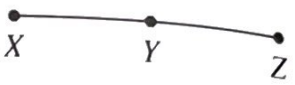
Written Exercises

Some information about the diagram is given. Tell whether the other statements can be deduced from what is given. (Write *yes* or *no*.)

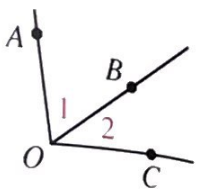
A

1. Given: Point Y lies between points X and Z.



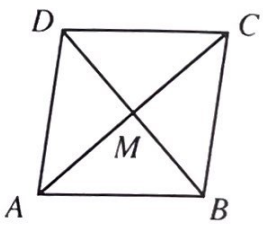
a. $XY = \frac{1}{2}XZ$ **b.** $XZ = XY + YZ$
c. $XZ > XY$ **d.** $YZ > XY$
e. $XZ > YZ$ **f.** $XZ > 2XY$

2. Given: Point B lies in the interior of $\angle AOC$.



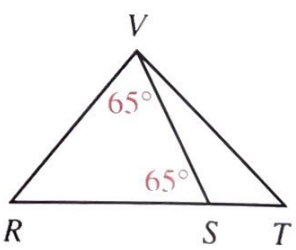
a. $m\angle 1 = m\angle 2$ **b.** $m\angle AOC = m\angle 1 + m\angle 2$
c. $m\angle AOC > m\angle 1$ **d.** $m\angle AOC > m\angle 2$
e. $m\angle 1 > m\angle 2$ **f.** $m\angle AOC > 90$

3. Given: $\square ABCD$; $AC > BD$



a. $AB > AD$ **b.** $AM > MC$
c. $DM = MB$ **d.** $AM > MB$

4. Given: $m\angle RVS = m\angle RSV = 65$

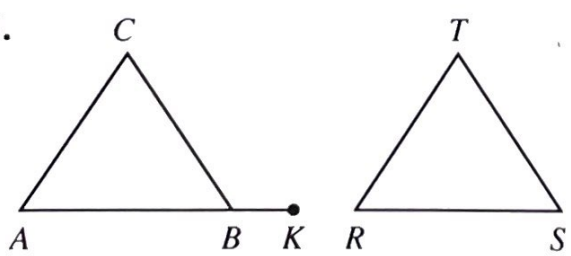


a. $RT > RS$ **b.** $RT > RV$
c. $RS > ST$ **d.** $VT < RS$

5. When some people are given that $j > k$ and $l > m$, they carelessly conclude that $j + k > l + m$. Find values for j, k, l, and m that show this conclusion is false.

Write the reasons that justify the statements.

6.

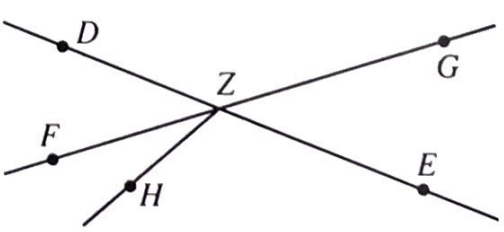


Given: $\triangle ABC \cong \triangle RST$
Prove: $AK > RS$

Statements of proof:

1. $\triangle ABC \cong \triangle RST$
2. $\overline{AB} \cong \overline{RS}$, or $AB = RS$
3. $AK = AB + BK$
4. $AK > AB$
5. $AK > RS$

7.



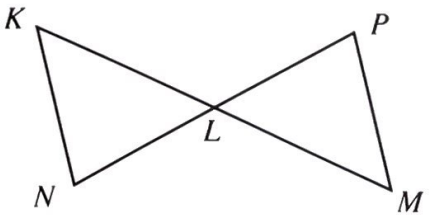
Given: $\overleftrightarrow{DE}$, $\overleftrightarrow{FG}$ and $\overrightarrow{ZH}$ contain point Z.
Prove: $m\angle DZH > m\angle GZE$

Statements of proof:

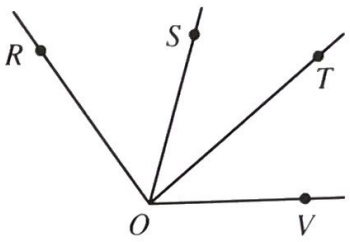
1. $\angle DZF \cong \angle GZE$, or $m\angle DZF = m\angle GZE$
2. $m\angle DZH = m\angle DZF + m\angle FZH$
3. $m\angle DZH > m\angle DZF$
4. $m\angle DZH > m\angle GZE$

Write proofs in two-column form.

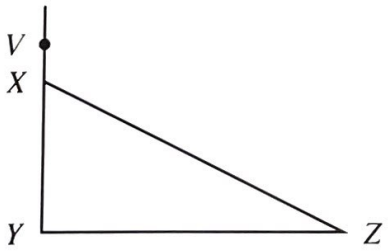
B **8.** Given: $KL > NL$; $LM > LP$
Prove: $KM > NP$



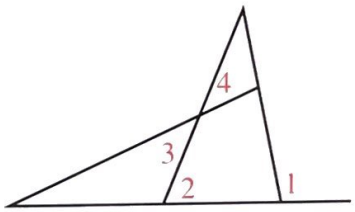
9. Given: $m\angle ROS > m\angle TOV$
Prove: $m\angle ROT > m\angle SOV$



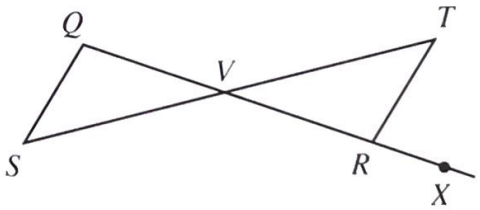
10. Given: $\overline{VY} \perp \overline{YZ}$
Prove: $\angle VXZ$ is an obtuse angle.



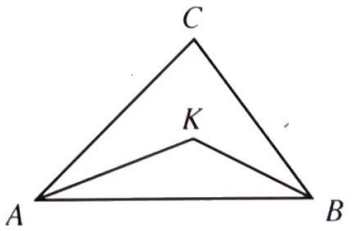
11. Given: The diagram
Prove: $m\angle 1 > m\angle 4$



12. Given: $\overline{QR}$ and $\overline{ST}$ bisect each other.
Prove: $m\angle XRT > m\angle S$



C **13.** Given: Point K lies inside $\triangle ABC$.
Prove: $m\angle K > m\angle C$

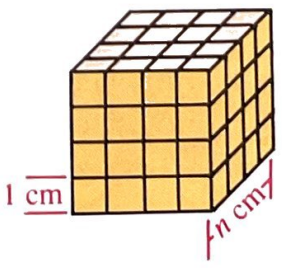


Challenge

A cube with sides n cm long is painted on all faces. It is then cut into cubes with sides 1 cm long. If $n = 4$, as the diagram at the right illustrates, how many of these smaller cubes will have paint on

a. 3 surfaces? **b.** 2 surfaces?
c. 1 surface? **d.** 0 surfaces?

Answer the questions for any positive integer n.



Classroom Exercises

1. State the contrapositive of each statement.
   a. If I can sing, then you can dance.
   b. If you can't play baseball, then I can't ride a horse.
   c. If $x = 4$, then $x^2 - 5 = 11$.
   d. If $y < 3$, then $y \neq 4$.
   e. If a polygon is a triangle, then the sum of the measures of its angles is 180.
2. State the inverse of each statement in Exercise 1.
3. A certain conditional is true. Must its converse be true? Must its inverse be true? Must its contrapositive be true?
4. A certain conditional is false. Must its converse be false? Must its inverse be false? Must its contrapositive be false?

Classify each conditional as true or false. Then state its inverse and contrapositive, and classify each of these as true or false.

5. If a triangle is equilateral, then it is equiangular.
6. If $\angle A$ is acute, then $m\angle A \neq 100$.
7. If a triangle is not isosceles, then it is not equilateral.
8. If two planes do not intersect, then they are parallel.

Express each statement in if-then form.

9. All squares are rhombuses.
10. No trapezoids are equiangular.
11. All marathoners have stamina.
12. Suppose "All marathoners have stamina" is a true conditional. What, if anything, can you conclude from each additional statement? If no conclusion is possible, say so.
    a. Nick is a marathoner.
    b. Heidi has stamina.
    c. Mimi does not have stamina.
    d. Arlo is not a marathoner.

Written Exercises

Write (a) the contrapositive and (b) the inverse of each statement.

A
1. If $n = 17$, then $4n = 68$.
2. If those are red and white, then this is blue.
3. If x is not even, then $x + 1$ is not odd.
4. If Abby is not here, then she is not well.

For each statement in Exercises 5–10 copy and complete a table like the one shown below.

	If ___?___, then ___?___.	True/False
Statement	?	?
Contrapositive	?	?
Converse	?	?
Inverse	?	?

5. If I live in Los Angeles, then I live in California.

6. If $\angle 1$ and $\angle 2$ are vertical angles, then $m\angle 1 = m\angle 2$.

7. If $AM = MB$, then M is the midpoint of $\overline{AB}$.

8. If a triangle is scalene, then it has no congruent sides.

B **9.** If $-2n < 6$, then $n > -3$.

10. If $x^2 > 1$, then $x > 1$.

Reword the given statement in if-then form and illustrate it with a Venn diagram. What can you conclude by using the given statement together with each additional statement? If no conclusion is possible, say so.

11. Given: All senators are at least 30 years old.
- **a.** Jose Avila is 48 years old.
- **b.** Rebecca Castelloe is a senator.
- **c.** Constance Brown is not a senator.
- **d.** Ling Chen is 29 years old.

12. Given: Math teachers assign hours of homework.
- **a.** Bridget Sullivan is a math teacher.
- **b.** August Campos assigns hours of homework.
- **c.** Andrew Byrnes assigns no homework at all.
- **d.** Jason Babler is not a math teacher.

What can you conclude by using the given statement together with each additional statement? If no conclusion is possible, say so.

13. Given: If it is not raining, then I am happy.
- **a.** I am not happy.
- **b.** It is not raining.
- **c.** I am overjoyed.
- **d.** It is raining.

14. Given: All my students love geometry.
- **a.** Stu is my student.
- **b.** Luis loves geometry.
- **c.** Stella is not my student.
- **d.** George does not love geometry.

15. Given: If two angles are vertical angles, then they are congruent.
- **a.** $\angle 1 \cong \angle 2$
- **b.** $m\angle ABC \neq m\angle DBF$
- **c.** $\angle 3$ and $\angle 4$ are adjacent angles.
- **d.** $\overline{RS}$ and $\overline{TU}$ intersect at V.

What can you conclude by using the given statement together with each additional statement? If no conclusion is possible, say so.

16. Given: The diagonals of a rhombus are perpendicular.
- **a.** $JKLM$ is a rhombus.
- **b.** In quad. $DIME$, $\overline{DM} \perp \overline{IE}$.
- **c.** $STUV$ is not a rhombus.
- **d.** In quad. $NOPQ$, $\overline{NP} \not\perp \overline{OQ}$.

17. Given: The diagonals of a rectangle are congruent.
- **a.** $PQRS$ is a rectangle.
- **b.** In quad. $ABCD$, $AC = BD$.
- **c.** $WXYZ$ is not a rectangle.
- **d.** In quad. $STAR$, $SA > TR$.

18. Given: Every square is a rhombus.
- **a.** $ABCD$ is a rhombus.
- **b.** In quad. $LAST$, $LA \neq LT$.
- **c.** $PQRS$ is a square.
- **d.** $GHIJ$ is not a square.

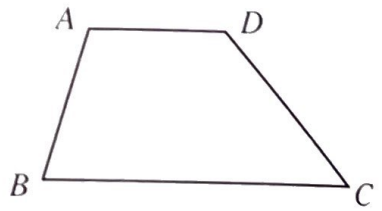
C **19.** What simpler name can be used for the converse of the inverse of a conditional?

20. Write the contrapositive of the converse of the inverse of the conditional: If r, then s.

Prove each of the following statements by proving its contrapositive. Begin by writing what is given and what is to be proved.

21. If $m\angle A + m\angle B \neq 180$,
then $m\angle D + m\angle C \neq 180$.

22. If n^2 is not a multiple of 3,
then n is not a multiple of 3.



Mixed Review Exercises

Complete each statement with the word *always*, *sometimes*, or *never*.

1. Two lines that do not intersect are ___?___ parallel.

2. Two lines parallel to the same plane ___?___ intersect.

3. The diagonals of a parallelogram ___?___ bisect each other.

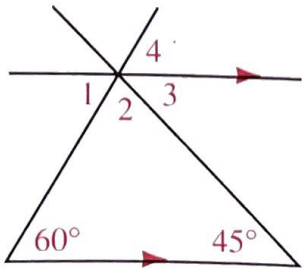
4. An acute triangle is ___?___ a right triangle.

5. Two lines parallel to a third line are ___?___ parallel.

6. A square is ___?___ a rectangle.

7. An altitude of a triangle is ___?___ a median.

8. Find the measures of $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ in the figure shown.



9. Find the value of x.