

5.1 Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

Vocabulary Check

Fill in the blank to complete the trigonometric identity.

1. $\frac{1}{\cos u} = \underline{\hspace{2cm}}$

2. $\frac{1}{\cot u} = \underline{\hspace{2cm}}$

3. $\frac{\cos u}{\sin u} = \underline{\hspace{2cm}}$

4. $\frac{1}{\sin u} = \underline{\hspace{2cm}}$

5. $1 + \underline{\hspace{2cm}} = \sec^2 u$

6. $1 + \cot^2 u = \underline{\hspace{2cm}}$

7. $\cos\left(\frac{\pi}{2} - u\right) = \underline{\hspace{2cm}}$

8. $\csc\left(\frac{\pi}{2} - u\right) = \underline{\hspace{2cm}}$

9. $\tan(-u) = \underline{\hspace{2cm}}$

10. $\cos(-u) = \underline{\hspace{2cm}}$

In Exercises 1–14, use the given values to evaluate (if possible) all six trigonometric functions.

1. $\sin x = \frac{1}{2}, \cos x = \frac{\sqrt{3}}{2}$

2. $\csc \theta = 2, \tan \theta = \frac{\sqrt{3}}{3}$

3. $\sec \theta = \sqrt{2}, \sin \theta = -\frac{\sqrt{2}}{2}$

4. $\tan x = \frac{\sqrt{3}}{3}, \cos x = -\frac{\sqrt{3}}{2}$

5. $\tan x = \frac{7}{24}, \sec x = -\frac{25}{24}$

6. $\cot \phi = -5, \sin \phi = \frac{\sqrt{26}}{26}$

7. $\sec \phi = -\frac{17}{15}, \sin \phi = \frac{8}{17}$

8. $\cos\left(\frac{\pi}{2} - x\right) = \frac{3}{5}, \cos x = \frac{4}{5}$

9. $\sin(-x) = -\frac{2}{3}, \tan x = -\frac{2\sqrt{5}}{5}$

10. $\csc(-x) = -5, \cos x = \frac{\sqrt{24}}{5}$

11. $\tan \theta = 2, \sin \theta < 0$

12. $\sec \theta = -3, \tan \theta < 0$

13. $\csc \theta$ is undefined, $\cos \theta < 0$

14. $\tan \theta$ is undefined, $\sin \theta > 0$

In Exercises 15–20, match the trigonometric expression with one of the following.

(a) $\sec x$

(b) -1

(c) $\cot x$

(d) 1

(e) $-\tan x$

(f) $\sin x$

15. $\sec x \cos x$

16. $\tan x \csc x$

17. $\cot^2 x - \csc^2 x$

18. $(1 - \cos^2 x)(\csc x)$

19. $\frac{\sin(-x)}{\cos(-x)}$

20. $\frac{\sin[(\pi/2) - x]}{\cos[(\pi/2) - x]}$

In Exercises 21–26, match the trigonometric expression with one of the following.

(a) $\csc x$

(b) $\tan x$

(c) $\sin^2 x$

(d) $\sin x \tan x$

(e) $\sec^2 x$

(f) $\sec^2 x + \tan^2 x$

21. $\sin x \sec x$

22. $\cos^2 x(\sec^2 x - 1)$

23. $\sec^4 x - \tan^4 x$

24. $\cot x \sec x$

25. $\frac{\sec^2 x - 1}{\sin^2 x}$

26. $\frac{\cos^2[(\pi/2) - x]}{\cos x}$

In Exercises 27–38, use the fundamental identities to simplify the expression. Use the *table* feature of a graphing utility to check your result numerically.

27. $\cot x \sin x$

28. $\cos \beta \tan \beta$

29. $\sin \phi(\csc \phi - \sin \phi)$

30. $\sec^2 x(1 - \sin^2 x)$

31. $\frac{\csc x}{\cot x}$

32. $\frac{\sec \theta}{\csc \theta}$

33. $\sec \alpha \cdot \frac{\sin \alpha}{\tan \alpha}$

34. $\frac{\tan^2 \theta}{\sec^2 \theta}$

35. $\sin\left(\frac{\pi}{2} - x\right)\csc x$

36. $\cot\left(\frac{\pi}{2} - x\right)\cos x$

37. $\frac{\cos^2 y}{1 - \sin y}$

38. $\frac{1}{\cot^2 x + 1}$

In Exercises 39–44, verify the identity algebraically. Use the *table* feature of a graphing utility to check your result numerically.

39. $\sin \theta + \cos \theta \cot \theta = \csc \theta$

40. $(\sec \theta - \tan \theta)(\csc \theta + 1) = \cot \theta$

41. $\frac{\cos \theta}{1 - \sin \theta} = \sec \theta + \tan \theta$

42. $\frac{1 + \csc \theta}{\cot \theta + \cos \theta} = \sec \theta$

43. $\frac{1 + \cos \theta}{\sin \theta} + \frac{\sin \theta}{1 + \cos \theta} = 2 \csc \theta$

44. $\frac{\sin \theta + \cos \theta}{\sin \theta} - \frac{\cos \theta - \sin \theta}{\cos \theta} = \sec \theta \csc \theta$

In Exercises 45–50, verify the identity algebraically. Use a graphing utility to check your result graphically.

45. $\csc \theta \tan \theta = \sec \theta$

46. $\sin \theta \csc \theta - \sin^2 \theta = \cos^2 \theta$

47. $1 - \frac{\sin^2 \theta}{1 - \cos \theta} = -\cos \theta$

48. $\frac{\tan \theta}{1 + \sec \theta} + \frac{1 + \sec \theta}{\tan \theta} = 2 \csc \theta$

49. $\frac{\cot(-\theta)}{\csc \theta} = -\cos \theta$

50. $\frac{\csc\left(\frac{\pi}{2} - \theta\right)}{\tan(-\theta)} = -\csc \theta$

In Exercises 51–60, factor the expression and use the fundamental identities to simplify. Use a graphing utility to check your result graphically.

51. $\cot^2 x - \cot^2 x \cos^2 x$

52. $\sec^2 x \tan^2 x + \sec^2 x$

53. $\frac{\cos^2 x - 4}{\cos x - 2}$

54. $\frac{\csc^2 x - 1}{\csc x - 1}$

55. $\tan^4 x + 2 \tan^2 x + 1$

56. $1 - 2 \sin^2 x + \sin^4 x$

57. $\sin^4 x - \cos^4 x$

58. $\sec^4 x - \tan^4 x$

59. $\csc^3 x - \csc^2 x - \csc x + 1$

60. $\sec^3 x - \sec^2 x - \sec x + 1$

In Exercises 61–68, perform the indicated operation and use the fundamental identities to simplify.

61. $(\sin x + \cos x)^2$

62. $(\tan x + \sec x)(\tan x - \sec x)$

63. $(\csc x + 1)(\csc x - 1)$

64. $(5 - 5 \sin x)(5 + 5 \sin x)$

65. $\frac{1}{1 + \cos x} + \frac{1}{1 - \cos x}$

66. $\frac{1}{\sec x + 1} - \frac{1}{\sec x - 1}$

67. $\tan x - \frac{\sec^2 x}{\tan x}$

68. $\frac{\cos x}{1 + \sin x} + \frac{1 + \sin x}{\cos x}$

 In Exercises 69–72, rewrite the expression so that it is *not* in fractional form.

69. $\frac{\sin^2 y}{1 - \cos y}$

70. $\frac{5}{\tan x + \sec x}$

71. $\frac{3}{\sec x - \tan x}$

72. $\frac{\tan^2 x}{\csc x + 1}$

Numerical and Graphical Analysis In Exercises 73–76, use a graphing utility to complete the table and graph the functions in the same viewing window. Make a conjecture about y_1 and y_2 .

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1							
y_2							

73. $y_1 = \cos\left(\frac{\pi}{2} - x\right), \quad y_2 = \sin x$

74. $y_1 = \cos x + \sin x \tan x, \quad y_2 = \sec x$

75. $y_1 = \frac{\cos x}{1 - \sin x}, \quad y_2 = \frac{1 + \sin x}{\cos x}$

76. $y_1 = \sec^4 x - \sec^2 x, \quad y_2 = \tan^2 x + \tan^4 x$

In Exercises 77–80, use a graphing utility to determine which of the six trigonometric functions is equal to the expression.

77. $\cos x \cot x + \sin x$

78. $\sin x(\cot x + \tan x)$

79. $\sec x - \frac{\cos x}{1 + \sin x}$

80. $\frac{1}{2} \left(\frac{1 + \sin \theta}{\cos \theta} + \frac{\cos \theta}{1 + \sin \theta} \right)$