In Exercises 15–22, use the Leading Coefficient Test to describe the right-hand and left-hand behavior of the graph of the polynomial function. Use a graphing utility to verify your result.

15. $f(x) = 2x^4 - 3x + 1$

16. $h(x) = 1 - x^6$

17. $g(x) = 5 - \frac{7}{2}x - 3x^2$

18. $f(x) = \frac{1}{3}x^3 + 5x$

19. $f(x) = \dfrac{6x^5 - 2x^4 + 4x^2 - 5x}{3}$

20. $f(x) = \dfrac{3x^7 - 2x^5 + 5x^3 + 6x^2}{4}$

21. $h(t) = -\frac{2}{3}(t^2 - 5t + 3)$

22. $f(s) = -\frac{7}{8}(s^3 + 5s^2 - 7s + 1)$

In Exercises 23–32, find all the real zeros of the polynomial function. Determine the multiplicity of each zero. Use a graphing utility to verify your result.

23. $f(x) = x^2 - 25$

24. $f(x) = 49 - x^2$

25. $h(t) = t^2 - 6t + 9$

26. $f(x) = x^2 + 10x + 25$

27. $f(x) = x^2 + x - 2$

28. $f(x) = 2x^2 - 14x + 24$

29. $f(t) = t^3 - 4t^2 + 4t$

30. $f(x) = x^4 - x^3 - 20x^2$

31. $f(x) = \frac{1}{2}x^2 + \frac{5}{2}x - \frac{3}{2}$

32. $f(x) = \frac{5}{3}x^2 + \frac{8}{3}x - \frac{4}{3}$

Graphical Analysis **In Exercises 33–44, (a) find the zeros algebraically, (b) use a graphing utility to graph the function, and (c) use the graph to approximate any zeros and compare them with those from part (a).**

33. $f(x) = 3x^2 - 12x + 3$

34. $g(x) = 5x^2 - 10x - 5$

35. $g(t) = \frac{1}{2}t^4 - \frac{1}{2}$

36. $y = \frac{1}{4}x^3(x^2 - 9)$

37. $f(x) = x^5 + x^3 - 6x$

38. $g(t) = t^5 - 6t^3 + 9t$

39. $f(x) = 2x^4 - 2x^2 - 40$

40. $f(x) = 5x^4 + 15x^2 + 10$

41. $f(x) = x^3 - 4x^2 - 25x + 100$

42. $y = 4x^3 + 4x^2 - 7x + 2$

43. $y = 4x^3 - 20x^2 + 25x$

44. $y = x^5 - 5x^3 + 4x$

In Exercises 45–48, use a graphing utility to graph the function and approximate (accurate to three decimal places) any real zeros and relative extrema.

45. $f(x) = 2x^4 - 6x^2 + 1$

46. $f(x) = -\frac{3}{8}x^4 - x^3 + 2x^2 + 5$

47. $f(x) = x^5 + 3x^3 - x + 6$

48. $f(x) = -3x^3 - 4x^2 + x - 3$

In Exercises 49–58, find a polynomial function that has the given zeros. (There are many correct answers.)

49. 0, 4

50. $-7, 2$

51. $0, -2, -3$

52. 0, 2, 5

53. $4, -3, 3, 0$

54. $-2, -1, 0, 1, 2$

55. $1 + \sqrt{3}, 1 - \sqrt{3}$

56. $6 + \sqrt{3}, 6 - \sqrt{3}$

57. $2, 4 + \sqrt{5}, 4 - \sqrt{5}$

58. $4, 2 + \sqrt{7}, 2 - \sqrt{7}$

In Exercises 59–64, find a polynomial function with the given zeros, multiplicities, and degree. (There are many correct answers.)

59. Zero: -2, multiplicity: 2
Zero: -1, multiplicity: 1
Degree: 3

60. Zero: 3, multiplicity: 1
Zero: 2, multiplicity: 3
Degree: 4

61. Zero: -4, multiplicity: 2
Zero: 3, multiplicity: 2
Degree: 4

62. Zero: -5, multiplicity: 3
Zero: 0, multiplicity: 2
Degree: 5

63. Zero: -1, multiplicity: 2
Zero: -2, multiplicity: 1
Degree: 3
Rises to the left,
Falls to the right

64. Zero: -1, multiplicity: 2
Zero: 4, multiplicity: 2
Degree: 4
Falls to the left,
Falls to the right

In Exercises 65–68, sketch the graph of a polynomial function that satisfies the given conditions. If not possible, explain your reasoning. (There are many correct answers.)

65. Third-degree polynomial with two real zeros and a negative leading coefficient

66. Fourth-degree polynomial with three real zeros and a positive leading coefficient

67. Fifth-degree polynomial with three real zeros and a positive leading coefficient

68. Fourth-degree polynomial with two real zeros and a negative leading coefficient

In Exercises 69–78, sketch the graph of the function by (a) applying the Leading Coefficient Test, (b) finding the zeros of the polynomial, (c) plotting sufficient solution points, and (d) drawing a continuous curve through the points.

69. $f(x) = x^3 - 9x$

70. $g(x) = x^4 - 4x^2$

71. $f(x) = x^3 - 3x^2$

72. $f(x) = 3x^3 - 24x^2$

73. $f(x) = -x^4 + 9x^2 - 20$

74. $f(x) = -x^6 + 7x^3 + 8$

75. $f(x) = x^3 + 3x^2 - 9x - 27$

76. $h(x) = x^5 - 4x^3 + 8x^2 - 32$

77. $g(t) = -\frac{1}{4}t^4 + 2t^2 - 4$

78. $g(x) = \frac{1}{10}(x^4 - 4x^3 - 2x^2 + 12x + 9)$

In Exercises 79–82, (a) use the Intermediate Value Theorem and a graphing utility to find graphically any intervals of length 1 in which the polynomial function is guaranteed to have a zero, and (b) use the *zero* or *root* feature of the graphing utility to approximate the real zeros of the function. Verify your answers in part (a) by using the *table* feature of the graphing utility.

79. $f(x) = x^3 - 3x^2 + 3$

80. $f(x) = -2x^3 - 6x^2 + 3$

81. $g(x) = 3x^4 + 4x^3 - 3$

82. $h(x) = x^4 - 10x^2 + 2$

In Exercises 83–90, use a graphing utility to graph the function. Identify any symmetry with respect to the *x*-axis, *y*-axis, or origin. Determine the number of *x*-intercepts of the graph.

83. $f(x) = x^2(x + 6)$

84. $h(x) = x^3(x - 4)^2$

85. $g(t) = -\frac{1}{2}(t - 4)^2(t + 4)^2$

86. $g(x) = \frac{1}{8}(x + 1)^2(x - 3)^3$

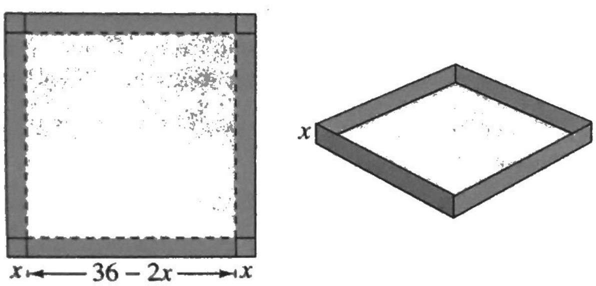
87. $f(x) = x^3 - 4x$

88. $f(x) = x^4 - 2x^2$

89. $g(x) = \frac{1}{5}(x + 1)^2(x - 3)(2x - 9)$

90. $h(x) = \frac{1}{5}(x + 2)^2(3x - 5)^2$

91. ***Numerical and Graphical Analysis*** An open box is to be made from a square piece of material 36 centimeters on a side by cutting equal squares with sides of length x from the corners and turning up the sides (see figure).



(a) Verify that the volume of the box is given by the function $V(x) = x(36 - 2x)^2$.

(b) Determine the domain of the function V.

(c) Use the *table* feature of a graphing utility to create a table that shows various box heights x and the corresponding volumes V. Use the table to estimate a range of dimensions within which the maximum volume is produced.

(d) Use a graphing utility to graph V and use the range of dimensions from part (c) to find the x-value for which $V(x)$ is maximum.

92. ***Geometry*** An open box with locking tabs is to be made from a square piece of material 24 inches on a side. This is done by cutting equal squares from the corners and folding along the dashed lines, as shown in the figure.

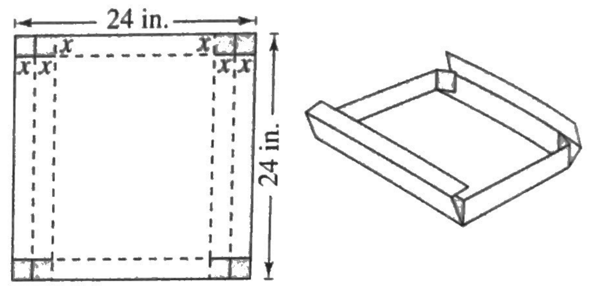


Figure for 92

(a) Verify that the volume of the box is given by the function $V(x) = 8x(6 - x)(12 - x)$.

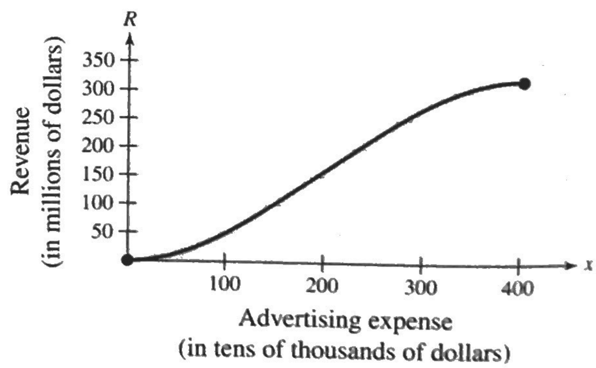
(b) Determine the domain of the function V.

(c) Sketch the graph of the function and estimate the value of x for which $V(x)$ is maximum.

93. ***Revenue*** The total revenue R (in millions of dollars) for a company is related to its advertising expense by the function

$$R = 0.00001(-x^3 + 600x^2),\ 0 \le x \le 400$$

where x is the amount spent on advertising (in tens of thousands of dollars). Use the graph of the function shown in the figure to estimate the point on the graph at which the function is increasing most rapidly. This point is called the **point of diminishing returns** because any expense above this amount will yield less return per dollar invested in advertising.



94. ***Environment*** The growth of a red oak tree is approximated by the function

$$G = -0.003t^3 + 0.137t^2 + 0.458t - 0.839$$

where G is the height of the tree (in feet) and t $(2 \le t \le 34)$ is its age (in years). Use a graphing utility to graph the function and estimate the age of the tree when it is growing most rapidly. This point is called the **point of diminishing returns** because the increase in growth will be less with each additional year. (*Hint:* Use a viewing window in which $0 \le x \le 35$ and $0 \le y \le 60$.)