6.1 Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

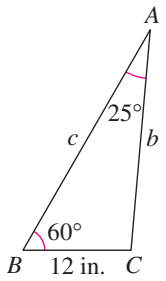
Vocabulary Check

Fill in the blanks.

1. An _____ triangle is one that has no right angles.
2. Law of Sines: $\frac{a}{\sin A} =$ _____ $= \frac{c}{\sin C}$
3. The Law of Sines can be used to solve a triangle for cases
   (a) _____ angle(s) and _____ side(s), which can be denoted _____ or _____ ,
   (b) _____ side(s) and _____ angle(s), which can be denoted _____ .
4. To find the area of any triangle, use one of the following three formulas: Area = _____ , _____ , or _____ .

In Exercises 1–18, use the Law of Sines to solve the triangle. If two solutions exist, find both.

1.



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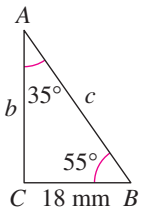

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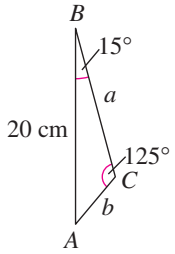

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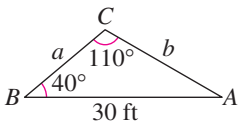

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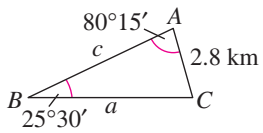

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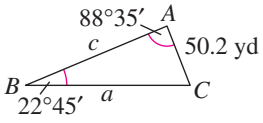


7. $A = 36°, \quad a = 8, \quad b = 5$

8. $A = 60°, \quad a = 9, \quad c = 10$

9. $A = 102.4°, \quad C = 16.7°, \quad a = 21.6$

10. $A = 24.3°, \quad C = 54.6°, \quad c = 2.68$

11. $A = 110° \, 15', \quad a = 48, \quad b = 16$

12. $B = 2° \, 45', \quad b = 6.2, \quad c = 5.8$

13. $A = 110°, \quad a = 125, \quad b = 100$

14. $A = 110°, \quad a = 125, \quad b = 200$

15. $A = 76°, \quad a = 18, \quad b = 20$

16. $A = 76°, \quad a = 34, \quad b = 21$

17. $A = 58°, \quad a = 11.4, \quad b = 12.8$

18. $A = 58°, \quad a = 4.5, \quad b = 12.8$

In Exercises 19–24, find the area of the triangle having the indicated angle and sides.

19. $C = 110°, \quad a = 6, \quad b = 10$

20. $B = 130°, \quad a = 92, \quad c = 30$

21. $A = 38° \, 45', \quad b = 67, \quad c = 85$

22. $A = 5° \, 15', \quad b = 4.5, \quad c = 22$

23. $B = 75° \, 15', \quad a = 103, \quad c = 58$

24. $C = 85° \, 45', \quad a = 16, \quad b = 20$

25. ***Height*** A flagpole at a right angle to the horizontal is located on a slope that makes an angle of 14° with the horizontal. The flagpole casts a 16-meter shadow up the slope when the angle of elevation from the tip of the shadow to the sun is 20°.

(a) Draw a triangle that represents the problem. Show the known quantities on the triangle and use a variable to indicate the height of the flagpole.

(b) Write an equation involving the unknown quantity.

(c) Find the height of the flagpole.

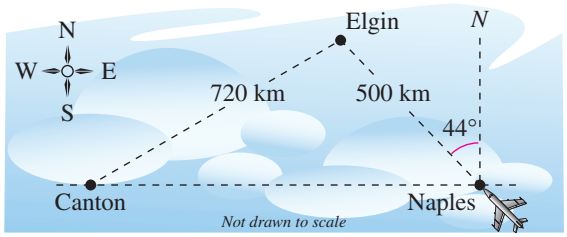
26. ***Height*** You are standing 40 meters from the base of a tree that is leaning 8° from the vertical away from you. The angle of elevation from your feet to the top of the tree is $20° \, 50'$.

(a) Draw a triangle that represents the problem. Show the known quantities on the triangle and use a variable to indicate the height of the tree.

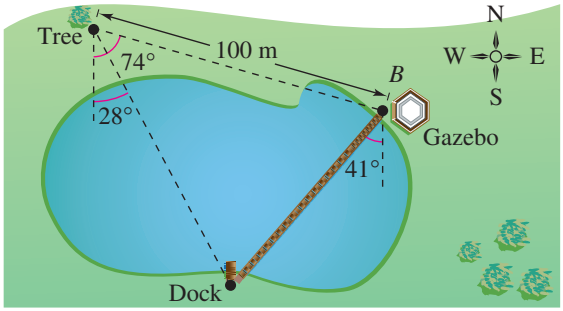
(b) Write an equation involving the unknown height of the tree.

(c) Find the height of the tree.

27. ***Flight Path*** A plane flies 500 kilometers with a bearing of 316° (clockwise from north) from Naples to Elgin (see figure). The plane then flies 720 kilometers from Elgin to Canton. Find the bearing of the flight from Elgin to Canton.



28. ***Bridge Design*** A bridge is to be built across a small lake from a gazebo to a dock (see figure). The bearing from the gazebo to the dock is S 41° W. From a tree 100 meters from the gazebo, the bearings to the gazebo and the dock are S 74° E and S 28° E, respectively. Find the distance from the gazebo to the dock.



29. ***Railroad Track Design*** The circular arc of a railroad curve has a chord of length 3000 feet and a central angle of 40°.

(a) Draw a diagram that visually represents the problem. Show the known quantities on the diagram and use the variables r and s to represent the radius of the arc and the length of the arc, respectively.

(b) Find the radius r of the circular arc.

(c) Find the length s of the circular arc.

30. ***Glide Path*** A pilot has just started on the glide path for landing at an airport with a runway of length 9000 feet. The angles of depression from the plane to the ends of the runway are 17.5° and 18.8°.

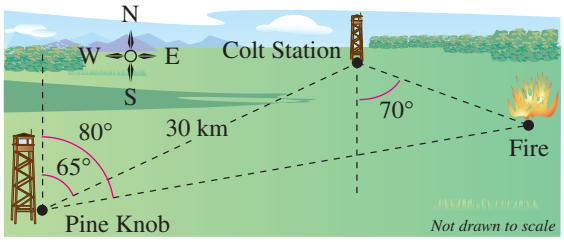
(a) Draw a diagram that visually represents the problem.

(b) Find the air distance the plane must travel until touching down on the near end of the runway.

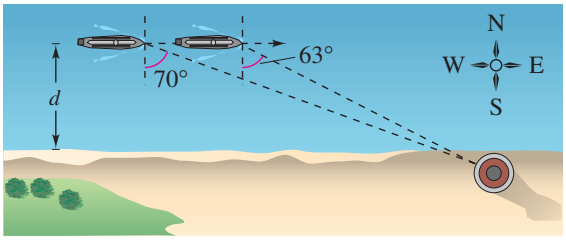
(c) Find the ground distance the plane must travel until touching down.

(d) Find the altitude of the plane when the pilot begins the descent.

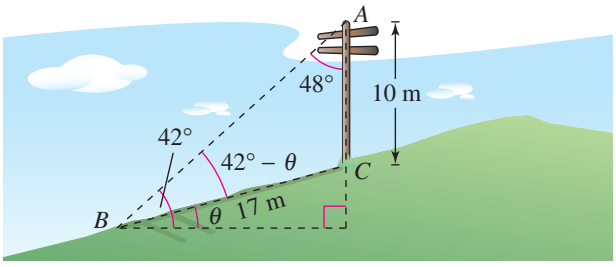
31. ***Locating a Fire*** The bearing from the Pine Knob fire tower to the Colt Station fire tower is N 65° E, and the two towers are 30 kilometers apart. A fire spotted by rangers in each tower has a bearing of N 80° E from Pine Knob and S 70° E from Colt Station. Find the distance of the fire from each tower.



32. ***Distance*** A boat is sailing due east parallel to the shoreline at a speed of 10 miles per hour. At a given time the bearing to a lighthouse is S 70° E, and 15 minutes later the bearing is S 63° E (see figure). The lighthouse is located at the shoreline. Find the distance from the boat to the shoreline.



33. ***Angle of Elevation*** A 10-meter telephone pole casts a 17-meter shadow directly down a slope when the angle of elevation of the sun is 42° (see figure). Find θ, the angle of elevation of the ground.

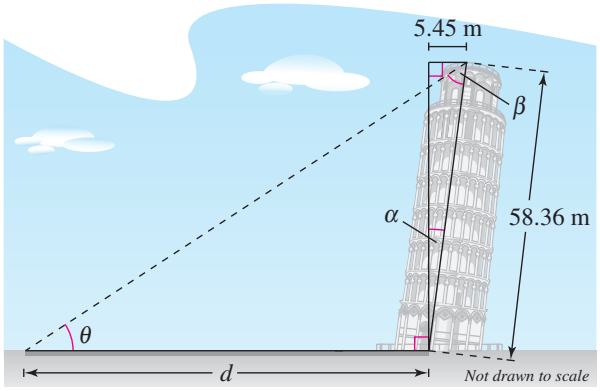


34. ***Distance*** The angles of elevation θ and ϕ to an airplane are being continuously monitored at two observation points A and B, respectively, which are 2 miles apart, and the airplane is east of both points in the same vertical plane.

(a) Draw a diagram that illustrates the problem.

(b) Write an equation giving the distance d between the plane and point B in terms of θ and ϕ.

35. ***Shadow Length*** The Leaning Tower of Pisa in Italy leans because it was built on unstable soil—a mixture of clay, sand, and water. The tower is approximately 58.36 meters tall from its foundation (see figure). The top of the tower leans about 5.45 meters off center.



(a) Find the angle of lean α of the tower.

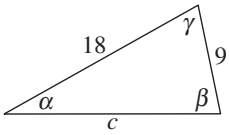
(b) Write β as a function of d and θ, where θ is the angle of elevation to the sun.

(c) Use the Law of Sines to write an equation for the length d of the shadow cast by the tower in terms of θ.

(d) Use a graphing utility to complete the table.

θ	10°	20°	30°	40°	50°	60°
d						

36. ***Graphical and Numerical Analysis*** In the figure, α and β are positive angles.



(a) Write α as a function of β.

(b) Use a graphing utility to graph the function. Determine its domain and range.

(c) Use the result of part (b) to write c as a function of β.

(d) Use a graphing utility to graph the function in part (c). Determine its domain and range.

(e) Use a graphing utility to complete the table. What can you conclude?

β	0.4	0.8	1.2	1.6	2.0	2.4	2.8
α							
c							

Synthesis

True or False? **In Exercises 37 and 38, determine whether the statement is true or false. Justify your answer.**

37. If any three sides or angles of an oblique triangle are known, then the triangle can be solved.

38. If a triangle contains an obtuse angle, then it must be oblique.

39. ***Writing*** Can the Law of Sines be used to solve a right triangle? If so, write a short paragraph explaining how to use the Law of Sines to solve the following triangle. Is there an easier way to solve the triangle? Explain.

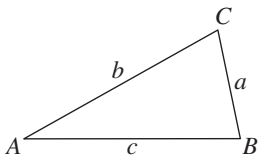
$B = 50°, C = 90°, a = 10$

40. ***Think About It*** Given $A = 36°$ and $a = 5$, find values of b such that the triangle has (a) one solution, (b) two solutions, and (c) no solution.

Mollweide's Formula **In Exercises 41 and 42, solve the triangle. Then use one of the two forms of Mollweide's Formula to verify the results.**

$$(a + b)\sin\left(\frac{C}{2}\right) = c\cos\left(\frac{A - B}{2}\right) \qquad \text{Form 1}$$

$$(a - b)\cos\left(\frac{C}{2}\right) = c\sin\left(\frac{A - B}{2}\right) \qquad \text{Form 2}$$



41. Solve the triangle for $A = 45°$, $B = 52°$, and $a = 16$. Then use form 1 of Mollweide's Formula to verify your solution.

42. Solve the triangle for $A = 42°$, $B = 60°$, and $a = 24$. Then use form 2 of Mollweide's Formula to verify your solution.

Skills Review

In Exercises 43 and 44, use the given values to find (if possible) the values of the remaining four trigonometric functions of θ.

43. $\cos\theta = \frac{5}{13}$, $\sin\theta = -\frac{12}{13}$

44. $\tan\theta = \frac{2}{9}$, $\csc\theta = -\frac{\sqrt{85}}{2}$

In Exercises 45–48, write the product as a sum or difference.

45. $6\sin 8\theta\cos 3\theta$

46. $2\cos 2\theta\cos 5\theta$

47. $3\cos\frac{\pi}{6}\sin\frac{5\pi}{3}$

48. $\frac{5}{2}\sin\frac{3\pi}{4}\sin\frac{5\pi}{6}$