5.3 Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

Vocabulary Check

Fill in the blanks.

1. The equation $2\cos x - 1 = 0$ has the solutions $x = \frac{\pi}{3} + 2n\pi$ and $x = \frac{5\pi}{3} + 2n\pi$, which are called _____ solutions.
2. The equation $\tan^2 x - 5\tan x + 6 = 0$ is an equation of _____ type.
3. A solution of an equation that does not satisfy the original equation is called an _____ solution.

In Exercises 1–6, verify that each x-value is a solution of the equation.

1. $2\cos x - 1 = 0$
   (a) $x = \frac{\pi}{3}$ (b) $x = \frac{5\pi}{3}$
2. $\sec x - 2 = 0$
   (a) $x = \frac{\pi}{3}$ (b) $x = \frac{5\pi}{3}$
3. $3\tan^2 2x - 1 = 0$
   (a) $x = \frac{\pi}{12}$ (b) $x = \frac{5\pi}{12}$
4. $4\cos^2 2x - 2 = 0$
   (a) $x = \frac{\pi}{8}$ (b) $x = \frac{7\pi}{8}$
5. $2\sin^2 x - \sin x - 1 = 0$
   (a) $x = \frac{\pi}{2}$ (b) $x = \frac{7\pi}{6}$
6. $\sec^4 x - 3\sec^2 x - 4 = 0$
   (a) $x = \frac{2\pi}{3}$ (b) $x = \frac{5\pi}{3}$

In Exercises 7–12, find all solutions of the equation in the interval $[0°, 360°)$.

7. $\sin x = \frac{1}{2}$
8. $\cos x = \frac{\sqrt{3}}{2}$
9. $\cos x = -\frac{1}{2}$
10. $\sin x = -\frac{\sqrt{2}}{2}$
11. $\tan x = 1$
12. $\tan x = -\sqrt{3}$

In Exercises 13–24, find all solutions of the equation in the interval $[0, 2\pi)$.

13. $\cos x = -\frac{\sqrt{3}}{2}$
14. $\sin x = -\frac{1}{2}$
15. $\cot x = -1$
16. $\sin x = \frac{\sqrt{3}}{2}$
17. $\tan x = -\frac{\sqrt{3}}{3}$
18. $\cos x = \frac{\sqrt{2}}{2}$
19. $\csc x = -2$
20. $\sec x = \sqrt{2}$
21. $\cot x = \sqrt{3}$
22. $\sec x = 2$
23. $\tan x = -1$
24. $\csc x = -\sqrt{2}$

In Exercises 25–34, solve the equation.

25. $2\cos x + 1 = 0$
26. $\sqrt{2}\sin x + 1 = 0$
27. $\sqrt{3}\sec x - 2 = 0$
28. $\cot x + 1 = 0$
29. $3\csc^2 x - 4 = 0$
30. $3\cot^2 x - 1 = 0$
31. $4\cos^2 x - 1 = 0$
32. $\cos x(\cos x - 1) = 0$
33. $\sin^2 x = 3\cos^2 x$
34. $(3\tan^2 x - 1)(\tan^2 x - 3) = 0$

In Exercises 35–48, find all solutions of the equation in the interval $[0, 2\pi)$ algebraically. Use the *table* feature of a graphing utility to check your answers numerically.

35. $\tan x + \sqrt{3} = 0$
36. $2\sin x + 1 = 0$
37. $\csc^2 x - 2 = 0$
38. $\tan^2 x - 1 = 0$
39. $3\tan^3 x = \tan x$
40. $2\sin^2 x = 2 + \cos x$
41. $\sec^2 x - \sec x = 2$
42. $\sec x \csc x = 2\csc x$
43. $2\sin x + \csc x = 0$
44. $\sec x + \tan x = 1$
45. $\cos x + \sin x \tan x = 2$
46. $\sin^2 x + \cos x + 1 = 0$
47. $\sec^2 x + \tan x = 3$
48. $2\cos^2 x + \cos x - 1 = 0$

In Exercises 49–56, use a graphing utility to approximate the solutions of the equation in the interval $[0, 2\pi)$ by setting the equation equal to 0, graphing the new equation, and using the *zero* or *root* feature to approximate the x-intercepts of the graph.

49. $2\sin^2 x + 3\sin x + 1 = 0$
50. $2\sec^2 x + \tan^2 x - 3 = 0$
51. $4\sin^2 x = 2\cos x + 1$
52. $\csc^2 x = 3\csc x + 4$

53. $\csc x + \cot x = 1$

54. $4 \sin x = \cos x - 2$

55. $\dfrac{\cos x \cot x}{1 - \sin x} = 3$

56. $\dfrac{1 + \sin x}{\cos x} + \dfrac{\cos x}{1 + \sin x} = 4$

In Exercises 57–60, (a) use a graphing utility to graph each function in the interval $[0, 2\pi)$, (b) write an equation whose solutions are the points of intersection of the graphs, and (c) use the *intersect* feature of the graphing utility to find the points of intersection (to four decimal places).

57. $y = \sin 2x, \quad y = x^2 - 2x$

58. $y = \cos x, \quad y = x + x^2$

59. $y = \sin^2 x, \quad y = e^x - 4x$

60. $y = \cos^2 x, \quad y = e^{-x} + x - 1$

In Exercises 61–72, solve the multiple-angle equation.

61. $\cos \dfrac{x}{4} = 0$

62. $\sin \dfrac{x}{2} = 0$

63. $\sin 4x = 1$

64. $\cos 2x = -1$

65. $\sin 2x = -\dfrac{\sqrt{3}}{2}$

66. $\sec 4x = 2$

67. $2 \sin^2 2x = 1$

68. $\tan^2 3x = 3$

69. $\tan 3x(\tan x - 1) = 0$

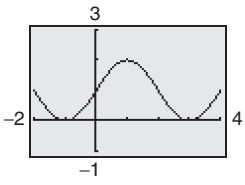
70. $\cos 2x(2 \cos x + 1) = 0$

71. $\cos \dfrac{x}{2} = \dfrac{\sqrt{2}}{2}$

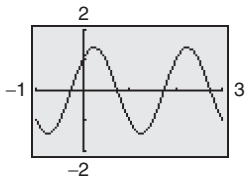
72. $\tan \dfrac{x}{3} = 1$

In Exercises 73–76, approximate the x-intercepts of the graph. Use a graphing utility to check your solutions.

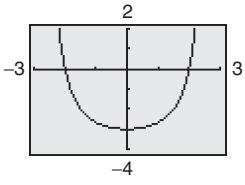
73. $y = \sin \dfrac{\pi x}{2} + 1$



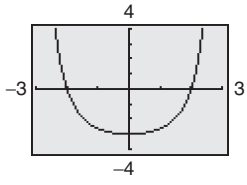
74. $y = \sin \pi x + \cos \pi x$



75. $y = \tan^2\left(\dfrac{\pi x}{6}\right) - 3$



76. $y = \sec^4\left(\dfrac{\pi x}{8}\right) - 4$



In Exercises 77–84, use a graphing utility to approximate the solutions of the equation in the interval $[0, 2\pi)$.

77. $2 \cos x - \sin x = 0$

78. $2 \sin x + \cos x = 0$

79. $x \tan x - 1 = 0$

80. $2x \sin x - 2 = 0$

81. $\sec^2 x + 0.5 \tan x = 1$

82. $\csc^2 x + 0.5 \cot x = 5$

83. $12 \sin^2 x - 13 \sin x + 3 = 0$

84. $3 \tan^2 x + 4 \tan x - 4 = 0$

In Exercises 85–88, use a graphing utility to approximate the solutions (to three decimal places) of the equation in the given interval.

85. $3 \tan^2 x + 5 \tan x - 4 = 0, \quad \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

86. $\cos^2 x - 2 \cos x - 1 = 0, \quad [0, \pi]$

87. $4 \cos^2 x - 2 \sin x + 1 = 0, \quad \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

88. $2 \sec^2 x + \tan x - 6 = 0, \quad \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

In Exercises 89–94, (a) use a graphing utility to graph the function and approximate the maximum and minimum points (to four decimal places) of the graph in the interval $[0, 2\pi]$, and (b) solve the trigonometric equation and verify that the x-coordinates of the maximum and minimum points of f are among its solutions (the trigonometric equation is found using calculus).

	Function	*Trigonometric Equation*
89.	$f(x) = \sin 2x$	$2 \cos 2x = 0$
90.	$f(x) = \cos 2x$	$-2 \sin 2x = 0$
91.	$f(x) = \sin^2 x + \cos x$	$2 \sin x \cos x - \sin x = 0$
92.	$f(x) = \cos^2 x - \sin x$	$-2 \sin x \cos x - \cos x = 0$
93.	$f(x) = \sin x + \cos x$	$\cos x - \sin x = 0$
94.	$f(x) = 2 \sin x + \cos 2x$	$2 \cos x - 4 \sin x \cos x = 0$

Fixed Point **In Exercises 95 and 96, find the smallest positive fixed point of the function f. [A *fixed point* of a function f is a real number c such that $f(c) = c$.]**

95. $f(x) = \tan \dfrac{\pi x}{4}$

96. $f(x) = \cos x$