ADDITIVE VERSUS MULTIPLICATIVE

... and how about interactions, etc.?

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REFER TO PREVIOUS EXAMPLE ON MORTALITY...

$$
\begin{aligned}
\text{RD} &= p_2 - p_1 = 0.093 \\
\text{RR} &= \frac{p_2}{p_1} = 1.94 \\
\text{OR} &= \frac{p_2/(1-p_2)}{p_1/(1-p_1)} = \frac{p_2}{p_1} \cdot \frac{1-p_1}{1-p_2} = 1.94 \cdot 1.11 = 2.16
\end{aligned}
$$

NOTE:

- RR and OR are very close when prevalence is low (say, less than 10%). This is because both $1 - p_1 \approx 1$ and $1 - p_2 \approx 1$.
- OR always farther away from 1 than RR (whether above or below 1). Here: $\frac{1-p_1}{1-p_2} = 1.11 > 1$

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THREE MOST IMPORTANT MEASURES FOR GROUP COMPARISON

1. Risk Difference (RD):
   - Intuitive (percentage points difference)
   - High public health relevance

2. Relative Risk (RR)
   - Intuitive (percentage increase in risk)
   - High relevance for risk assessment in research

3. Odds Ratio (OR)
   - Percentage increase in odds
   - Of high practical utility and relevance in research
   - Applicable also in **case-control studies**

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How do they handle selection?

	Males	**Females**	Total
X (deaths)	398	188	586
n - X (non-deaths)	1688	1726	3414
n (total)	2086	1914	4000

Select 50% of males:

	Males	**Females**	Total
X (deaths)	199	188	387
n - X (non-deaths)	844	1726	2570
n (total)	1043	1914	2957

	Before	After
RD	0.093	0.093
RR	1.94	1.94
OR	2.16	2.16

But remember:

- Confidence intervals get wider
- I.e. less significant difference

How do they handle selection?

	Males	**Females**	Total
X (deaths)	398	188	586
n - X (non-deaths)	1688	1726	3414
n (total)	2086	1914	4000

Select 50% of controls (non-deaths):

	Males	**Females**	Total
X (deaths)	398	188	586
n - X (non-deaths)	844	863	1707
n (total)	1242	1051	2293

	Before	After
RD	0.093	0.14
RR	1.94	1.79
OR	2.16	2.16

Thus:

- OR is unbiased
- RD and RR are biased

FLIP THE COIN

What if we want to compare *survival* probabilities, not death?

	Males	**Females**	Total
X (deaths)	398	188	586
n - X (non-deaths)	1688	1726	3414
n (total)	2086	1914	4000

Females versus males:

$$\begin{aligned} \text{RD} &= (1-p_1)-(1-p_2) = \frac{1726}{1914} - \frac{1688}{2086} = 0.093 \\ \text{RR} &= \frac{1-p_1}{1-p_2} = \frac{1726/1914}{1688/2086} = 1.11 \\ \text{OR} &= \frac{(1-p_1)/p_1}{(1-p_2)/p_2} = \frac{p_2}{p_1} \cdot \frac{1-p_1}{1-p_2} = 1.94 \cdot 1.11 = 2.16 \end{aligned}$$

RD and OR unaffected!

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graphs $-$ $>$

Myth I

"For high prevalences, OR cannot be used, only RR is correct"

Wrong!

- For instance, when $\mathrm{p}_1 > 0.5$, then $\mathrm{RR} = 2$ is mathematically impossible.
- RR gets squeezed down to 1 when p_1 increases, since p_2 hits the roof of 1.
- So for very large p_1 you get $\mathrm{RR} = 1.01$ and similarly uninformative results.
- Estimation of the RR with binomial regression becomes increasingly unstable when p_1 increases.

OR handles all this very nicely.

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Myth II

"Using OR is cheating, since it's bigger than RR"

Wrong!

- If you state that it is the OR, then there's no cheating.
- The significance is the same (at least for two groups).
- In a case-control study, you would need to use the OR anyway.
- The "true" reason for presenting the RR is usually slight ease of interpretation since "people" are more familiar with risk than odds.

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Myth III

"Additive risk scale is biologically correct"

Wrong!

- There is nothing inherently biological about an additiv risk scale.
- However, choice of scale *will* have implications for for instance causal inference, and an additive scale is frequently used.

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SIGH...

Figur F001: Sammenhengen mellom samfunnets endringshastighet og omstillingskrav overfor individ og samfunn

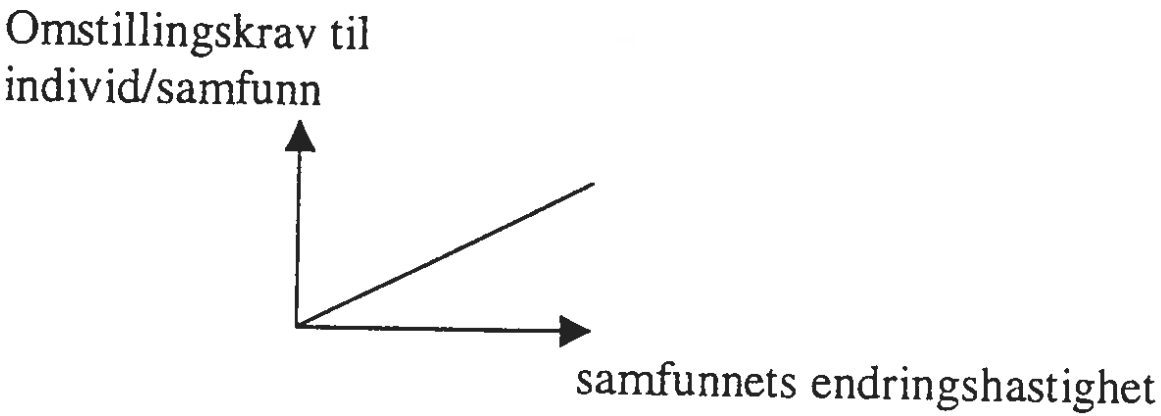

Fig F001: Hypotese: omstillingskravene antas å øke proporsjonalt ved samfunnets endringshastighet

Scale dependence

- Measurement scale dependence pervades all of statistical modeling
- “Problem without solution” (at least no simple, canonical solution)
- Results will depend on measurement scale
- Some methods are robust to monotone scale change, such as methods based on observation rank, i.e. many non-parametric methods.

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Pragmatic viewpoint

1. Fit a statistical model:
   - Incorporate structural relationships, such as growth curves, additive, linear structures, etc.
   - Take into consideration causal aspects, confounders, interactions etc.
   - Add a stochastic model for random variability (i.e. how randomness is generated)
2. Check if model fits to data.
3. If model fits, interpret data in light of model.
4. Problem:
   - If two different plausible models with different interpretations both fit, what then?
   - Perhaps too little data to distinguish

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