

4.7 Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

Vocabulary Check

Fill in the blanks.

Function	Alternative Notation	Domain	Range
1. $y = \arcsin x$	_____	_____	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
2. _____	$y = \cos^{-1} x$	$-1 \leq x \leq 1$	_____
3. $y = \arctan x$	_____	_____	_____

Library of Parent Functions In Exercises 1–9, find the exact value of each expression without using a calculator.

- (a) $\arcsin \frac{1}{2}$ (b) $\arcsin 0$
- (a) $\arccos \frac{1}{2}$ (b) $\arccos 0$
- (a) $\arcsin 1$ (b) $\arccos 1$
- (a) $\arctan 1$ (b) $\arctan 0$
- (a) $\arctan \frac{\sqrt{3}}{3}$ (b) $\arctan(-1)$
- (a) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ (b) $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$
- (a) $\arctan(-\sqrt{3})$ (b) $\arctan \sqrt{3}$
- (a) $\arccos\left(-\frac{1}{2}\right)$ (b) $\arcsin \frac{\sqrt{2}}{2}$
- (a) $\sin^{-1} \frac{\sqrt{3}}{2}$ (b) $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

10. Numerical and Graphical Analysis Consider the function $y = \arcsin x$.

(a) Use a graphing utility to complete the table.

x	-1	-0.8	-0.6	-0.4	-0.2
y					

x	0	0.2	0.4	0.6	0.8	1
y						

- Plot the points from the table in part (a) and graph the function. (Do not use a graphing utility.)
- Use a graphing utility to graph the inverse sine function and compare the result with your hand-drawn graph in part (b).
- Determine any intercepts and symmetry of the graph.

11. Numerical and Graphical Analysis Consider the function $y = \arccos x$.

(a) Use a graphing utility to complete the table.

x	-1	-0.8	-0.6	-0.4	-0.2
y					

x	0	0.2	0.4	0.6	0.8	1
y						

- Plot the points from the table in part (a) and graph the function. (Do not use a graphing utility.)
- Use a graphing utility to graph the inverse cosine function and compare the result with your hand-drawn graph in part (b).
- Determine any intercepts and symmetry of the graph.

12. Numerical and Graphical Analysis Consider the function $y = \arctan x$.

(a) Use a graphing utility to complete the table.

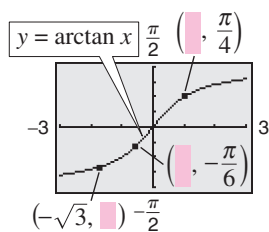
x	-10	-8	-6	-4	-2
y					

x	0	2	4	6	8	10
y						

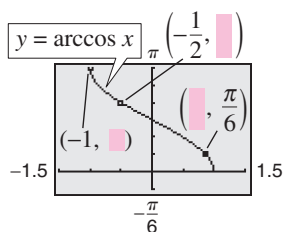
- Plot the points from the table in part (a) and graph the function. (Do not use a graphing utility.)
- Use a graphing utility to graph the inverse tangent function and compare the result with your hand-drawn graph in part (b).
- Determine the horizontal asymptotes of the graph.

In Exercises 13 and 14, determine the missing coordinates of the points on the graph of the function.

13.



14.



In Exercises 15–20, use a calculator to approximate the value of the expression. Round your answer to the nearest hundredth.

15. $\cos^{-1} 0.75$

16. $\sin^{-1} 0.56$

17. $\arcsin(-0.75)$

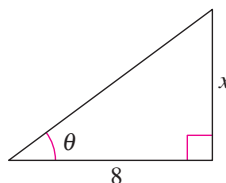
18. $\arccos(-0.7)$

19. $\arctan(-6)$

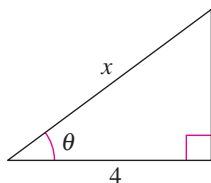
20. $\tan^{-1} 5.9$

In Exercises 21–24, use an inverse trigonometric function to write θ as a function of x .

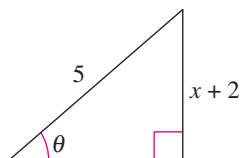
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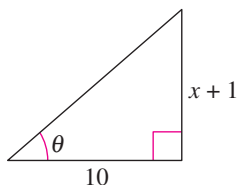
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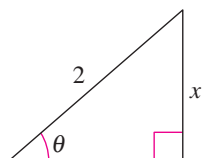


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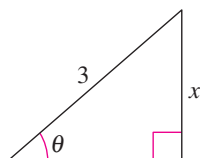


In Exercises 25–28, find the length of the third side of the triangle in terms of x . Then find θ in terms of x for all three inverse trigonometric functions.

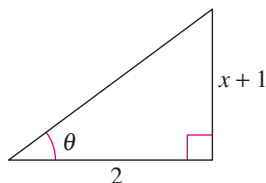
25.



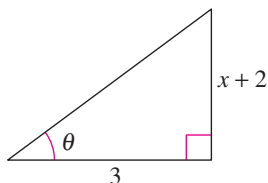
26.



27.



28.



In Exercises 29–46, use the properties of inverse functions to find the exact value of the expression.

29. $\sin(\arcsin 0.7)$

30. $\tan(\arctan 35)$

31. $\cos[\arccos(-0.3)]$

32. $\sin[\arcsin(-0.1)]$

33. $\arcsin(\sin 3\pi)$

34. $\arccos\left(\cos \frac{7\pi}{2}\right)$

35. $\tan^{-1}\left(\tan \frac{11\pi}{6}\right)$

36. $\sin^{-1}\left(\sin \frac{7\pi}{4}\right)$

37. $\sin^{-1}\left(\sin \frac{5\pi}{2}\right)$

38. $\cos^{-1}\left(\cos \frac{3\pi}{2}\right)$

39. $\sin^{-1}\left(\tan \frac{5\pi}{4}\right)$

40. $\cos^{-1}\left(\tan \frac{3\pi}{4}\right)$

41. $\tan(\arcsin 0)$

42. $\cos[\arctan(-1)]$

43. $\sin(\arctan 1)$

44. $\sin[\arctan(-1)]$

45. $\arcsin\left[\cos\left(-\frac{\pi}{6}\right)\right]$

46. $\arccos\left[\sin\left(-\frac{\pi}{6}\right)\right]$

In Exercises 47–54, find the exact value of the expression. Use a graphing utility to verify your result. (Hint: Make a sketch of a right triangle.)

47. $\sin(\arctan \frac{4}{3})$

48. $\sec(\arcsin \frac{3}{5})$

49. $\cos(\arcsin \frac{24}{25})$

50. $\csc[\arctan(-\frac{12}{5})]$

51. $\sec[\arctan(-\frac{3}{5})]$

52. $\tan[\arcsin(-\frac{3}{4})]$

53. $\sin[\arccos(-\frac{2}{3})]$

54. $\cot(\arctan \frac{5}{8})$

In Exercises 55–62, write an algebraic expression that is equivalent to the expression. (Hint: Sketch a right triangle, as demonstrated in Example 7.)

55. $\cot(\arctan x)$

56. $\sin(\arctan x)$

57. $\sin[\arccos(x+2)]$

58. $\sec[\arcsin(x-1)]$

59. $\tan\left(\arccos \frac{x}{5}\right)$

60. $\cot\left(\arctan \frac{4}{x}\right)$

61. $\csc\left(\arctan \frac{x}{\sqrt{7}}\right)$

62. $\cos\left(\arcsin \frac{x-h}{r}\right)$

In Exercises 63–66, complete the equation.

63. $\arctan \frac{14}{x} = \arcsin(\quad), \quad x > 0$

64. $\arcsin \frac{\sqrt{36-x^2}}{6} = \arccos(\quad), \quad 0 \leq x \leq 6$

65. $\arccos \frac{3}{\sqrt{x^2-2x+10}} = \arcsin(\quad)$

66. $\arccos \frac{x-2}{2} = \arctan(\quad), \quad 2 < x < 4$