5.5 Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

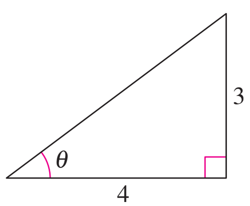
Vocabulary Check

Fill in the blank to complete the trigonometric formula.

1. $\sin 2u =$ _______
2. $\cos^2 u =$ _______
3. _______ $= 1 - 2\sin^2 u$
4. _______ $= \dfrac{\sin u}{1 + \cos u}$
5. $\tan 2u =$ _______
6. $\cos u \cos v =$ _______
7. _______ $= \dfrac{1 - \cos 2u}{2}$
8. _______ $= \pm\sqrt{\dfrac{1 + \cos u}{2}}$
9. $\sin u \cos v =$ _______
10. $\sin u + \sin v =$ _______

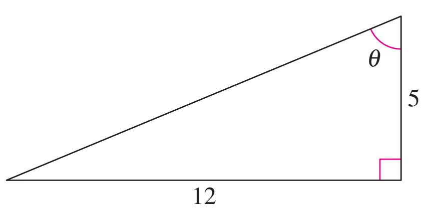
In Exercises 1 and 2, use the figure to find the exact value of each trigonometric function.

1.



(a) $\sin\theta$ (b) $\cos\theta$
(c) $\cos 2\theta$ (d) $\sin 2\theta$
(e) $\tan 2\theta$ (f) $\sec 2\theta$
(g) $\csc 2\theta$ (h) $\cot 2\theta$

2.



(a) $\sin\theta$ (b) $\cos\theta$
(c) $\sin 2\theta$ (d) $\cos 2\theta$
(e) $\tan 2\theta$ (f) $\cot 2\theta$
(g) $\sec 2\theta$ (h) $\csc 2\theta$

In Exercises 3–12, use a graphing utility to approximate the solutions of the equation in the interval $[0, 2\pi)$. If possible, find the exact solutions algebraically.

3. $\sin 2x - \sin x = 0$
4. $\sin 2x + \cos x = 0$
5. $4\sin x \cos x = 1$
6. $\sin 2x \sin x = \cos x$
7. $\cos 2x - \cos x = 0$
8. $\tan 2x - \cot x = 0$
9. $\sin 4x = -2\sin 2x$
10. $(\sin 2x + \cos 2x)^2 = 1$
11. $\cos 2x + \sin x = 0$
12. $\tan 2x - 2\cos x = 0$

In Exercises 13–18, find the exact values of $\sin 2u$, $\cos 2u$, and $\tan 2u$ using the double-angle formulas.

13. $\sin u = \frac{3}{5}, \quad 0 < u < \pi/2$
14. $\cos u = -\frac{2}{7}, \quad \pi/2 < u < \pi$
15. $\tan u = \frac{1}{2}, \quad \pi < u < 3\pi/2$
16. $\cot u = -6, \quad 3\pi/2 < u < 2\pi$
17. $\sec u = -\frac{5}{2}, \quad \pi/2 < u < \pi$
18. $\csc u = 3, \quad \pi/2 < u < \pi$

In Exercises 19–22, use a double-angle formula to rewrite the expression. Use a graphing utility to graph both expressions to verify that both forms are the same.

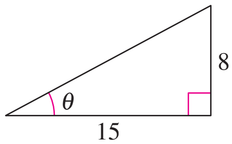
19. $8\sin x \cos x$
20. $4\sin x \cos x + 1$
21. $6 - 12\sin^2 x$
22. $(\cos x + \sin x)(\cos x - \sin x)$

In Exercises 23–36, rewrite the expression in terms of the first power of the cosine. Use a graphing utility to graph both expressions to verify that both forms are the same.

23. $\cos^4 x$
24. $\sin^4 x$
25. $\sin^2 x \cos^2 x$
26. $\cos^6 x$
27. $\sin^2 x \cos^4 x$
28. $\sin^4 x \cos^2 x$
29. $\sin^2 2x$
30. $\cos^2 2x$
31. $\cos^2 \dfrac{x}{2}$
32. $\sin^2 \dfrac{x}{2}$
33. $\sin^2 2x \cos^2 2x$
34. $\sin^2 \dfrac{x}{2} \cos^2 \dfrac{x}{2}$
35. $\sin^4 \dfrac{x}{2}$
36. $\cos^4 \dfrac{x}{2}$

In Exercises 37 and 38, use the figure to find the exact value of each trigonometric function.

37.



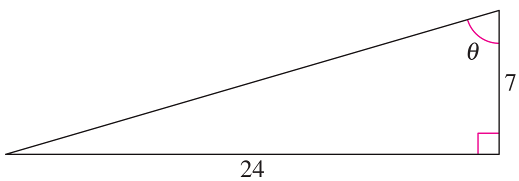
(a) $\cos \frac{\theta}{2}$ (b) $\sin \frac{\theta}{2}$

(c) $\tan \frac{\theta}{2}$ (d) $\sec \frac{\theta}{2}$

(e) $\csc \frac{\theta}{2}$ (f) $\cot \frac{\theta}{2}$

(g) $2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}$ (h) $2 \cos \frac{\theta}{2} \tan \frac{\theta}{2}$

38.



(a) $\sin \frac{\theta}{2}$ (b) $\cos \frac{\theta}{2}$

(c) $\tan \frac{\theta}{2}$ (d) $\cot \frac{\theta}{2}$

(e) $\sec \frac{\theta}{2}$ (f) $\csc \frac{\theta}{2}$

(g) $2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}$ (h) $\cos 2\theta$

In Exercises 39–46, use the half-angle formulas to determine the exact values of the sine, cosine, and tangent of the angle.

39. 15° **40.** 165°

41. $112^\circ\, 30'$ **42.** $157^\circ\, 30'$

43. $\frac{\pi}{8}$ **44.** $\frac{\pi}{12}$

45. $\frac{3\pi}{8}$ **46.** $\frac{7\pi}{12}$

In Exercises 47–52, find the exact values of $\sin(u/2)$, $\cos(u/2)$, and $\tan(u/2)$ using the half-angle formulas.

47. $\sin u = \frac{5}{13}, \quad \pi/2 < u < \pi$

48. $\cos u = \frac{7}{25}, \quad 0 < u < \pi/2$

49. $\tan u = -\frac{8}{5}, \quad 3\pi/2 < u < 2\pi$

50. $\cot u = 7, \quad \pi < u < 3\pi/2$

51. $\csc u = -\frac{5}{3}, \quad \pi < u < 3\pi/2$

52. $\sec u = -\frac{7}{2}, \quad \pi/2 < u < \pi$

In Exercises 53–56, use the half-angle formulas to simplify the expression.

53. $\sqrt{\dfrac{1 - \cos 6x}{2}}$

54. $\sqrt{\dfrac{1 + \cos 4x}{2}}$

55. $-\sqrt{\dfrac{1 - \cos 8x}{1 + \cos 8x}}$

56. $-\sqrt{\dfrac{1 - \cos(x - 1)}{2}}$

In Exercises 57–60, find the solutions of the equation in the interval $[0, 2\pi)$. Use a graphing utility to verify your answers.

57. $\sin \frac{x}{2} - \cos x = 0$

58. $\sin \frac{x}{2} + \cos x - 1 = 0$

59. $\cos \frac{x}{2} - \sin x = 0$

60. $\tan \frac{x}{2} - \sin x = 0$

In Exercises 61–72, use the product-to-sum formulas to write the product as a sum or difference.

61. $6 \sin \frac{\pi}{3} \cos \frac{\pi}{3}$

62. $4 \sin \frac{\pi}{3} \cos \frac{5\pi}{6}$

63. $\sin 5\theta \cos 3\theta$

64. $5 \sin 3\alpha \sin 4\alpha$

65. $10 \cos 75^\circ \cos 15^\circ$

66. $6 \sin 45^\circ \cos 15^\circ$

67. $5 \cos(-5\beta) \cos 3\beta$

68. $\cos 2\theta \cos 4\theta$

69. $\sin(x + y) \sin(x - y)$

70. $\sin(x + y) \cos(x - y)$

71. $\cos(\theta - \pi) \sin(\theta + \pi)$

72. $\sin(\theta + \pi) \sin(\theta - \pi)$

In Exercises 73–80, use the sum-to-product formulas to write the sum or difference as a product.

73. $\sin 5\theta - \sin \theta$

74. $\sin 3\theta + \sin \theta$

75. $\cos 6x + \cos 2x$

76. $\sin x + \sin 7x$

77. $\sin(\alpha + \beta) - \sin(\alpha - \beta)$

78. $\cos(\phi + 2\pi) + \cos \phi$

79. $\cos\left(\theta + \frac{\pi}{2}\right) - \cos\left(\theta - \frac{\pi}{2}\right)$

80. $\sin\left(x + \frac{\pi}{2}\right) + \sin\left(x - \frac{\pi}{2}\right)$

In Exercises 81–84, use the sum-to-product formulas to find the exact value of the expression.

81. $\sin 195° + \sin 105°$

82. $\cos 165° - \cos 75°$

83. $\cos \frac{5\pi}{12} + \cos \frac{\pi}{12}$

84. $\sin \frac{11\pi}{12} - \sin \frac{7\pi}{12}$

In Exercises 85–88, find the solutions of the equation in the interval $[0, 2\pi)$. Use a graphing utility to verify your answers.

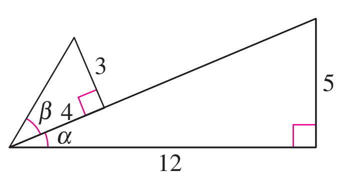
85. $\sin 6x + \sin 2x = 0$

86. $\cos 2x - \cos 6x = 0$

87. $\frac{\cos 2x}{\sin 3x - \sin x} - 1 = 0$

88. $\sin^2 3x - \sin^2 x = 0$

In Exercises 89–92, use the figure and trigonometric identities to find the exact value of the trigonometric function in two ways.



89. $\sin^2 \alpha$

90. $\cos^2 \alpha$

91. $\sin \alpha \cos \beta$

92. $\cos \alpha \sin \beta$

In Exercises 93–110, verify the identity algebraically. Use a graphing utility to check your result graphically.

93. $\csc 2\theta = \frac{\csc \theta}{2 \cos \theta}$

94. $\sec 2\theta = \frac{\sec^2 \theta}{2 - \sec^2 \theta}$

95. $\cos^2 2\alpha - \sin^2 2\alpha = \cos 4\alpha$

96. $\cos^4 x - \sin^4 x = \cos 2x$

97. $(\sin x + \cos x)^2 = 1 + \sin 2x$

98. $\sin \frac{\alpha}{3} \cos \frac{\alpha}{3} = \frac{1}{2} \sin \frac{2\alpha}{3}$

99. $1 + \cos 10y = 2 \cos^2 5y$

100. $\frac{\cos 3\beta}{\cos \beta} = 1 - 4 \sin^2 \beta$

101. $\sec \frac{u}{2} = \pm \sqrt{\frac{2 \tan u}{\tan u + \sin u}}$

102. $\tan \frac{u}{2} = \csc u - \cot u$

103. $\cos 3\beta = \cos^3 \beta - 3 \sin^2 \beta \cos \beta$

104. $\sin 4\beta = 4 \sin \beta \cos \beta(1 - 2 \sin^2 \beta)$

105. $\frac{\cos 4x - \cos 2x}{2 \sin 3x} = -\sin x$

106. $\frac{\cos 3x - \cos x}{\sin 3x - \sin x} = -\tan 2x$

107. $\frac{\cos 4x + \cos 2x}{\sin 4x + \sin 2x} = \cot 3x$

108. $\frac{\cos t + \cos 3t}{\sin 3t - \sin t} = \cot t$

109. $\sin\left(\frac{\pi}{6} + x\right) + \sin\left(\frac{\pi}{6} - x\right) = \cos x$

110. $\cos\left(\frac{\pi}{3} + x\right) + \cos\left(\frac{\pi}{3} - x\right) = \cos x$

In Exercises 111–114, rewrite the function using the power-reducing formulas. Then use a graphing utility to graph the function.

111. $f(x) = \sin^2 x$

112. $f(x) = \cos^2 x$

113. $f(x) = \cos^4 x$

114. $f(x) = \sin^3 x$

In Exercises 115–120, write the trigonometric expression as an algebraic expression.

115. $\sin(2 \arcsin x)$

116. $\cos(2 \arccos x)$

117. $\cos(2 \arcsin x)$

118. $\sin(2 \arccos x)$

119. $\cos(2 \arctan x)$

120. $\sin(2 \arctan x)$