5.2 Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

Vocabulary Check

In Exercises 1 and 2, fill in the blanks.

1. An equation that is true for only some values in its domain is called a _______ equation.
2. An equation that is true for all real values in its domain is called an _______ .

In Exercises 3–10, fill in the blank to complete the trigonometric identity.

3. $\frac{1}{\tan u} =$ _______
4. $\frac{1}{\csc u} =$ _______
5. $\frac{\sin u}{\cos u} =$ _______
6. $\frac{1}{\sec u} =$ _______
7. $\sin^2 u +$ _______ $= 1$
8. $\tan\left(\frac{\pi}{2} - u\right) =$ _______
9. $\sin(-u) =$ _______
10. $\sec(-u) =$ _______

In Exercises 1–10, verify the identity.

1. $\sin t \csc t = 1$
2. $\sec y \cos y = 1$
3. $\frac{\csc^2 x}{\cot x} = \csc x \sec x$
4. $\frac{\sin^2 t}{\tan^2 t} = \cos^2 t$
5. $\cos^2 \beta - \sin^2 \beta = 1 - 2\sin^2 \beta$
6. $\cos^2 \beta - \sin^2 \beta = 2\cos^2 \beta - 1$
7. $\tan^2 \theta + 6 = \sec^2 \theta + 5$
8. $2 - \csc^2 z = 1 - \cot^2 z$
9. $(1 + \sin x)(1 - \sin x) = \cos^2 x$
10. $\tan^2 y(\csc^2 y - 1) = 1$

Numerical, Graphical, and Algebraic Analysis **In Exercises 11–18, use a graphing utility to complete the table and graph the functions in the same viewing window. Use both the table and the graph as evidence that $y_1 = y_2$. Then verify the identity algebraically.**

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1							
y_2							

11. $y_1 = \frac{1}{\sec x \tan x}, \quad y_2 = \csc x - \sin x$
12. $y_1 = \frac{\csc x - 1}{1 - \sin x}, \quad y_2 = \csc x$
13. $y_1 = \csc x - \sin x, \quad y_2 = \cos x \cot x$
14. $y_1 = \sec x - \cos x, \quad y_2 = \sin x \tan x$
15. $y_1 = \sin x + \cos x \cot x, \quad y_2 = \csc x$
16. $y_1 = \cos x + \sin x \tan x, \quad y_2 = \sec x$
17. $y_1 = \frac{1}{\tan x} + \frac{1}{\cot x}, \quad y_2 = \tan x + \cot x$
18. $y_1 = \frac{1}{\sin x} - \frac{1}{\csc x}, \quad y_2 = \csc x - \sin x$

Error Analysis **In Exercises 19 and 20, describe the error.**

19. $$\begin{aligned}(1 + \tan x)[1 + \cot(-x)] &= (1 + \tan x)(1 + \cot x)\\ &= 1 + \cot x + \tan x + \tan x \cot x\\ &= 1 + \cot x + \tan x + 1\\ &= 2 + \cot x + \tan x\end{aligned}$$

20. $$\begin{aligned}\frac{1 + \sec(-\theta)}{\sin(-\theta) + \tan(-\theta)} &= \frac{1 - \sec\theta}{\sin\theta - \tan\theta}\\ &= \frac{1 - \sec\theta}{(\sin\theta)\left[1 - \left(\frac{1}{\cos\theta}\right)\right]}\\ &= \frac{1 - \sec\theta}{\sin\theta(1 - \sec\theta)}\\ &= \frac{1}{\sin\theta} = \csc\theta\end{aligned}$$

In Exercises 21–30, verify the identity.

21. $\sin^{1/2} x \cos x - \sin^{5/2} x \cos x = \cos^3 x\sqrt{\sin x}$

22. $\sec^6 x(\sec x \tan x) - \sec^4 x(\sec x \tan x) = \sec^5 x \tan^3 x$

23. $\cot\left(\frac{\pi}{2} - x\right)\csc x = \sec x$

24. $\frac{\sec[(\pi/2) - x]}{\tan[(\pi/2) - x]} = \sec x$

25. $\frac{\csc(-x)}{\sec(-x)} = -\cot x$

26. $(1 + \sin y)[1 + \sin(-y)] = \cos^2 y$

27. $\frac{\cos(-\theta)}{1 + \sin(-\theta)} = \sec \theta + \tan \theta$

28. $\frac{1 + \csc(-\theta)}{\cos(-\theta) + \cot(-\theta)} = \sec \theta$

29. $\frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y} = \frac{\tan x + \tan y}{1 - \tan x \tan y}$

30. $\frac{\tan x + \tan y}{1 - \tan x \tan y} = \frac{\cot x + \cot y}{\cot x \cot y - 1}$

In Exercises 31–38, verify the identity algebraically. Use the *table* feature of a graphing utility to check your result numerically.

31. $\frac{\cos x - \cos y}{\sin x + \sin y} + \frac{\sin x - \sin y}{\cos x + \cos y} = 0$

32. $\frac{\tan x + \cot y}{\tan x \cot y} = \tan y + \cot x$

33. $\sqrt{\frac{1 + \sin \theta}{1 - \sin \theta}} = \frac{1 + \sin \theta}{|\cos \theta|}$

34. $\sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} = \frac{1 - \cos \theta}{|\sin \theta|}$

35. $\sin^2\left(\frac{\pi}{2} - x\right) + \sin^2 x = 1$

36. $\sec^2 y - \cot^2\left(\frac{\pi}{2} - y\right) = 1$

37. $\sin x \csc\left(\frac{\pi}{2} - x\right) = \tan x$

38. $\sec^2\left(\frac{\pi}{2} - x\right) - 1 = \cot^2 x$

In Exercises 39–50, verify the identity algebraically. Use a graphing utility to check your result graphically.

39. $2 \sec^2 x - 2 \sec^2 x \sin^2 x - \sin^2 x - \cos^2 x = 1$

40. $\csc x(\csc x - \sin x) + \frac{\sin x - \cos x}{\sin x} + \cot x = \csc^2 x$

41. $\frac{\cot x \tan x}{\sin x} = \csc x$

42. $\frac{1 + \csc \theta}{\sec \theta} - \cot \theta = \cos \theta$

43. $\csc^4 x - 2 \csc^2 x + 1 = \cot^4 x$

44. $\sin x(1 - 2 \cos^2 x + \cos^4 x) = \sin^5 x$

45. $\sec^4 \theta - \tan^4 \theta = 1 + 2 \tan^2 \theta$

46. $\csc^4 \theta - \cot^4 \theta = 2 \csc^2 \theta - 1$

47. $\frac{\sin \beta}{1 - \cos \beta} = \frac{1 + \cos \beta}{\sin \beta}$

48. $\frac{\cot \alpha}{\csc \alpha - 1} = \frac{\csc \alpha + 1}{\cot \alpha}$

49. $\frac{\tan^3 \alpha - 1}{\tan \alpha - 1} = \tan^2 \alpha + \tan \alpha + 1$

50. $\frac{\sin^3 \beta + \cos^3 \beta}{\sin \beta + \cos \beta} = 1 - \sin \beta \cos \beta$

Conjecture **In Exercises 51–54, use a graphing utility to graph the trigonometric function. Use the graph to make a conjecture about a simplification of the expression. Verify the resulting identity algebraically.**

51. $y = \frac{1}{\cot x + 1} + \frac{1}{\tan x + 1}$

52. $y = \frac{\cos x}{1 - \tan x} + \frac{\sin x \cos x}{\sin x - \cos x}$

53. $y = \frac{1}{\sin x} - \frac{\cos^2 x}{\sin x}$

54. $y = \sin t + \frac{\cot^2 t}{\csc t}$

In Exercises 55–58, use the properties of logarithms and trigonometric identities to verify the identity.

55. $\ln|\cot \theta| = \ln|\cos \theta| - \ln|\sin \theta|$

56. $\ln|\sec \theta| = -\ln|\cos \theta|$

57. $-\ln(1 + \cos \theta) = \ln(1 - \cos \theta) - 2 \ln|\sin \theta|$

58. $-\ln|\csc \theta + \cot \theta| = \ln|\csc \theta - \cot \theta|$

In Exercises 59–62, use the cofunction identities to evaluate the expression without using a calculator.

59. $\sin^2 35° + \sin^2 55°$

60. $\cos^2 14° + \cos^2 76°$

61. $\cos^2 20° + \cos^2 52° + \cos^2 38° + \cos^2 70°$

62. $\sin^2 18° + \sin^2 40° + \sin^2 50° + \sin^2 72°$