

$$\begin{aligned}
 \text{Solution 2} \quad 2ab - 6ac + 3b - 9c &= (2ab + 3b) - (6ac + 9c) \\
 &= b(2a + 3) - 3c(2a + 3) \\
 &= (b - 3c)(2a + 3) \quad \text{Answer}
 \end{aligned}$$

Example 4 uses what you know about factoring perfect square trinomials and differences of squares.

$$\text{Example 4} \quad \text{Factor } 4p^2 - 4q^2 + 4qr - r^2.$$

$$\begin{aligned}
 \text{Solution} \quad 4p^2 - 4q^2 + 4qr - r^2 &= 4p^2 - (4q^2 - 4qr + r^2) \leftarrow \text{A trinomial square} \\
 &= (2p)^2 - (2q - r)^2 \leftarrow \text{The difference of two squares.} \\
 &= [2p + (2q - r)][2p - (2q - r)] \\
 &= (2p + 2q - r)(2p - 2q + r) \quad \text{Answer}
 \end{aligned}$$

In Example 4 you could have tried the grouping $(4p^2 - 4q^2) + (4qr - r^2)$ and factored the groups to obtain $4(p + q)(p - q) + r(4q - r)$. But this doesn't lead anywhere. There are different approaches to factoring a polynomial. You may need to try several before arriving at one that works.

Oral Exercises

Factor.

- | | | |
|---------------------------|--------------------------|---------------------------|
| 1. $a(a - 2) + 3(a - 2)$ | 2. $p(q + 1) - 4(q + 1)$ | 3. $2r(r - 3) - 5(r - 3)$ |
| 4. $x(x + 2y) - (x + 2y)$ | 5. $u(u + v) - v(u + v)$ | 6. $h(h - 2) + 2(2 - h)$ |
| 7. $x(x - 4) - (4 - x)$ | 8. $m(n - m) - n(m - n)$ | 9. $2r(r - s) + s(s - r)$ |

Written Exercises

Factor. Check by multiplying the factors.

- | | |
|---|---------------------------------------|
| A 1. $3(x + y) + z(x + y)$ | 2. $7(r - s) + t(r - s)$ |
| 3. $e(f - g) - 4(f - g)$ | 4. $w(x - y) - 8(x - y)$ |
| 5. $7(r - s) + t(s - r)$ | 6. $7(m - n) + p(n - m)$ |
| 7. $2a(a + 3) - (3 + a)$ | 8. $u(v - 2) + 2(2 - v)$ |
| 9. $2x(x - y) + y(y - x)$ | 10. $3p(2q - p) - 2q(p - 2q)$ |
| 11. $2u(u - 2v) + v(u - 2v) + (u - 2v)$ | 12. $a(a - b) + 4b(a - b) - a(a - b)$ |
| 13. $x(2w - 3v + u) - (2w - 3v + u)$ | 14. $r(r - s - 2t) + s(r - s - 2t)$ |
| 15. $(s^2 - 2ps + 2s) - (2s - 4p + 4)$ | 16. $(x^2 - xy + x) - (y - x - 1)$ |
| 17. $(3t - 3st) + (rs - r)$ | 18. $(9p - 3pq) + (2nq - 6n)$ |
| 19. $(12x^2 - 8xy) - 5(3xz - 2yz)$ | 20. $(p^2 - 2pq) - 2(2qr - pr)$ |

Factor. Check by multiplying the factors.

21. $3a + ab + 3c + bc$

23. $x^2 - 2x + xy - 2y$

25. $h^2 - hk + hr - kr$

27. $p^3 - 2p^2 + 4p - 8$

29. $p^2 - 2pq + pr - 2qr$

31. $3hk - 2k - 12h + 8$

33. $4z^3 - 6z^2 - 6z + 9$

35. $(h^2k^2 + 4k^2) + (h^2k + 4k)$

37. $x^3 - 3x^2 - x + 3$

22. $rs + 5r + st + 5t$

24. $u^2 - 2u + uv - 2v$

26. $x^2 - 2xy + 4xz - 8yz$

28. $3a^3 + a^2 + 6a + 2$

30. $u^2 - 3uv - 6uw + 18vw$

32. $3ab - b - 4 + 12a$

34. $3u^3 - u^2 - 9u + 3$

36. $(a^2b^2 + 2a^2) - (2ab^2 + 4a)$

38. $n^3 + 2n^2 - 4n - 8$

Factor each expression as a difference of squares.

39. $x^2 - (y - z)^2$

41. $(u - 2v)^2 - 4w^2$

40. $(a + 2b)^2 - 9c^2$

42. $4p^2 - (q - 2r)^2$

B 43. $(a + 2b)^2 - (2b + c)^2$

45. $a^2 + 4a + 4 - b^2$

47. $u^2 - v^2 + 2v - 1$

49. $h^2 - 4k^2 - 4h + 4$

51. $p^2 - q^2 + r^2 - 2pr$

44. $4(x + y)^2 - (2y - z)^2$

46. $x^2 - 2xy + y^2 - 4$

48. $m^2 - n^2 - 2m + 1$

50. $a^2 - b^2 - 2a + 1$

52. $4s^2 - 4t^2 + 4s + 1$

Factor.

53. $x^2 - 4y^2 + 4z^2 - 4xz$

55. $a^2 + b^2 + 2ab + 2a + 2b$

57. $x^4 - y^4 - 4x^2 + 4$

59. $p^2 + q^2 - r^2 - 2pq + 2r - 1$

54. $m^2 - 9n^2 + 9 - 6m$

56. $p^2 - q^2 - 2p + 2q$

58. $a^4 + b^4 - c^4 + 2a^2b^2$

60. $h^2 - 4k^2 + 4h - 8k$

C 61. Factor $x^4 + 4$ by writing it as $(x^4 + 4x^2 + 4) - 4x^2$, a difference of two squares.

62. Use the method of Exercise 61 to factor (a) $64x^4 + 1$ and (b) $x^4 + 4a^4$.

63. Factor $a^{2n+1} + b^{2n+1} + a^{2n}b^{2n} + ab$, where n is a positive integer.

Mixed Review Exercises

Solve.

1. $-12 + x = -29$

2. $-n + 10 = 2$

3. $18 + x = 32$

4. $16 = 1 + 3z$

5. $10m - 6m = 36$

6. $5n - 2n + 8 = 9$

7. $14x = 700$

8. $-13n = 156$

9. $9b = 108$

10. $10n = 2n - 24$

11. $19m = 55 + 14m$

12. $10y + 6 = 4(19 - y)$