5.4 Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

Vocabulary Check

Fill in the blank to complete the trigonometric formula.

1. $\sin(u - v) =$ _______
2. $\cos(u + v) =$ _______
3. $\tan(u + v) =$ _______
4. $\sin(u + v) =$ _______
5. $\cos(u - v) =$ _______
6. $\tan(u - v) =$ _______

In Exercises 1–6, find the exact value of each expression.

1. (a) $\cos(240° - 0°)$ (b) $\cos 240° - \cos 0°$
2. (a) $\sin(405° + 120°)$ (b) $\sin 405° + \sin 120°$
3. (a) $\cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$ (b) $\cos \frac{\pi}{4} + \cos \frac{\pi}{3}$
4. (a) $\sin\left(\frac{2\pi}{3} + \frac{5\pi}{6}\right)$ (b) $\sin \frac{2\pi}{3} + \sin \frac{5\pi}{6}$
5. (a) $\sin(315° - 60°)$ (b) $\sin 315° - \sin 60°$
6. (a) $\sin\left(\frac{7\pi}{6} - \frac{\pi}{3}\right)$ (b) $\sin \frac{7\pi}{6} - \sin \frac{\pi}{3}$

In Exercises 7–22, find the exact values of the sine, cosine, and tangent of the angle.

7. $105° = 60° + 45°$
8. $165° = 135° + 30°$
9. $195° = 225° - 30°$
10. $255° = 300° - 45°$
11. $\frac{11\pi}{12} = \frac{3\pi}{4} + \frac{\pi}{6}$
12. $\frac{17\pi}{12} = \frac{7\pi}{6} + \frac{\pi}{4}$
13. $-\frac{\pi}{12} = \frac{\pi}{6} - \frac{\pi}{4}$
14. $-\frac{19\pi}{12} = \frac{2\pi}{3} - \frac{9\pi}{4}$
15. $75°$
16. $15°$
17. $-225°$
18. $-165°$
19. $\frac{13\pi}{12}$
20. $\frac{5\pi}{12}$
21. $-\frac{7\pi}{12}$
22. $-\frac{13\pi}{12}$

In Exercises 23–30, write the expression as the sine, cosine, or tangent of an angle.

23. $\cos 60° \cos 20° - \sin 60° \sin 20°$
24. $\sin 110° \cos 80° + \cos 110° \sin 80°$
25. $\frac{\tan 325° - \tan 86°}{1 + \tan 325° \tan 86°}$
26. $\frac{\tan 154° - \tan 49°}{1 + \tan 154° \tan 49°}$
27. $\sin 3.5 \cos 1.2 - \cos 3.5 \sin 1.2$
28. $\cos 0.96 \cos 0.42 + \sin 0.96 \sin 0.42$
29. $\cos \frac{\pi}{9} \cos \frac{\pi}{7} - \sin \frac{\pi}{9} \sin \frac{\pi}{7}$
30. $\sin \frac{4\pi}{9} \cos \frac{\pi}{8} + \cos \frac{4\pi}{9} \sin \frac{\pi}{8}$

Numerical, Graphical, and Algebraic Analysis **In Exercises 31–34, use a graphing utility to complete the table and graph the two functions in the same viewing window. Use both the table and the graph as evidence that $y_1 = y_2$. Then verify the identity algebraically.**

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1							
y_2							

31. $y_1 = \sin\left(\frac{\pi}{6} + x\right), \quad y_2 = \frac{1}{2}\left(\cos x + \sqrt{3} \sin x\right)$
32. $y_1 = \cos\left(\frac{5\pi}{4} - x\right), \quad y_2 = -\frac{\sqrt{2}}{2}(\cos x + \sin x)$
33. $y_1 = \cos(x + \pi) \cos(x - \pi), \quad y_2 = \cos^2 x$
34. $y_1 = \sin(x + \pi) \sin(x - \pi), \quad y_2 = \sin^2 x$

In Exercises 35–38, find the exact value of the trigonometric function given that $\sin u = \frac{5}{13}$ and $\cos v = -\frac{3}{5}$. (Both u and v are in Quadrant II.)

35. $\sin(u + v)$
36. $\cos(v - u)$
37. $\tan(u + v)$
38. $\sin(u - v)$

In Exercises 39–42, find the exact value of the trigonometric function given that $\sin u = -\frac{8}{17}$ and $\cos v = -\frac{4}{5}$. (Both u and v are in Quadrant III.)

39. $\cos(u + v)$
40. $\tan(u + v)$
41. $\sin(v - u)$
42. $\cos(u - v)$

In Exercises 43–46, write the trigonometric expression as an algebraic expression.

43. $\sin(\arcsin x + \arccos x)$

44. $\cos(\arccos x - \arcsin x)$

45. $\sin(\arctan 2x - \arccos x)$

46. $\cos(\arcsin x - \arctan 2x)$

In Exercises 47–54, find the value of the expression without using a calculator.

47. $\sin(\sin^{-1} 1 + \cos^{-1} 1)$

48. $\cos[\sin^{-1}(-1) + \cos^{-1} 0]$

49. $\sin[\sin^{-1} 1 - \cos^{-1}(-1)]$

50. $\cos[\cos^{-1}(-1) - \cos^{-1} 1]$

51. $\sin\left(\sin^{-1} \frac{1}{2} - \cos^{-1} \frac{1}{2}\right)$

52. $\cos\left[\cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1} 1\right]$

53. $\tan\left(\sin^{-1} 0 + \sin^{-1} \frac{1}{2}\right)$

54. $\tan\left(\cos^{-1} \frac{\sqrt{2}}{2} - \sin^{-1} 0\right)$

In Exercises 55–58, evaluate the trigonometric function without using a calculator.

55. $\sin\left[\frac{\pi}{2} + \sin^{-1}(-1)\right]$

56. $\sin[\cos^{-1}(-1) + \pi]$

57. $\cos(\pi + \sin^{-1} 1)$

58. $\cos[\pi - \cos^{-1}(-1)]$

In Exercises 59–62, use right triangles to evaluate the expression.

59. $\sin\left(\cos^{-1} \frac{3}{5} - \sin^{-1} \frac{5}{13}\right)$

60. $\cos\left(\sin^{-1} \frac{12}{13} + \cos^{-1} \frac{8}{17}\right)$

61. $\sin\left(\tan^{-1} \frac{3}{4} + \sin^{-1} \frac{3}{5}\right)$

62. $\tan\left(\sin^{-1} \frac{4}{5} - \cos^{-1} \frac{5}{13}\right)$

In Exercises 63–70, verify the identity.

63. $\sin\left(\frac{\pi}{2} + x\right) = \cos x$

64. $\sin(3\pi - x) = \sin x$

65. $\tan(x + \pi) - \tan(\pi - x) = 2 \tan x$

66. $\tan\left(\frac{\pi}{4} - \theta\right) = \frac{1 - \tan \theta}{1 + \tan \theta}$

67. $\sin(x + y) + \sin(x - y) = 2 \sin x \cos y$

68. $\cos(x + y) + \cos(x - y) = 2 \cos x \cos y$

69. $\cos(x + y) \cos(x - y) = \cos^2 x - \sin^2 y$

70. $\sin(x + y) \sin(x - y) = \sin^2 x - \sin^2 y$

In Exercises 71–74, find the solution(s) of the equation in the interval $[0, 2\pi)$. Use a graphing utility to verify your results.

71. $\sin\left(x + \frac{\pi}{3}\right) + \sin\left(x - \frac{\pi}{3}\right) = 1$

72. $\cos\left(x + \frac{\pi}{6}\right) - \cos\left(x - \frac{\pi}{6}\right) = 1$

73. $\tan(x + \pi) + 2 \sin(x + \pi) = 0$

74. $2 \sin\left(x + \frac{\pi}{2}\right) + 3 \tan(\pi - x) = 0$

In Exercises 75–78, use a graphing utility to approximate the solutions of the equation in the interval $[0, 2\pi)$.

75. $\cos\left(x + \frac{\pi}{4}\right) + \cos\left(x - \frac{\pi}{4}\right) = 1$

76. $\sin\left(x + \frac{\pi}{2}\right) - \cos\left(x + \frac{3\pi}{2}\right) = 0$

77. $\tan(x + \pi) - \cos\left(x + \frac{\pi}{2}\right) = 0$

78. $\tan(\pi - x) + 2 \cos\left(x + \frac{3\pi}{2}\right) = 0$

79. ***Standing Waves*** The equation of a standing wave is obtained by adding the displacements of two waves traveling in opposite directions (see figure). Assume that each of the waves has amplitude A, period T, and wavelength λ. If the models for these waves are

$$y_1 = A \cos 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \text{ and } y_2 = A \cos 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)$$

show that

$$y_1 + y_2 = 2A \cos \frac{2\pi t}{T} \cos \frac{2\pi x}{\lambda}.$$