4.8 Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

Vocabulary Check

Fill in the blanks.

1. An angle that measures from the horizontal upward to an object is called the angle of _______ , whereas an angle that measures from the horizontal downward to an object is called the angle of _______ .

2. A _______ measures the acute angle a path or line of sight makes with a fixed north-south line.

3. A point that moves on a coordinate line is said to be in simple _______ if its distance from the origin at time t is given by either $d = a \sin \omega t$ or $d = a \cos \omega t$.

In Exercises 1–10, solve the right triangle shown in the figure.

1. $A = 30°,\ b = 10$

2. $B = 60°,\ c = 15$

3. $B = 71°,\ b = 14$

4. $A = 7.4°,\ a = 20.5$

5. $a = 6,\ b = 12$

6. $a = 25,\ c = 45$

7. $b = 16,\ c = 54$

8. $b = 1.32,\ c = 18.9$

9. $A = 12° 15',\ c = 430.5$

10. $B = 65° 12',\ a = 145.5$

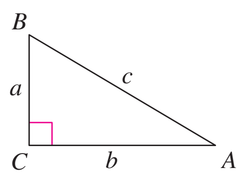


Figure for 1–10

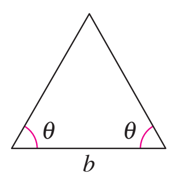


Figure for 11–14

In Exercises 11–14, find the altitude of the isosceles triangle shown in the figure.

11. $\theta = 52°,\ b = 8$ inches

12. $\theta = 18°,\ b = 12$ meters

13. $\theta = 41.6°,\ b = 18.5$ feet

14. $\theta = 72.94°,\ b = 3.26$ centimeters

15. ***Length*** A shadow of length L is created by a 60-foot silo when the sun is $\theta°$ above the horizon.

(a) Draw a right triangle that gives a visual representation of the problem. Label the known and unknown quantities.

(b) Write L as a function of θ.

(c) Use a graphing utility to complete the table.

θ	10°	20°	30°	40°	50°
L					

(d) The angle measure increases in equal increments in the table. Does the length of the shadow change in equal increments? Explain.

16. ***Length*** A shadow of length L is created by an 850-foot building when the sun is $\theta°$ above the horizon.

(a) Draw a right triangle that gives a visual representation of the problem. Label the known and unknown quantities.

(b) Write L as a function of θ.

(c) Use a graphing utility to complete the table.

θ	10°	20°	30°	40°	50°
L					

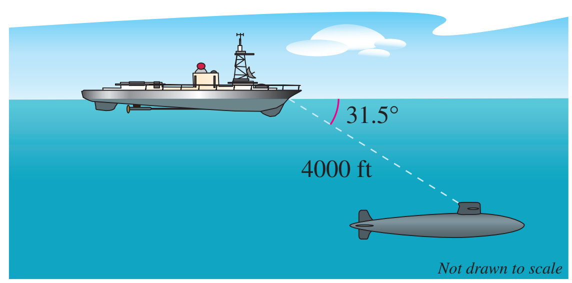
(d) The angle measure increases in equal increments in the table. Does the length of the shadow change in equal increments? Explain.

17. ***Height*** A ladder 20 feet long leans against the side of a house. The angle of elevation of the ladder is 80°. Find the height from the top of the ladder to the ground.

18. ***Height*** The angle of elevation from the base to the top of a waterslide is 13°. The slide extends horizontally 58.2 meters. Approximate the height of the waterslide.

19. ***Height*** A 100-foot line is attached to a kite. When the kite has pulled the line taut, the angle of elevation to the kite is approximately 50°. Approximate the height of the kite.

20. ***Depth*** The sonar of a navy cruiser detects a submarine that is 4000 feet from the cruiser. The angle between the water level and the submarine is 31.5°. How deep is the submarine?



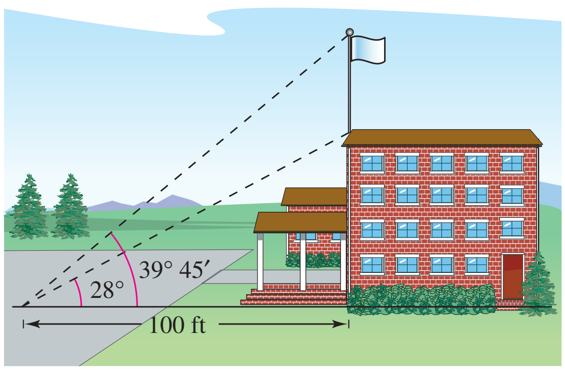
21. ***Height*** From a point 50 feet in front of a church, the angles of elevation to the base of the steeple and the top of the steeple are 35° and 47° 40′, respectively.

(a) Draw right triangles that give a visual representation of the problem. Label the known and unknown quantities.

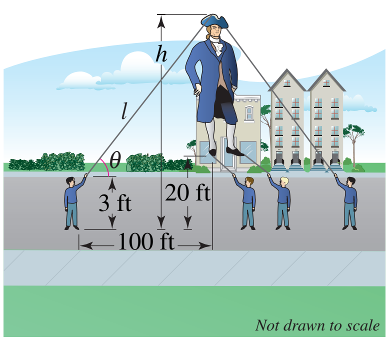
(b) Use a trigonometric function to write an equation involving the unknown quantity.

(c) Find the height of the steeple.

22. ***Height*** From a point 100 feet in front of a public library, the angles of elevation to the base of the flagpole and the top of the flagpole are 28° and 39° 45′, respectively. The flagpole is mounted on the front of the library's roof. Find the height of the flagpole.



23. ***Height*** You are holding one of the tethers attached to the top of a giant character balloon in a parade. Before the start of the parade the balloon is upright and the bottom is floating approximately 20 feet above ground level. You are standing approximately 100 feet ahead of the balloon (see figure).

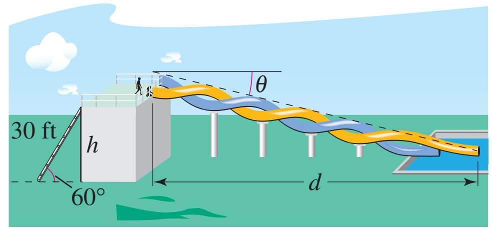


(a) Find the length ℓ of the tether you will be holding while walking, in terms of h, the height of the balloon.

(b) Find an expression for the angle of elevation θ from you to the top of the balloon.

(c) Find the height of the balloon from top to bottom if the angle of elevation to the top of the balloon is 35°.

24. ***Height*** The designers of a water park are creating a new slide and have sketched some preliminary drawings. The length of the ladder is 30 feet, and its angle of elevation is 60° (see figure).



(a) Find the height h of the slide.

(b) Find the angle of depression θ from the top of the slide to the end of the slide at the ground in terms of the horizontal distance d the rider travels.

(c) The angle of depression of the ride is bounded by safety restrictions to be no less than 25° and not more than 30°. Find an interval for how far the rider travels horizontally.

25. ***Angle of Elevation*** An engineer erects a 75-foot vertical cellular-phone tower. Find the angle of elevation to the top of the tower from a point on level ground 95 feet from its base.

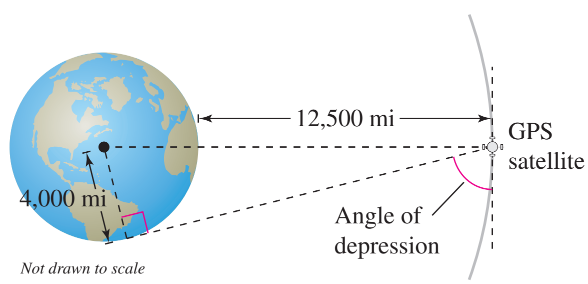
26. ***Angle of Elevation*** The height of an outdoor basketball backboard is $12\frac{1}{2}$ feet, and the backboard casts a shadow $17\frac{1}{3}$ feet long.

(a) Draw a right triangle that gives a visual representation of the problem. Label the known and unknown quantities.

(b) Use a trigonometric function to write an equation involving the unknown quantity.

(c) Find the angle of elevation of the sun.

27. ***Angle of Depression*** A Global Positioning System satellite orbits 12,500 miles above Earth's surface (see figure). Find the angle of depression from the satellite to the horizon. Assume the radius of Earth is 4000 miles.

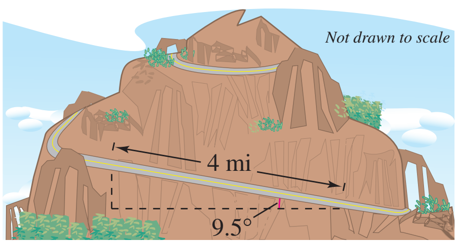


28. ***Angle of Depression*** Find the angle of depression from the top of a lighthouse 250 feet above water level to the water line of a ship $2\frac{1}{2}$ miles offshore.

29. ***Airplane Ascent*** When an airplane leaves the runway, its angle of climb is 18° and its speed is 275 feet per second. Find the plane's altitude after 1 minute.

30. ***Airplane Ascent*** How long will it take the plane in Exercise 29 to climb to an altitude of 10,000 feet? 16,000 feet?

31. ***Mountain Descent*** A sign on the roadway at the top of a mountain indicates that for the next 4 miles the grade is 9.5° (see figure). Find the change in elevation for a car descending the mountain.

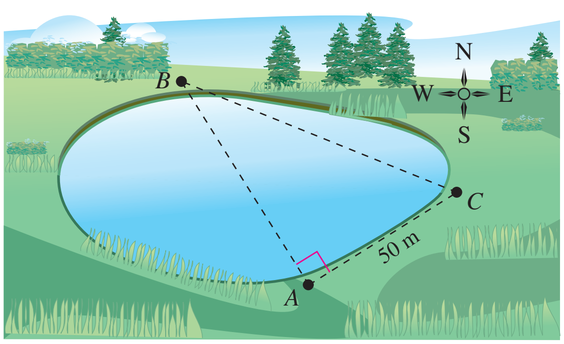


32. ***Ski Slope*** A ski slope on a mountain has an angle of elevation of 25.2°. The vertical height of the slope is 1808 feet. How long is the slope?

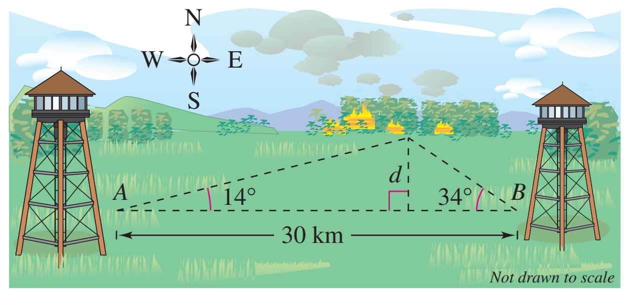
33. ***Navigation*** A ship leaves port at noon and has a bearing of S 29° W. The ship sails at 20 knots. How many nautical miles south and how many nautical miles west will the ship have traveled by 6:00 P.M.?

34. ***Navigation*** An airplane flying at 600 miles per hour has a bearing of 52°. After flying for 1.5 hours, how far north and how far east has the plane traveled from its point of departure?

35. ***Surveying*** A surveyor wants to find the distance across a pond (see figure). The bearing from A to B is N 32° W. The surveyor walks 50 meters from A, and at the point C the bearing to B is N 68° W. Find (a) the bearing from A to C and (b) the distance from A to B.



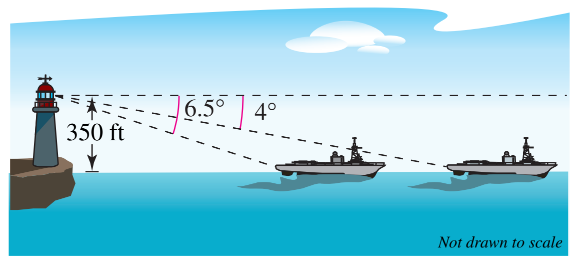
36. ***Location of a Fire*** Two fire towers are 30 kilometers apart, where tower A is due west of tower B. A fire is spotted from the towers, and the bearings from A and B are E 14° N and W 34° N, respectively (see figure). Find the distance d of the fire from the line segment AB.



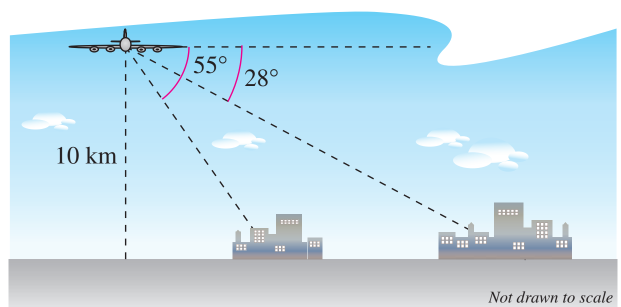
37. ***Navigation*** A ship is 45 miles east and 30 miles south of port. The captain wants to sail directly to port. What bearing should be taken?

38. ***Navigation*** A plane is 160 miles north and 85 miles east of an airport. The pilot wants to fly directly to the airport. What bearing should be taken?

39. ***Distance*** An observer in a lighthouse 350 feet above sea level observes two ships directly offshore. The angles of depression to the ships are 4° and 6.5° (see figure). How far apart are the ships?



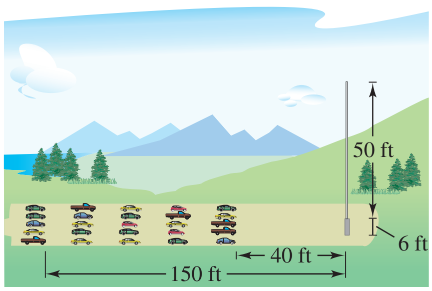
40. ***Distance*** A passenger in an airplane flying at an altitude of 10 kilometers sees two towns directly to the east of the plane. The angles of depression to the towns are 28° and 55° (see figure). How far apart are the towns?



41. ***Altitude*** A plane is observed approaching your home and you assume its speed is 550 miles per hour. The angle of elevation to the plane is 16° at one time and 57° one minute later. Approximate the altitude of the plane.

42. ***Height*** While traveling across flat land, you notice a mountain directly in front of you. The angle of elevation to the peak is 2.5°. After you drive 18 miles closer to the mountain, the angle of elevation is 10°. Approximate the height of the mountain.

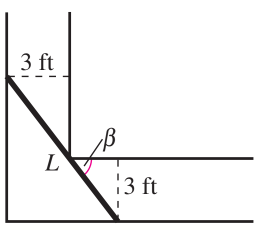
43. ***Angle of Elevation*** The top of a drive-in theater screen is 50 feet high and is mounted on a 6-foot-high cement wall. The nearest row of parking is 40 feet from the base of the wall. The furthest row of parking is 150 feet from the base of the wall.



(a) Find the angles of elevation to the *top* of the screen from both the closest row and the furthest row.

(b) How far from the base of the wall should you park if you want to have to look up to the top of the screen at an angle of 45°?

44. ***Moving*** A mattress of length L is being moved through two hallways that meet at right angles. Each hallway has a width of three feet (see figure).



(a) Show that the length of the mattress can be written as $L(\beta) = 3 \csc \beta + 3 \sec \beta$.

(b) Graph the function in part (a) for the interval $0 < \beta < \frac{\pi}{2}$.

(c) For what value(s) of β is the value of L the least ?

Geometry **In Exercises 45 and 46, find the angle α between the two nonvertical lines L_1 and L_2. The angle α satisfies the equation**

$$\tan \alpha = \left| \frac{m_2 - m_1}{1 + m_2 m_1} \right|$$

where m_1 and m_2 are the slopes of L_1 and L_2, respectively. (Assume $m_1 m_2 \neq -1$.)

45. L_1: $3x - 2y = 5$
L_2: $x + y = 1$

46. L_1: $2x + y = 8$
L_2: $x - 5y = -4$

47. ***Geometry*** Determine the angle between the diagonal of a cube and the diagonal of its base, as shown in the figure.

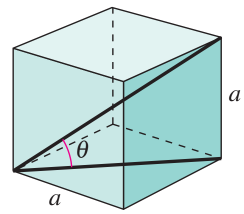


Figure for 47

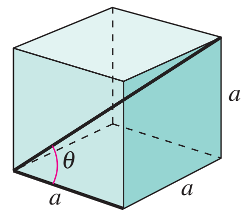
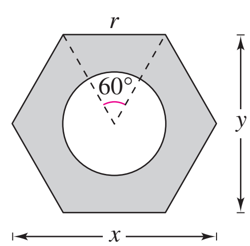


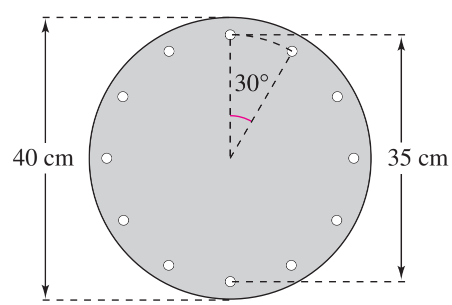
Figure for 48

48. ***Geometry*** Determine the angle between the diagonal of a cube and its edge, as shown in the figure.

49. ***Hardware*** Write the distance y across the flat sides of a hexagonal nut as a function of r, as shown in the figure.



50. ***Hardware*** The figure shows a circular piece of sheet metal of diameter 40 centimeters. The sheet contains 12 equally spaced bolt holes. Determine the straight-line distance between the centers of two consecutive bolt holes.



Harmonic Motion **In Exercises 51–54, find a model for simple harmonic motion satisfying the specified conditions.**

	Displacement ($t = 0$)	*Amplitude*	*Period*
51.	0	8 centimeters	2 seconds
52.	0	3 meters	6 seconds
53.	3 inches	3 inches	1.5 seconds
54.	2 feet	2 feet	10 seconds

Harmonic Motion **In Exercises 55–58, for the simple harmonic motion described by the trigonometric function, find (a) the maximum displacement, (b) the frequency, (c) the value of d when $t = 5$, and (d) the least positive value of t for which $d = 0$. Use a graphing utility to verify your results.**

55. $d = 4 \cos 8\pi t$

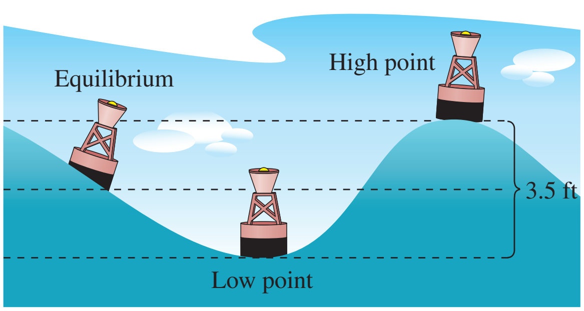
56. $d = \frac{1}{2} \cos 20\pi t$

57. $d = \frac{1}{16} \sin 140\pi t$

58. $d = \frac{1}{64} \sin 792\pi t$

59. ***Tuning Fork*** A point on the end of a tuning fork moves in the simple harmonic motion described by $d = a \sin \omega t$. Find ω given that the tuning fork for middle C has a frequency of 264 vibrations per second.

60. ***Wave Motion*** A buoy oscillates in simple harmonic motion as waves go past. At a given time it is noted that the buoy moves a total of 3.5 feet from its low point to its high point (see figure), and that it returns to its high point every 10 seconds. Write an equation that describes the motion of the buoy if it is at its high point at time $t = 0$.



61. ***Springs*** A ball that is bobbing up and down on the end of a spring has a maximum displacement of 3 inches. Its motion (in ideal conditions) is modeled by

$$y = \frac{1}{4} \cos 16t, \quad t > 0$$

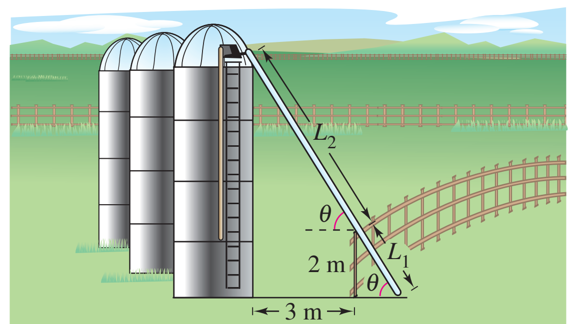
where y is measured in feet and t is the time in seconds.

(a) Use a graphing utility to graph the function.

(b) What is the period of the oscillations?

(c) Determine the first time the ball passes the point of equilibrium ($y = 0$).

62. ***Numerical and Graphical Analysis*** A two-meter-high fence is 3 meters from the side of a grain storage bin. A grain elevator must reach from ground level outside the fence to the storage bin (see figure). The objective is to determine the shortest elevator that meets the constraints.



(a) Complete four rows of the table.

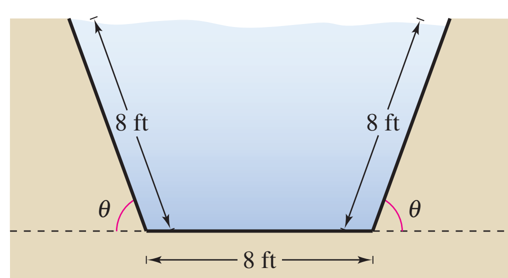
θ	L_1	L_2	$L_1 + L_2$
0.1	$\frac{2}{\sin 0.1}$	$\frac{3}{\cos 0.1}$	23.05
0.2	$\frac{2}{\sin 0.2}$	$\frac{3}{\cos 0.2}$	13.13

(b) Use the *table* feature of a graphing utility to generate additional rows of the table. Use the table to estimate the minimum length of the elevator.

(c) Write the length $L_1 + L_2$ as a function of θ.

(d) Use a graphing utility to graph the function. Use the graph to estimate the minimum length. How does your estimate compare with that in part (b)?

63. ***Numerical and Graphical Analysis*** The cross sections of an irrigation canal are isosceles trapezoids, where the lengths of three of the sides are 8 feet (see figure). The objective is to find the angle θ that maximizes the area of the cross sections. [*Hint:* The area of a trapezoid is given by $(h/2)(b_1 + b_2)$.]



(a) Complete seven rows of the table.

Base 1	*Base 2*	*Altitude*	*Area*
8	$8 + 16 \cos 10°$	$8 \sin 10°$	22.06
8	$8 + 16 \cos 20°$	$8 \sin 20°$	42.46

(b) Use the *table* feature of a graphing utility to generate additional rows of the table. Use the table to estimate the maximum cross-sectional area.

(c) Write the area A as a function of θ.

(d) Use a graphing utility to graph the function. Use the graph to estimate the maximum cross-sectional area. How does your estimate compare with that in part (b)?

64. ***Data Analysis*** The table shows the average sales S (in millions of dollars) of an outerwear manufacturer for each month t, where $t = 1$ represents January.

Month, t	**Sales, S**
1	13.46
2	11.15
3	8.00
4	4.85
5	2.54
6	1.70
7	2.54
8	4.85
9	8.00
10	11.15
11	13.46
12	14.30

(a) Create a scatter plot of the data.

(b) Find a trigonometric model that fits the data. Graph the model on your scatter plot. How well does the model fit the data?

(c) What is the period of the model? Do you think it is reasonable given the context? Explain your reasoning.

(d) Interpret the meaning of the model's amplitude in the context of the problem.

65. ***Data Analysis*** The times S of sunset (Greenwich Mean Time) at 40° north latitude on the 15th of each month are: 1(16:59), 2(17:35), 3(18:06), 4(18:38), 5(19:08), 6(19:30), 7(19:28), 8(18:57), 9(18:09), 10(17:21), 11(16:44), and 12(16:36). The month is represented by t, with $t = 1$ corresponding to January. A model (in which minutes have been converted to the decimal parts of an hour) for the data is given by

$$S(t) = 18.09 + 1.41 \sin\left(\frac{\pi t}{6} + 4.60\right).$$

(a) Use a graphing utility to graph the data points and the model in the same viewing window.

(b) What is the period of the model? Is it what you expected? Explain.

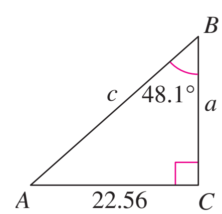
(c) What is the amplitude of the model? What does it represent in the context of the problem? Explain.

66. ***Writing*** Is it true that N 24° E means 24 degrees north of east? Explain.

Synthesis

True or False? **In Exercises 67 and 68, determine whether the statement is true or false. Justify your answer.**

67. In the right triangle shown below, $a = \dfrac{22.56}{\tan 41.9°}$.



68. For the harmonic motion of a ball bobbing up and down on the end of a spring, one period can be described as the length of one coil of the spring.

Skills Review

In Exercises 69–72, write the standard form of the equation of the line that has the specified characteristics.

69. $m = 4$, passes through $(-1, 2)$

70. $m = -\frac{1}{2}$, passes through $\left(\frac{1}{3}, 0\right)$

71. Passes through $(-2, 6)$ and $(3, 2)$

72. Passes through $\left(\frac{1}{4}, -\frac{2}{3}\right)$ and $\left(-\frac{1}{2}, \frac{1}{3}\right)$

In Exercises 73–76, find the domain of the function.

73. $f(x) = 3x + 8$

74. $f(x) = -x^2 - 1$

75. $g(x) = \sqrt[3]{x + 2}$

76. $g(x) = \sqrt{7 - x}$