6.2 Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

Vocabulary Check

Fill in the blanks.

1. The standard form of the Law of Cosines for $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$ is ________ .

2. ________ Formula is established by using the Law of Cosines.

3. Three different formulas for the area of a triangle are given by Area = ________ , Area $= \frac{1}{2}bc \sin A = \frac{1}{2}ab \sin C = \frac{1}{2}ac \sin B$, and Area = ________ .

In Exercises 1–20, use the Law of Cosines to solve the triangle.

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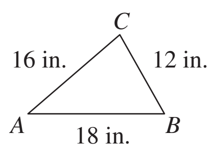

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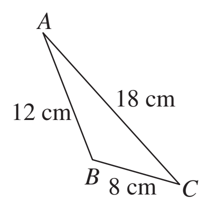

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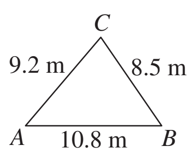

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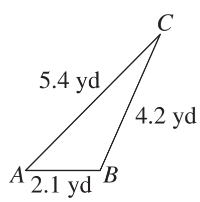

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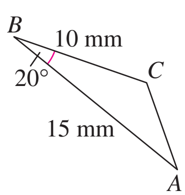

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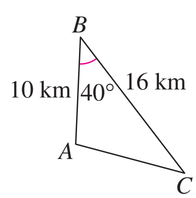

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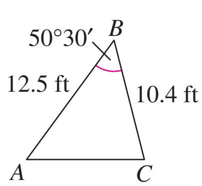

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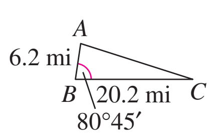


9. $a = 6, \quad b = 8, \quad c = 12$

10. $a = 9, \quad b = 3, \quad c = 11$

11. $A = 50°, \quad b = 15, \quad c = 30$

12. $C = 108°, \quad a = 10, \quad b = 7$

13. $a = 9, \quad b = 12, \quad c = 15$

14. $a = 45, \quad b = 30, \quad c = 72$

15. $a = 75.4, \quad b = 48, \quad c = 48$

16. $a = 1.42, \quad b = 0.75, \quad c = 1.25$

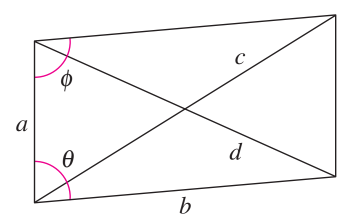
17. $B = 8° \, 15', \quad a = 26, \quad c = 18$

18. $B = 10° \, 35', \quad a = 40, \quad c = 30$

19. $B = 75° \, 20', \quad a = 6.2, \quad c = 9.5$

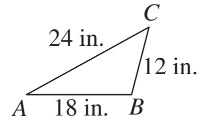
20. $C = 15° \, 15', \quad a = 6.25, \quad b = 2.15$

In Exercises 21–26, complete the table by solving the parallelogram shown in the figure. (The lengths of the diagonals are given by *c* and *d*.)



	a	b	c	d	θ	ϕ
21.	4	8			30°	
22.	25	35				120°
23.	10	14	20			
24.	40	60		80		
25.	15		25	20		
26.		25	50	35		

In Exercises 27–36, use Heron's Area Formula to find the area of the triangle.

27.


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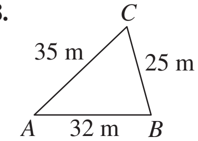
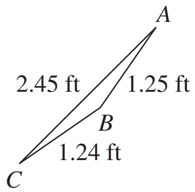


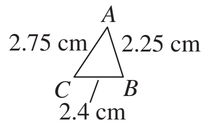
29. $a = 5, \quad b = 8, \quad c = 10$

30. $a = 14, \quad b = 17, \quad c = 7$

31.



32.



33. $a = 3.5, \quad b = 10.2, \quad c = 9$

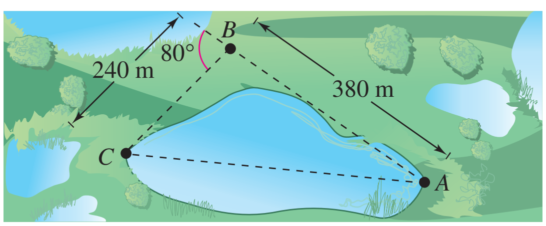
34. $a = 75.4, \quad b = 52, \quad c = 52$

35. $a = 10.59, \quad b = 6.65, \quad c = 12.31$

36. $a = 4.45, \quad b = 1.85, \quad c = 3.00$

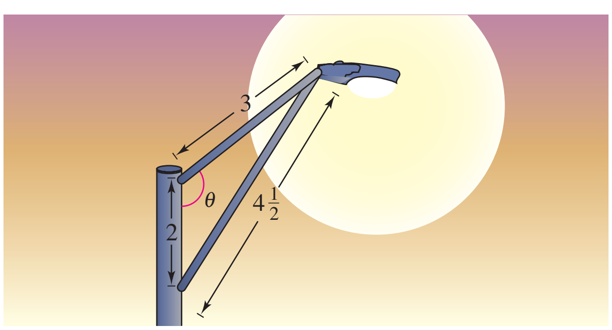
37. ***Navigation*** A plane flies 810 miles from Franklin to Centerville with a bearing of 75° (clockwise from north). Then it flies 648 miles from Centerville to Rosemont with a bearing of 32°. Draw a diagram that visually represents the problem, and find the straight-line distance and bearing from Rosemont to Franklin.

38. ***Surveying*** To approximate the length of a marsh, a surveyor walks 380 meters from point A to point B. Then the surveyor turns 80° and walks 240 meters to point C (see figure). Approximate the length AC of the marsh.



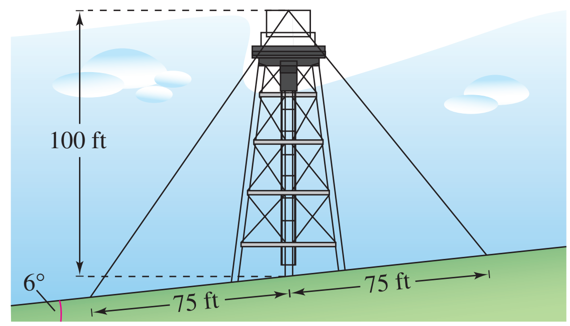
39. ***Navigation*** A boat race runs along a triangular course marked by buoys A, B, and C. The race starts with the boats headed west for 3600 meters. The other two sides of the course lie to the north of the first side, and their lengths are 1500 meters and 2800 meters. Draw a diagram that visually represents the problem, and find the bearings for the last two legs of the race.

40. ***Streetlight Design*** Determine the angle θ in the design of the streetlight shown in the figure.

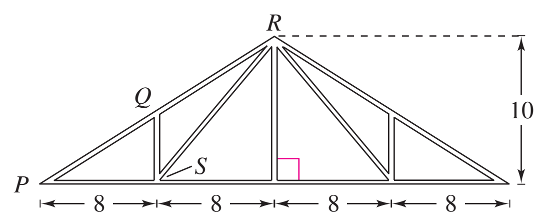


41. ***Distance*** Two ships leave a port at 9 A.M. One travels at a bearing of N 53° W at 12 miles per hour, and the other travels at a bearing of S 67° W at 16 miles per hour. Approximate how far apart the ships are at noon that day.

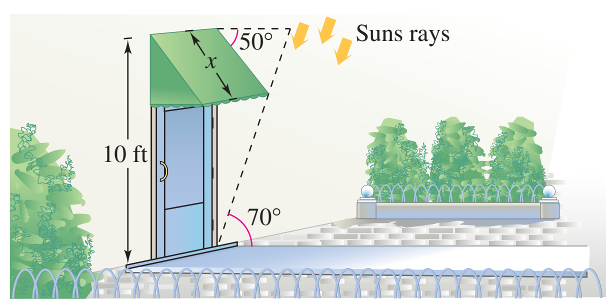
42. ***Length*** A 100-foot vertical tower is to be erected on the side of a hill that makes a 6° angle with the horizontal (see figure). Find the length of each of the two guy wires that will be anchored 75 feet uphill and downhill from the base of the tower.



43. ***Trusses*** Q is the midpoint of the line segment $\overline{PR}$ in the truss rafter shown in the figure. What are the lengths of the line segments $\overline{PQ}$, $\overline{QS}$, and $\overline{RS}$?

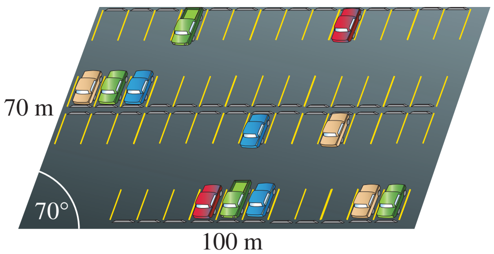


44. ***Awning Design*** A retractable awning above a patio lowers at an angle of 50° from the exterior wall at a height of 10 feet above the ground (see figure). No direct sunlight is to enter the door when the angle of elevation of the sun is greater than 70°. What is the length x of the awning?



45. ***Landau Building*** The Landau Building in Cambridge, Massachusetts has a triangular-shaped base. The lengths of the sides of the triangular base are 145 feet, 257 feet, and 290 feet. Find the area of the base of the building.

46. ***Geometry*** A parking lot has the shape of a parallelogram (see figure). The lengths of two adjacent sides are 70 meters and 100 meters. The angle between the two sides is 70°. What is the area of the parking lot?



47. ***Engine Design*** An engine has a seven-inch connecting rod fastened to a crank (see figure).

(a) Use the Law of Cosines to write an equation giving the relationship between x and θ.

(b) Write x as a function of θ. (Select the sign that yields positive values of x.)

(c) Use a graphing utility to graph the function in part (b).

(d) Use the graph in part (c) to determine the total distance the piston moves in one cycle.

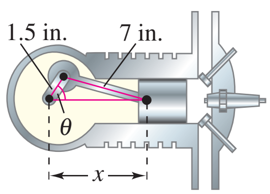


Figure for 47

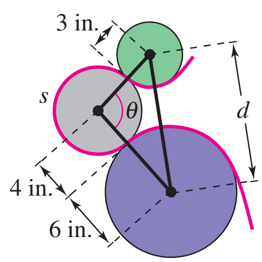


Figure for 48

48. ***Manufacturing*** In a process with continuous paper, the paper passes across three rollers of radii 3 inches, 4 inches, and 6 inches (see figure). The centers of the three-inch and six-inch rollers are d inches apart, and the length of the arc in contact with the paper on the four-inch roller is s inches.

(a) Use the Law of Cosines to write an equation giving the relationship between d and θ.

(b) Write θ as a function of d.

(c) Write s as a function of θ.

(d) Complete the table.

d (inches)	9	10	12	13	14	15	16
θ (degrees)							
s (inches)							

Synthesis

True or False? **In Exercises 49–51, determine whether the statement is true or false. Justify your answer.**

49. A triangle with side lengths of 10 feet, 16 feet, and 5 feet can be solved using the Law of Cosines.

50. Two sides and their included angle determine a unique triangle.

51. In Heron's Area Formula, s is the average of the lengths of the three sides of the triangle.

Proofs **In Exercises 52–54, use the Law of Cosines to prove each of the following.**

52. $\frac{1}{2}bc(1 + \cos A) = \left(\frac{a + b + c}{2}\right)\left(\frac{-a + b + c}{2}\right)$

53. $\frac{1}{2}bc(1 - \cos A) = \left(\frac{a - b + c}{2}\right)\left(\frac{a + b - c}{2}\right)$

54. $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} = \frac{a^2 + b^2 + c^2}{2abc}$

55. ***Proof*** Use a half-angle formula and the Law of Cosines to show that, for any triangle,

$$\cos\left(\frac{C}{2}\right) = \sqrt{\frac{s(s - c)}{ab}}$$

where $s = \frac{1}{2}(a + b + c)$.

56. ***Proof*** Use a half-angle formula and the Law of Cosines to show that, for any triangle,

$$\sin\left(\frac{C}{2}\right) = \sqrt{\frac{(s - a)(s - b)}{ab}}$$

where $s = \frac{1}{2}(a + b + c)$.

57. ***Writing*** Describe how the Law of Cosines can be used to solve the ambiguous case of the oblique triangle ABC, where $a = 12$ feet, $b = 30$ feet, and $A = 20°$. Is the result the same as when the Law of Sines is used to solve the triangle? Describe the advantages and the disadvantages of each method.

58. ***Writing*** In Exercise 57, the Law of Cosines was used to solve a triangle in the two-solution case of SSA. Can the Law of Cosines be used to solve the no-solution and single-solution cases of SSA? Explain.

Skills Review

In Exercises 59–62, evaluate the expression without using a calculator.

59. $\arcsin(-1)$

60. $\cos^{-1} 0$

61. $\tan^{-1} \sqrt{3}$

62. $\arcsin\left(-\frac{\sqrt{3}}{2}\right)$