

## EXERCISES

Find all the zeros of the function. State the multiplicity of each zero.

1.  $f(x) = x(x-1)(x+2)$

2.  $f(x) = -x(x+5)(x-4)$

3.  $f(x) = x^2(2x+1)(3x-6)^3$

4.  $f(x) = (x+4)^2(x^2-9)^4$

5.  $p(x) = (x^2+x-12)^3(9x^2-1)^2$

6.  $p(x) = (2x+3)(4x^2-9)^5$

7.  $p(x) = (x^2+3x+2)^2(x+2)^3$

8.  $p(x) = x^4 - 5x^2 - 36$

9.  $f(x) = x^2(x^2-1)(x^2+1)^3$

10.  $f(x) = 3x^3(x-4)^2(x^2-16)$

Find the polynomial equation  $f(x) = 0$  that has the indicated zeros.

11.  $x = 1, -4$

12.  $x = -1, 4$

13.  $x = -2, 5$

14.  $x = 2, -5$

15.  $x = 0, 2, -3$

16.  $x = 0, 3, -2$

17.  $x = -4, -5, 2$

18.  $x = 2i, -2i, 1$

19.  $x = 4i, -4i, -3$ (multiplicity 2)

20.  $x = 1+i, 1-i, 2$ (multiplicity 3)

Find the polynomial function  $f(x)$  that has the indicated zeros and satisfies the given condition. (Hint: Find  $a$  from  $f(x) = a(x-r_1)(x-r_2)(x-r_3)\cdots$ ).

21.  $x = 1, -4$ ;  $f(2) = 18$

22.  $x = -1, 4$ ;  $f(-3) = 28$

23.  $x = -2, 5$ ;  $f(1) = -24$

24.  $x = 2, -5$ ;  $f(-2) = -6$

25.  $x = 0, 2, -3$ ;  $f(3) = -6$

26.  $x = 0, 3, -2$ ;  $f(4) = 24$

27.  $x = -4, -5, 2$ ;  $f(-1) = 72$

28.  $x = 2i, -2i, 1$ ;  $f(2) = 48$

29.  $x = 4i, -4i, -3$ (multiplicity 2);  $f(-2) = 40$

30.  $x = 1+i, 1-i, 2$ (multiplicity 3);  $f(3) = 30$

Use Descartes' Rule of Signs to determine the number of possible positive, negative, and complex solutions (with imaginary roots) of the equation.

31.  $x^3 - x^2 - 6x - 8 = 0$ .

32.  $x^3 + 6x^2 + 11x + 6 = 0$ .

33.  $x^4 - x^3 - 5x^2 - x - 6 = 0$ .

34.  $x^5 - 2x^3 - x + 6 = 0$ .

35.  $t^7 - 6 = 0$ .

36.  $y^8 - 9 = 0$ .

37. What is the maximum number of points can the graph of  $y = x^5 - x^4 + 3x^3 - 2x^2 + x$  cross the  $x$ -axis?

Use synthetic division and the rules on upper and lower bounds to determine if each value of  $x$  is an upper bound, lower bound, or neither of the roots of the equation.

38.  $x^2 + 2x - 3 = 0$ .

a)  $x = -3$

b)  $x = -1$

c)  $x = 1$

39.  $x^3 + 6x^2 + 11x + 6 = 0$ .

a)  $x = -7$

b)  $x = -6$

c)  $x = 1$

40.  $2x^3 + 9x^2 + 7x - 6 = 0$ .

a)  $x = -5$

b)  $x = -3$

c)  $x = 0$

41.  $x^4 - 2x^3 - 23x^2 - 12x + 36 = 0$ .

a)  $x = -7$

b)  $x = -3$

c)  $x = 6$

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Find the smallest and largest integers that are upper and lower bound on the real roots (zeros).

42.  $x^3 + 7x^2 + 2x - 40 = 0$

43.  $x^4 - 2x^3 - 23x^2 - 12x + 36 = 0$

44.  $p(x) = x^4 - 4x^2 - 5$

45.  $f(x) = 2x^3 + 5x^2 - x - 4$

Identify whether each function or relation is even, odd, or neither.

46.  $f(x) = 4x^2$

47.  $f(x) = 5x^3$

48.  $f(x) = \frac{5}{x}$

49.  $y - x = 0$

50.  $f(x) = 4x^2 - 4$

51.  $f(x) = 5x^3 - 4$

52.  $x^2 - y^2 = 4$

53.  $f = \{(1, 2), (3, 6), (0, 0), (-1, -2), (-3, -6)\}$

Identify the general shape of the graph of  $y = f(x)$ .

54.  $y = x^3$

55.  $y = -x^3 + 3x^2 - 3x + 1$

56.  $y = -x^4$

57.  $y = x^3 + x^2 - 1$

58.  $y = x^4 + x^2 - 1$

59.  $y = \frac{1}{4}x^4$

60.  $y = x^5 + x^3 - x$

Solve each polynomial equation in the complex number system.

61.  $x^2 - 9 = 0$

62.  $x^2 + 9 = 0$

63.  $x^2 - 16 = 0$

64.  $x^2 + 16 = 0$

65.  $x^2 + 64 = 0$

66.  $x^2 - 64 = 0$

67.  $2x^2 - 16 = 0$

68.  $x^2 - 4x = 0$

69.  $3x^2 + 5x - 2 = 0$

70.  $x^2 + 4x + 8 = 0$

71.  $x^2 - 6x + 10 = 0$

72.  $x^2 + x - 1 = 0$

73.  $x^2 + x + 1 = 0$

74.  $x^2 + x + 5 = 0$

75.  $x^2 + 6x + 11 = 0$

76.  $x^2 - 2x + 5 = 0$

77.  $3x^2 + 2x - 2 = 0$

78.  $3x^2 + 2x + 2 = 0$

79.  $5x^2 - 2x + 1 = 0$

80.  $x^3 - 8 = 0$

81.  $x^2 + 8 = 0$

82.  $x^4 - 16 = 0$

83.  $x^4 - 1 = 0$

84.  $x^3 - 7x^2 + 16x - 10 = 0$

85.  $x^3 + x^2 - x + 15 = 0$

86.  $3x^3 + 5x^2 - 2 = 0$

87.  $5x^3 - 12x^2 + 5x - 2 = 0$

88.  $x^4 + 3x^2 - 4 = 0$

89.  $x^4 + 4x^3 + 2x^2 - x + 6 = 0$

90.  $2x^4 + 11x^3 - 6x^2 + 64x + 32 = 0$

91. Solve  $x^3 - 4x^2 + 2x - 8 = 0$  if one root is  $\sqrt{2}i$ .

92. Solve  $x^3 - 3x^2 + 7x - 5 = 0$  if one root is  $1 - 2i$ .

93. Solve  $x^4 + 2x^2 - 8 = 0$  if one root is  $2i$ .

94. Solve  $x^4 - x^3 + 4x^2 - 3x + 3 = 0$  if one root is  $\frac{1+\sqrt{3}i}{2}$ .

95. Is 3 a zero of  $f(x) = x^5 - 3x^4 + 5x^3 + 2x^2 - x + 7$ ?

96. Is  $\frac{2}{3}$  a zero of  $f(x) = 2x^4 + 5x^3 - 2x^2 - 4x + 1$ ?

97. A quadratic equation with real coefficients has one root  $4 + i$ . Find such an equation.

98. A cubic equation with real coefficients has two roots 3 and  $1 - 2i$ . Find such an equation.

99. Prove that the function  $f(x) = x^3 + x^2 + x - 5$  has a zero in the interval  $[1, 2]$ .

100. Prove that the function  $f(x) = x^4 + x^2 - 1$  has a zero in the interval  $[-1, 0]$ , and a zero in the interval  $[0, -1]$ .

101. Estimate the zeros of  $f(x) = x^3 + x^2 + x - 5$  correct to two decimal places.

102. Estimate the zeros of  $f(x) = x^4 + x^2 - 1$  correct to two decimal places.

103. If  $\frac{2}{3}$  is one root of  $9x^2 + bx + 8 = 0$ , find the other root and the value of  $b$ .

104. Find the sum and the product of the roots of  $x^3 + 6x^2 + 11x + 6 = 0$ .

105. Find the sum of the product of the roots of  $2x^3 + x^2 - 10x - 5 = 0$ , take 2 roots at a time.