

人工智能原理⁵

Ricklab

A little bit of this, A little bit of that



王曲苑

Welcome to Ricklab with some of our works.

Logic in Artificial intelligence

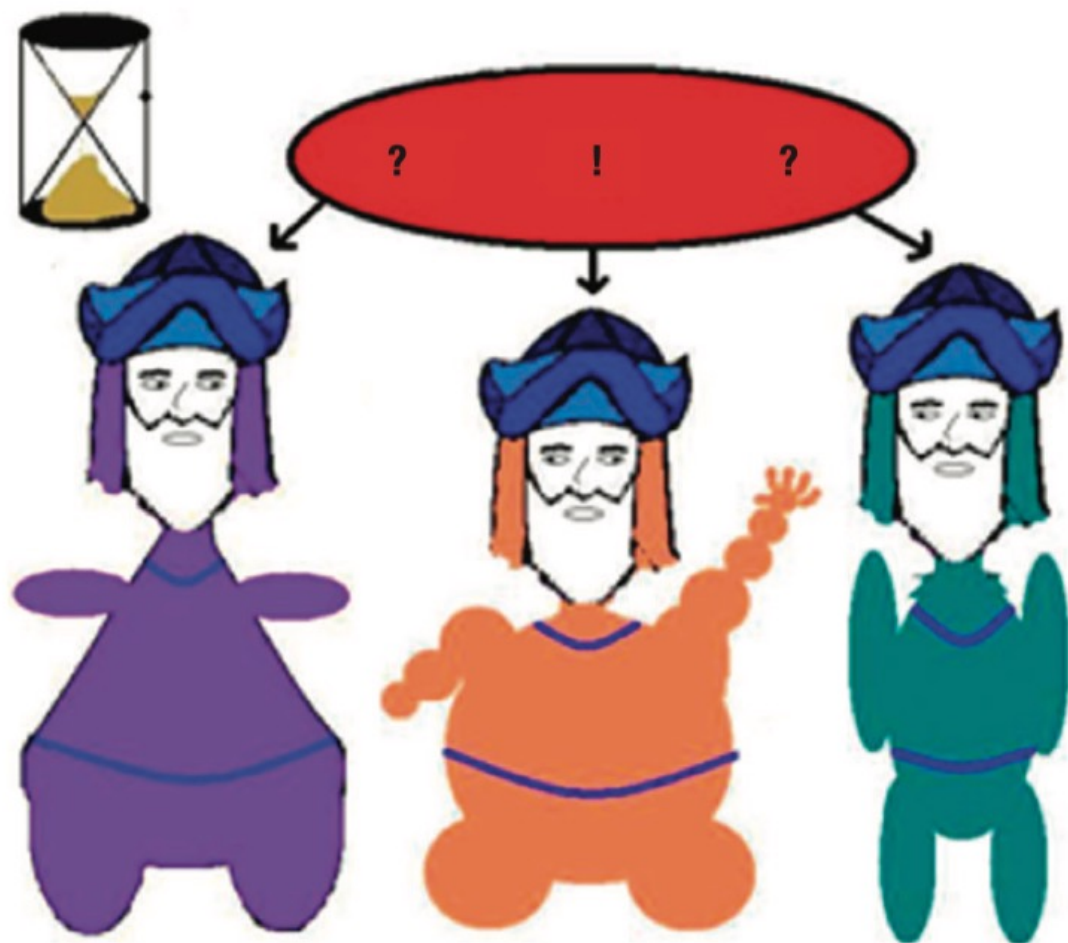
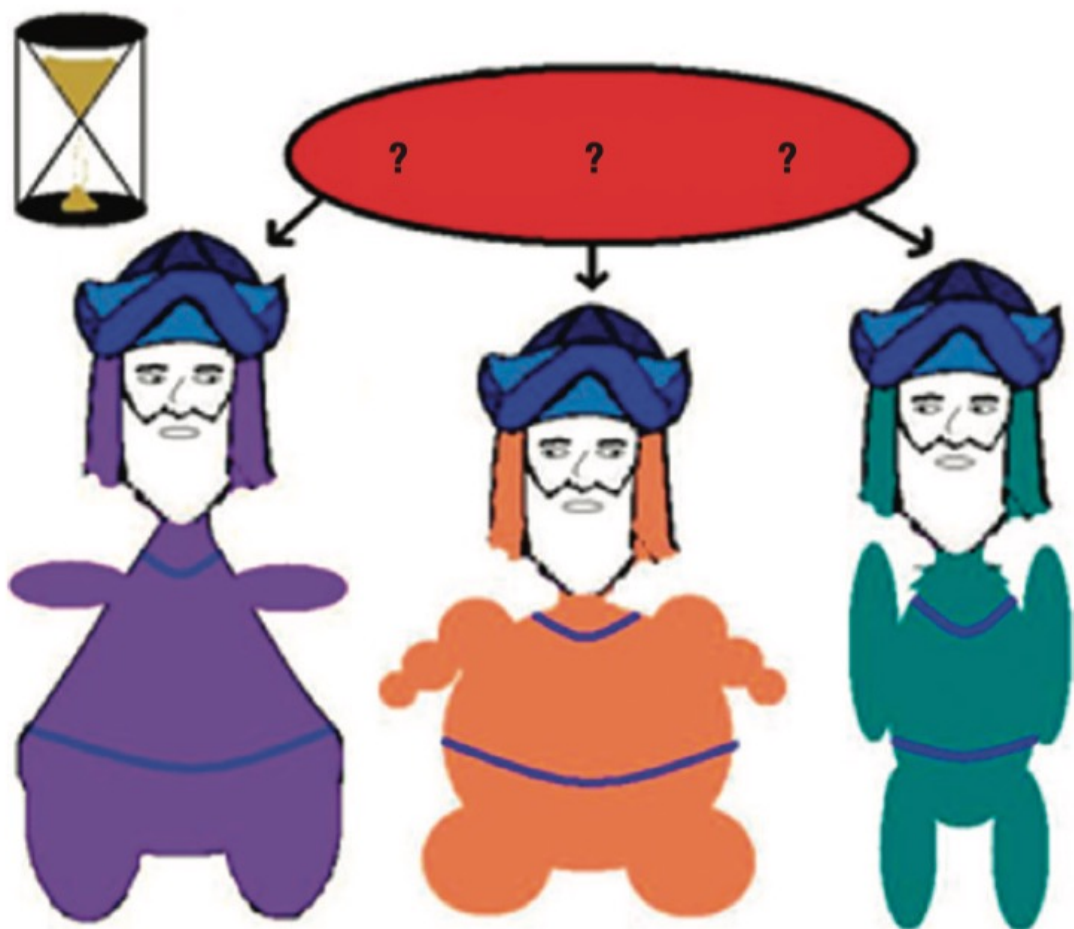


CREWMATE

There are 3 **Impostors** among us



Logic is a concise language for
knowledge representation



WM_1()

$$WM_1(W) \wedge WM_2(W) \wedge WM_3(B)$$

Propositional Logic

1. He is taking the bus home tonight.
2. The music was beautiful.
3. Watch out.



The **atomic sentences** consist of a single proposition symbol.

propositional logic variables: $p, q, r, \dots$

Complex sentences are constructed from simpler sentences, using parentheses and operators called **logical connectives**.

Symbol	Name	Example	English Equivalent
$\wedge$	conjunction	$\mathbf{p} \wedge \mathbf{q}$	p and q
$\vee$	disjunction	$\mathbf{p} \vee \mathbf{q}$	p or q
$\sim$	negation	$\sim \mathbf{p}$	not p
$\Rightarrow$	implication	$\mathbf{p} \Rightarrow \mathbf{q}$	if p then q or p implies q
$\Leftrightarrow$	biconditional	$\mathbf{p} \Leftrightarrow \mathbf{q}$	p if and only if q or p is equivalent to q

Truth Table

(a) AND function					(b) OR function					(c) NOT function				
p	q	$p \wedge q$			p	q	$p \vee q$							
F	F	F			F	F	F				p	$\sim p$		
F	T	F			F	T	T				F	T		
T	F	F			T	F	T				T	F		
T	T	T			T	T	T							

What is XOR

Exclusive OR = **XOR**

(b) OR function		
p	q	$p \vee q$
F	F	F
F	T	T
T	F	T
T	T	T

$\oplus$		
p	q	$p \underline{\vee} q$
F	F	F
F	T	T
T	F	T
T	T	F

p (前项、前件、前提) $\Rightarrow$ q (后项、后件、结论)

p	q	$p \Rightarrow q$
F	F	T
F	T	T
T	F	F
T	T	T

There is **only one false** when the case in which **p is true and q is false**.

converse、inverse、contrapositive

1	2	3	4	5	6
p	q	$p \Rightarrow q$	$q \Rightarrow p$	$\sim p \Rightarrow \sim q$	$\sim q \Rightarrow \sim p$
F	F	T	T	T	T
F	T	T	F	F	T
T	F	F	T	T	F
T	T	T	T	T	T

Two tautologies in the propositional logic

p	q	$(p \Rightarrow q)$	$(\sim p \vee q)$	$(p \Rightarrow q) \equiv (\sim p \vee q)$	$(\sim p \vee \sim q) \equiv \sim(p \wedge q)$
F	F	T	T	T	T
F	T	T	T	T	T
T	F	F	F	T	T
T	T	T	T	T	T

$$(p \Rightarrow q) \equiv \sim p \vee q$$

Two tautologies in the propositional logic

p	q	$(p \Rightarrow q)$	$(\sim p \vee q)$	$(p \Rightarrow q) \equiv (\sim p \vee q)$	$(\sim p \vee \sim q) \equiv \sim(p \wedge q)$
F	F	T	T	T	T
F	T	T	T	T	T
T	F	F	F	T	T
T	T	T	T	T	T

$$\neg(\alpha \wedge \beta) \equiv (\neg\alpha \vee \neg\beta) \quad \text{De Morgan}$$

$$\neg(\alpha \vee \beta) \equiv (\neg\alpha \wedge \neg\beta) \quad \text{De Morgan}$$

$$(\alpha \wedge \beta) \equiv (\beta \wedge \alpha) \quad \text{commutativity of } \wedge$$

$$(\alpha \vee \beta) \equiv (\beta \vee \alpha) \quad \text{commutativity of } \vee$$

$$((\alpha \wedge \beta) \wedge \gamma) \equiv (\alpha \wedge (\beta \wedge \gamma)) \quad \text{associativity of } \wedge$$

$$((\alpha \vee \beta) \vee \gamma) \equiv (\alpha \vee (\beta \vee \gamma)) \quad \text{associativity of } \vee$$

$$\neg(\neg\alpha) \equiv \alpha \quad \text{double-negation elimination}$$

$$(\alpha \Rightarrow \beta) \equiv (\neg\beta \Rightarrow \neg\alpha) \quad \text{contraposition}$$

$$(\alpha \Rightarrow \beta) \equiv (\neg\alpha \vee \beta) \quad \text{implication elimination}$$

$$(\alpha \Leftrightarrow \beta) \equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) \quad \text{biconditional elimination}$$

$$\neg(\alpha \wedge \beta) \equiv (\neg\alpha \vee \neg\beta) \quad \text{De Morgan}$$

$$\neg(\alpha \vee \beta) \equiv (\neg\alpha \wedge \neg\beta) \quad \text{De Morgan}$$

$$(\alpha \wedge (\beta \vee \gamma)) \equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) \quad \text{distributivity of } \wedge \text{ over } \vee$$

$$(\alpha \vee (\beta \wedge \gamma)) \equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) \quad \text{distributivity of } \vee \text{ over } \wedge$$

证明 $[(\sim p \vee q) \wedge \sim q] \Rightarrow \sim p$ 是一个重言式

An argument in the propositional logic has the form:

A: P_1
 P_2
.
.
.
 P_r

C // **Conclusion**

$$(P_1 \wedge P_2 \wedge \dots \wedge P_r) \Rightarrow C \text{ is a tautology.}$$

Let **g** represent: “the moon is made of green cheese” and
r represent: “I am rich.” This argument has the form:

$$\frac{\mathbf{g}}{\therefore \mathbf{r}}$$



$$1. \quad \mathbf{p} \Rightarrow \mathbf{q}$$

$$2. \quad \mathbf{q} \Rightarrow \sim \mathbf{r}$$

$$3. \quad \sim \mathbf{p} \Rightarrow \sim \mathbf{r}$$

$$\therefore \sim \mathbf{r}$$



1	2	3	4	5	6	7	8
p	q	r	$p \Rightarrow q$	$q \Rightarrow \sim r$	$\sim p \Rightarrow \sim r$	$4 \wedge 5 \wedge 6$	$7 \Rightarrow \sim r$
F	F	F	T	T	T	T	T
F	F	T	T	T	F	F	T
F	T	F	T	T	T	T	T
F	T	T	T	F	F	F	T
T	F	F	F	T	T	F	T
T	F	T	F	T	T	F	T
T	T	F	T	T	T	T	T
T	T	T	T	F	T	F	T

This approach assumes that the **premises are true** and that the **conclusion is false**

There is no:

1. implication
2. conjunction
3. **double negation**

$$1. \quad \mathbf{p} \Rightarrow \mathbf{q}$$

$$2. \quad \mathbf{q} \Rightarrow \sim \mathbf{r}$$

$$3. \quad \sim \mathbf{p} \Rightarrow \sim \mathbf{r}$$

$$\therefore \sim \mathbf{r}$$

$$1') \sim \mathbf{p} \vee \mathbf{q}$$

$$2') \sim \mathbf{q} \vee \sim \mathbf{r}$$

$$3') \mathbf{p} \vee \sim \mathbf{r}$$

$$4') \mathbf{r}$$

Predicate Logic

A predicate logic expression consists of a **predicate name** followed by **a list (possibly empty) of arguments**.

WM_1()

Two quantifiers

$\exists$: existential quantifier, read as: “there exists an x ”

$\forall$: universal quantifier, reads as: “for all x ”

Because **$\forall$ is really a conjunction** over the universe of objects and **$\exists$ is a disjunction**

$$\neg \exists x P \equiv \forall x \neg P$$

$$\neg \forall x P \equiv \exists x \neg P$$

Setting (sun) and $\sim$ Setting (sun) is a contradiction

Beautiful (Day) and $\sim$ Beautiful (Night)

In order to find contradictions, we require a matching procedure that compares two literals to detect whether there exists a set of substitutions that makes them identical. This procedure is called **unification**.

If two literals are to unify, then first, **their predicate symbols must match**

If the predicate symbols match, then **you check the arguments one pair at a time.**