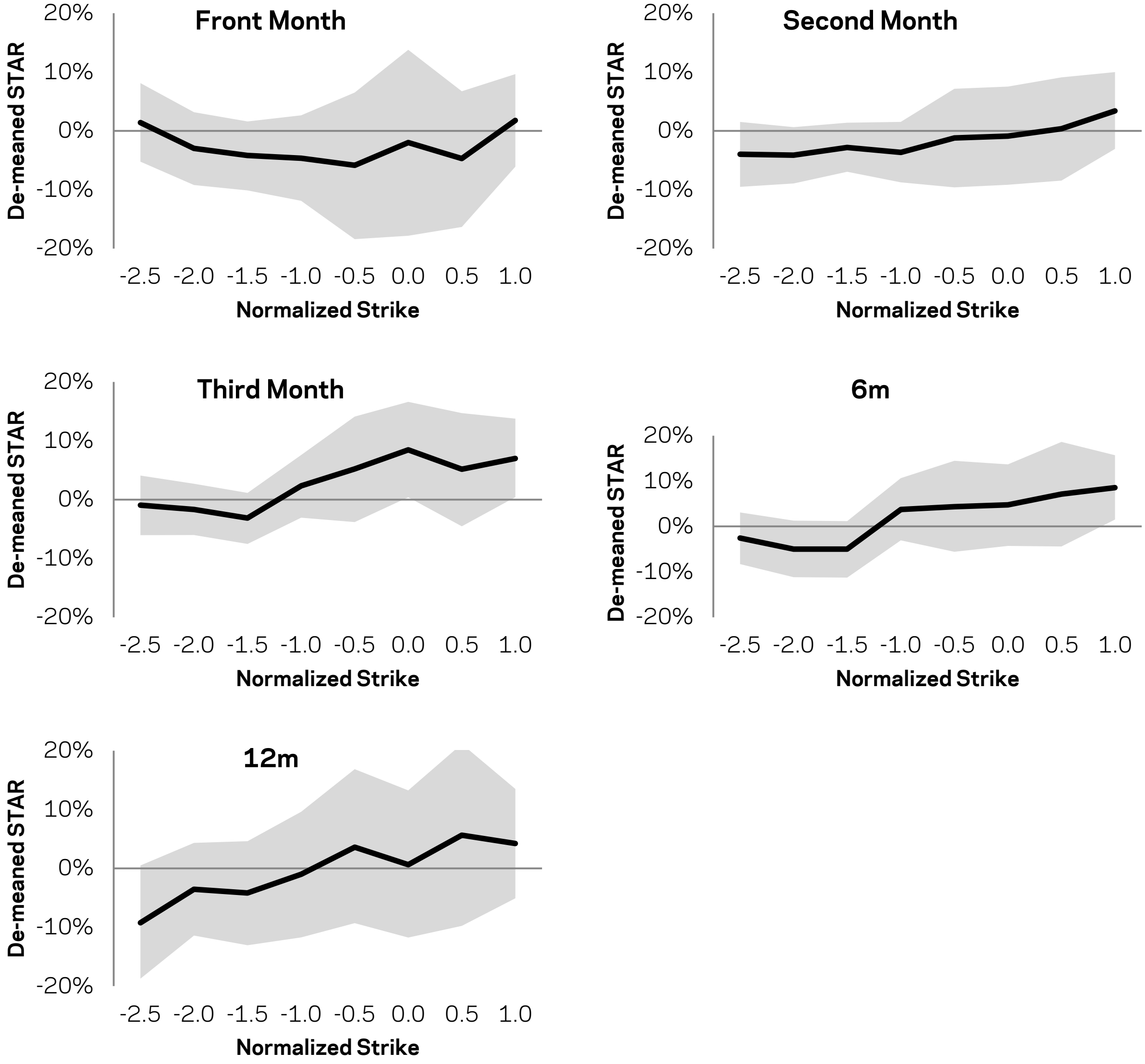


We bucket S&P 500 options on each date by moneyness (standard deviations) and time until expiration. Then for each option in a bucket, on each day we calculate the next day's return of a delta-hedged portfolio that held just one contract of that option, and we average across the options in a bucket to get one "delta-hedged return" value per bucket per date.

We then run 10,000 bootstraps. For each bootstrap we compute the beta of each bucket's return series to the S&P 500. We then define "alpha" on each day as "delta-hedged return" minus full-period beta * S&P 500 return. For each bootstrap, we divide the annualized alpha by the 0.1% CVaR of alphas to calculate a Conditional Value at Risk Appraisal Ratio (CVaRAR) for each bucket. We then report the average cross-sectionally de-meaned CVaRAR across bootstraps as a solid line and the 90% confidence interval as the shaded region.

In the chart titles, '6m' refers to the option maturity nearest 6 months to expiration, and '12m' refers to the option maturity nearest 12 months to expiration. The data source is OptionMetrics. For illustrative purposes only.

Figure 14: Surface De-meaned Stress-Test Appraisal Ratio (STAR) Subsample Analysis
Difference between Second Half (2006-2015) and First Half (1996-2005) of Sample


Front Month

Second Month

Third Month

6m

12m

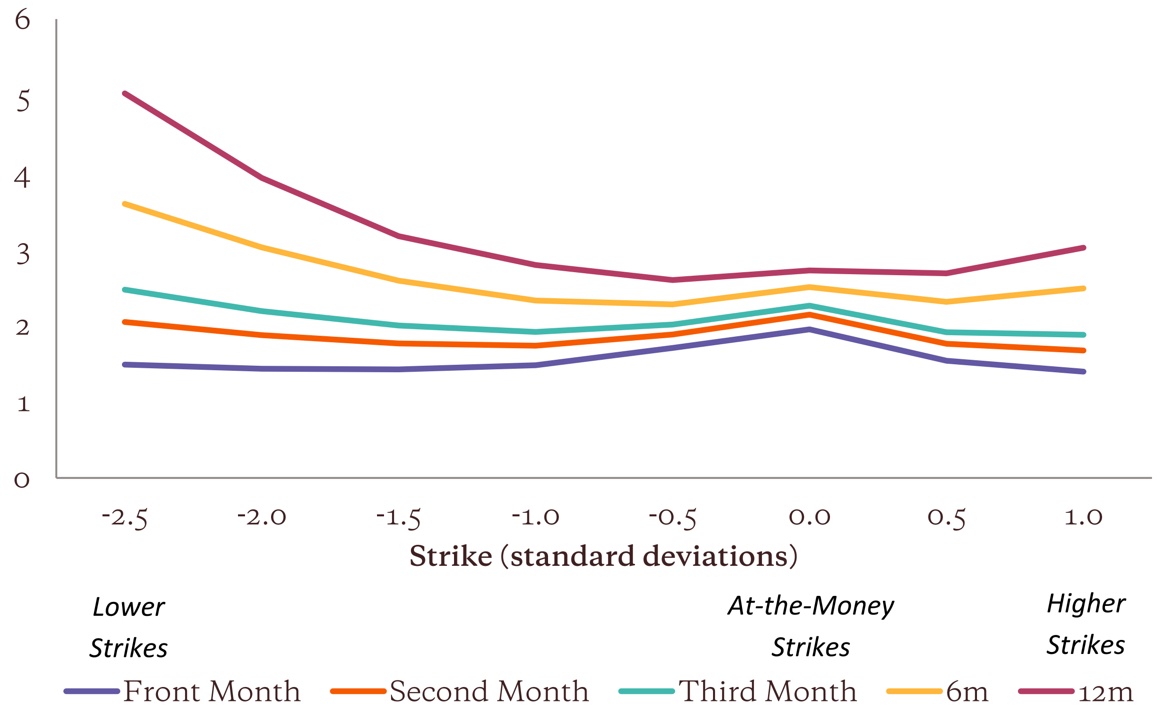
We bucket S&P 500 options on each date by moneyness (standard deviations) and time until expiration. Then for each option in a bucket, on each day we calculate the next day's return of a delta-hedged portfolio that held just one contract of that option, and we average across the options in a bucket to get one "delta-hedged return" value per bucket per date. For each option in a bucket, on each day we also calculate the stress-test loss of a beta-adjusted portfolio that held just one contract of that option, and we average across the options in a bucket to get one "stress-test loss" value per bucket per date.

We split the sample into two periods: 1996 – 2005 and 2006 – 2015. We run 10,000 bootstraps to form a distribution of difference in average cross-sectionally de-meaned Stress-Test Appraisal Ratios (STARs) between the two periods. For each bootstrap we compute the sub-period beta of each bucket's return series to the S&P 500. We then define "alpha" on each day as "delta-hedged return" minus full-period beta * S&P 500 return. On each date, we divide the annualized alpha by the stress-test loss to calculate a STAR per bucket per date. We then calculate each sub-period's time series average of cross-sectionally de-meaned STARs. Lastly we take the difference between each sub-period's cross-sectionally de-meaned average STAR. We report the average difference as a solid line and the 90% confidence interval as the shaded region.

In the chart titles, '6m' refers to the option maturity nearest 6 months to expiration, and '12m' refers to the option maturity nearest 12 months to expiration. The data source is OptionMetrics. For illustrative purposes only.

Figure 15: Average Option Notional Leverage for 20% Stress-Test Loss Budget Across Surface

1996-2015



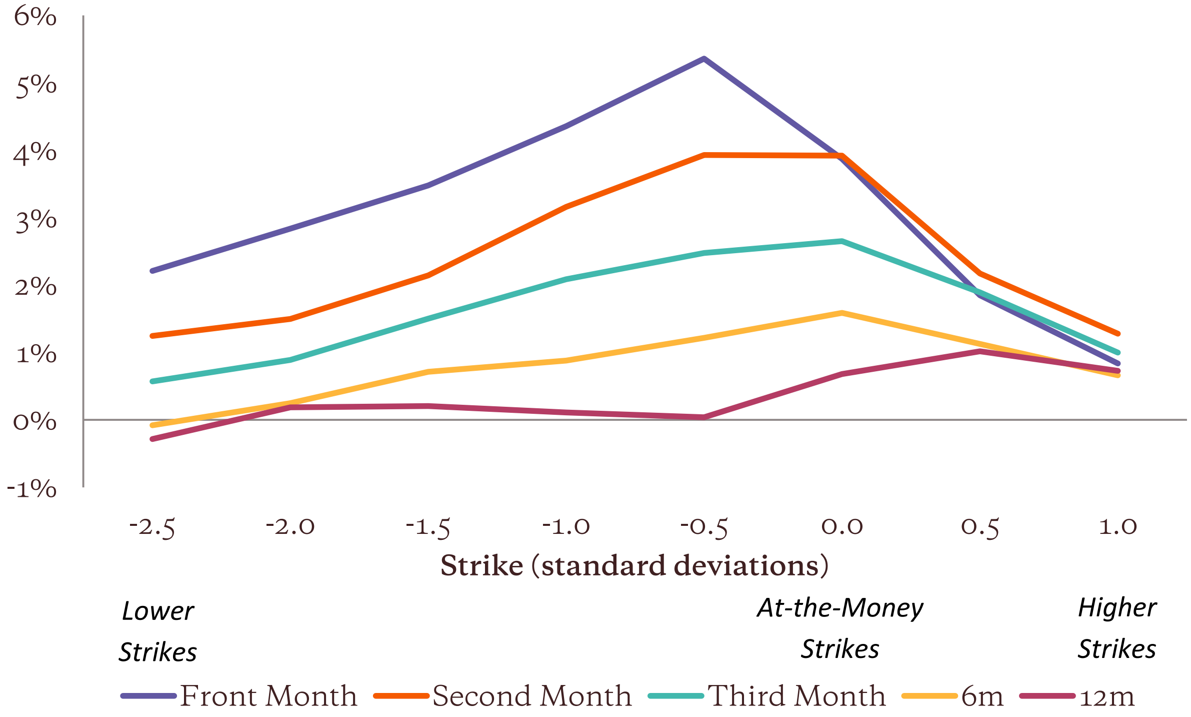
We bucket S&P 500 options on each date by moneyness (standard deviations) and time until expiration. Then for each option in a bucket, on each day we calculate the stress-test loss of a beta-adjusted portfolio that held just one contract of that option. Using this, we derive the leverage that would be needed for the portfolio to attain a 20% stress-test loss, and we average these values across the options in a bucket to get one "leverage" value per bucket per date. For each bucket, this series is then averaged over time to produce the chart above. In the legend, '6m' refers to the option maturity nearest 6 months to expiration, and '12m' refers to the option maturity nearest 12 months to expiration.

Stress-test loss is defined as the worst simulated one-day loss during 2 extreme one-day shock scenarios:
1. -20% S&P 500 crash with +60% IV spike
2. +20% S&P 500 rally with +20% IV spike

The data source is OptionMetrics. For illustrative purposes only.

Figure 16: Average Annualized Short Option Alpha for 20% Stress-Test Loss Budget Across Surface

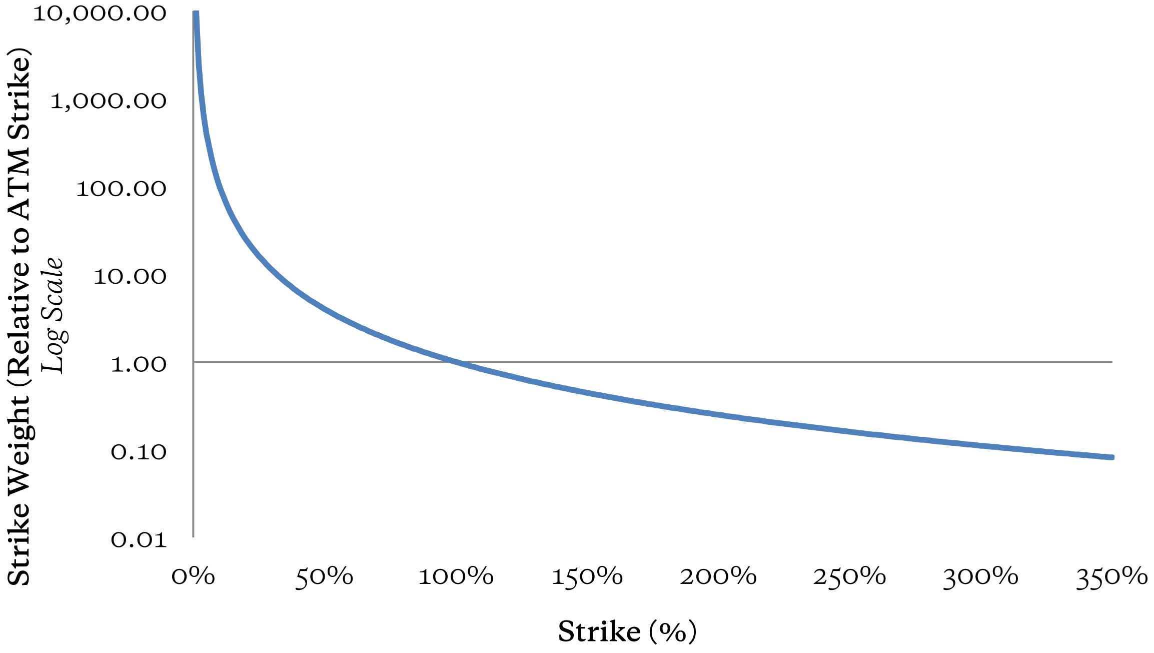
1996-2015



We bucket S&P 500 options on each date by moneyness (standard deviations) and time until expiration. Then for each option in a bucket, on each day we calculate the next day's annualized return of a delta-hedged portfolio that held just one contract of that option, and we average across the options in a bucket to get one "delta-hedged return" value per bucket per date. For each bucket's return series, we then compute the full-period beta to the S&P 500. We then define "alpha" on each day as "delta-hedged return" minus full-period beta * S&P 500 return. Then for each option in a bucket, on each day we calculate the stress-test loss of a beta-adjusted portfolio that held just one contract of that option, and we average across the options in a bucket to get one "stress-test loss" value per bucket per date. For each bucket on each date we then divide the annualized alpha by the stress-test loss to calculate a Stress-Test Appraisal Ratio (STAR), multiply by 20%, and report the full-period average in the chart above. In the legend, '6m' refers to the option maturity nearest 6 months to expiration, and '12m' refers to the option maturity nearest 12 months to expiration.

The data source is OptionMetrics. For illustrative purposes only.

Figure 17: Illustrative Variance Swap Strike Weighting



A variance swap is equivalent to a portfolio of options across an infinite range of strikes, with 1/K^2 weighting. We show an illustrative relative weighting across these strikes from 1% of the current index value to 350% of the current index value using 1% strike increments. By definition, weight on the 100% at-the-money strike is weighted with a weight of 1.

Chart is for illustrative purposes only.