Fixed Effects

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Methods we have studied

- Control variables
- RCTs
- IVs
- Regression Discontinuity
- Fixed-effects

Nomenclature

- What is Panel Data/Longitudinal?

What is Panel Data/Longitudinal Data?

- Panel Data or Longitudinal Data refers to a database in which we observe a given unit of observation in several time periods.

Example

	id	year	age	sex	race	region	weight	height
25	2	1979	20	Female	Non-Black, Non-Hispanic	Northeast	.	.
26	2	1980	21	Female	Non-Black, Non-Hispanic	Northeast	.	.
27	2	1981	22	Female	Non-Black, Non-Hispanic	Northeast	120	50.200000763
28	2	1982	23	Female	Non-Black, Non-Hispanic	Northeast	125	62
29	2	1983	24	Female	Non-Black, Non-Hispanic	Northeast	.	.
30	2	1984	25	Female	Non-Black, Non-Hispanic	Northeast	.	.
31	2	1985	26	Female	Non-Black, Non-Hispanic	Northeast	118	62
32	2	1986	27	Female	Non-Black, Non-Hispanic	Northeast	115	62
33	2	1987	28	Female	Non-Black, Non-Hispanic	Northeast	.	62
34	2	1988	29	Female	Non-Black, Non-Hispanic	Northeast	120	62
35	2	1989	30	Female	Non-Black, Non-Hispanic	Northeast	120	62
36	2	1990	31	Female	Non-Black, Non-Hispanic	Northeast	128	62
37	2	1991	32	Female	Non-Black, Non-Hispanic	Northeast	.	62
38	2	1992	33	Female	Non-Black, Non-Hispanic	Northeast	160	62
39	2	1993	34	Female	Non-Black, Non-Hispanic	Northeast	145	62
40	2	1994	35	Female	Non-Black, Non-Hispanic	Northeast	158	62
41	2	1996	37	Female	Non-Black, Non-Hispanic	Northeast	150	62
42	2	1998	39	Female	Non-Black, Non-Hispanic	Northeast	158	62
43	2	2000	41	Female	Non-Black, Non-Hispanic	Northeast	160	62
44	2	2002	43	Female	Non-Black, Non-Hispanic	Northeast	158	62
45	2	2004	45	Female	Non-Black, Non-Hispanic	Northeast	158	62
46	2	2006	47	Female	Non-Black, Non-Hispanic	Northeast	160	62
47	2	2008	49	Female	Non-Black, Non-Hispanic	Northeast	160	62
48	2	2010	51	Female	Non-Black, Non-Hispanic	Northeast	165	62
49	3	1979	17	Female	Non-Black, Non-Hispanic	Northeast	.	.
50	3	1980	18	Female	Non-Black, Non-Hispanic	Northeast	.	.
51	3	1981	.	Female	Non-Black, Non-Hispanic	.	.	.
52	3	1982	20	Female	Non-Black, Non-Hispanic	Northeast	160	70
53	3	1984	22	Female	Non-Black, Non-Hispanic	Northeast	.	.
54	3	1985	23	Female	Non-Black, Non-Hispanic	Northeast	152	70
55	3	1986	24	Female	Non-Black, Non-Hispanic	Northeast	170	70
56	3	1987	25	Female	Non-Black, Non-Hispanic	Northeast	.	70
57	3	1988	26	Female	Non-Black, Non-Hispanic	Northeast	155	70
58	3	1989	27	Female	Non-Black, Non-Hispanic	Northeast	165	70

Framework

- This type of data has two dimensions: i and j
- Usually, i represents the panel variable (e.g. city, individual, state, company, legislator)
- Usually j represents the time variable (e.g. year, quarter, month, etc)

Real Examples

- National Longitudinal Survey of Youth of 1979.
  - In this data we observe the same individual over time for about 30 years (and counting)
- Other examples are:
  - Panel Study of Income Dynamics (PSID)
  - Medical Expenditure Panel Survey (MEPS)
  - Consumer Expenditure Panel (CES)

What's so great?

- Following a given unit (i.e individual) over time is very powerful because we can then draw inference to what happens to that particular person, when a policy changes.
- We are also able to account for other unobservable factors that before we couldn't, helping us to mitigate more sources of bias.
- What is not so great?
  - They are not as popular.
  - Their N is low relative to cross-sectional data.

What is Cross Sectional Data?

- Cross-Sectional Data refers to a database in which we observe a given unit of observation in only one time period.
  - Think of it as a "slice" of the panel data.

How does Cross Sectional Data look?

	intvid	idate	_state	genhlth	hlthplan	asthma
748808	191	05152002	1	Very Good	1	.
748809	191	05152002	1	Very Good	1	.
748810	191	05152002	1	Very Good	1	.
748811	191	05152002	1	Fair	1	.
748812	191	05152006	1	Poor	1	.
748813	191	05182002	1	Good	1	.
748814	191	05192008	1	Very Good	1	.
748815	191	05192008	1	Good	1	.
748816	191	05202002	1	Very Good	2	.
748817	191	05202002	1	Fair	2	.
748818	191	05202008	1	Excellent	1	.
748819	191	05212002	1	Excellent	1	.
748820	191	05212002	1	Good	1	.
748821	191	05222006	1	Fair	1	.
748822	191	05232006	1	Poor	1	.
748823	191	06012002	1	Fair	1	.
748824	191	06072006	1	Excellent	1	.
748825	191	06072006	1	Very Good	1	.
748826	191	06072006	1	Very Good	2	.
748827	191	06082006	1	Excellent	2	.
748828	191	06202006	1	Excellent	1	.
748829	191	07292008	1	Good	1	.
748830	191	07302008	1	Very Good	1	.
748831	191	08062003	1	Poor	1	.
748832	191	09302008	1	Poor	1	.
748833	191	10222007	1	Excellent	1	.
748834	191	11182008	1	Good	1	.
748835	191	11242008	1	Excellent	1	.
748836	191	12172008	1	Fair	1	.
748837	191	08012000	5	Good	1	2
748838	191	03162007	10	Excellent	1	.
748839	191	04042007	10	Excellent	1	.
748840	191	04042007	10	Good	1	.
748841	191	04042007	10	Good	2	.
748842	191	04042007	10	Good	2	.
748843	191	04042007	10	Fair	1	.
748844	191	04042007	10	Fair	1	.
748845	191	04102007	10	Good	1	.
748846	191	08062007	12	Very Good	1	.
748847	191	[illegible]	[illegible]	[illegible]	1	[illegible]

Cross Sectional Examples

- Examples of Cross-Sectional Data:
  - NHIS, BRFSS, CPS and other well know surveys ask questions to a representative population each year.
- Each year we are getting a random set of the population. Therefore, they may not be the same person.
- Drawback: We can't follow individuals over time
- Advantage: More Sample Size

Quick Note

- Note that the level of observation doesn't have to be an individual it could be a business, a household, a city, etc.

Practice Question: What type of data is this?

county	year	month	pop
ANDERSON	2006	3	73,117
ANDERSON	2006	4	73,117
ANDERSON	2006	5	73,117
ANDERSON	2006	6	73,117
ANDERSON	2006	7	73,117
ANDERSON	2006	8	73,117
ANDERSON	2006	9	73,117
ANDERSON	2006	10	73,117
ANDERSON	2006	11	73,117
ANDERSON	2006	12	73,117
ANDERSON	2007	1	73,580
ANDERSON	2007	2	73,580
ANDERSON	2007	3	73,580
ANDERSON	2007	4	73,580
ANDERSON	2007	5	73,580
ANDERSON	2007	6	73,580
ANDERSON	2007	7	73,580
ANDERSON	2007	8	73,580
ANDERSON	2007	9	73,580
ANDERSON	2007	10	73,580
ANDERSON	2007	11	73,580
ANDERSON	2007	12	73,580
ANDERSON	2008	1	74,397
ANDERSON	2008	2	74,397
ANDERSON	2008	3	74,397
ANDERSON	2008	4	74,397
ANDERSON	2008	5	74,397
BENTON	2007	1	16,490
BENTON	2007	2	16,490
BENTON	2007	3	16,490
BENTON	2007	4	16,490
BENTON	2007	5	16,490
BENTON	2007	6	16,490
BENTON	2007	7	16,490
BENTON	2007	8	16,490
BENTON	2007	9	16,490
BENTON	2007	10	16,490
BENTON	2007	11	16,490
BENTON	2007	12	16,490
BENTON	2008	1	16,431
BENTON	2008	2	16,431
BENTON	2008	3	16,431
BENTON	2008	4	16,431
BENTON	2008	5	16,431
BENTON	2008	6	16,431
BENTON	2008	7	16,431
BENTON	2008	8	16,431
BENTON	2008	9	16,431
BENTON	2008	10	16,431
BENTON	2008	11	16,431
BENTON	2008	12	16,431
BENTON	2009	1	16,424
BENTON	2009	2	16,424
BENTON	2009	3	16,424

What type of data is this?

_state	iyear	age	marital
1	2000	61	Divorced
1	2000	25	Married
1	2000	40	Married
1	2000	36	Married
1	2000	48	Divorced
1	2000	55	Married
1	2000	58	Married
1	2000	33	Married
1	2000	21	Never Married
1	2000	46	Divorced
1	2000	64	Married
1	2000	32	Married
1	2000	34	Married
1	2000	54	Married
1	2000	46	Divorced
1	2000	23	Divorced
1	2000	43	Married
1	2000	21	Married
1	2000	42	Married
1	2000	42	Married
1	2000	27	Married
1	2000	70	Widowed
1	2000	48	Married
1	2000	49	Widowed
1	2000	43	Divorced
1	2000	39	Never Married
1	2000	71	Married

What type of data is this?

	_state	iyear	age	marital
1	1	2003	38	Married
2	1	2009	44	Married
3	1	2009	54	Divorced
4	1	2010	68	Married
5	1	2011	74	Married
6	1	2011	69	Married
7	5	2002	73	Married
8	5	2004	59	Divorced
9	5	2007	55	Married
10	5	2008	72	Widowed
11	5	2009	86	Widowed
12	5	2010	80	Widowed
13	5	2010	76	Married
14	5	2012	25	Never Married
15	10	2003	23	Never Married
16	10	2008	57	Married
17	10	2011	85	Married
18	10	2012	35	Divorced
19	11	2011	40	Married
20	11	2012	82	Married
21	12	2002	60	Married
22	12	2003	45	Married
23	12	2007	65	Married
24	12	2007	70	Married
25	12	2007	66	Married
26	12	2007	27	Married
27	12	2008	64	Never Married

Can a cross-section become a panel?

- Can a cross-sectional data become a panel?
  - Imagine you have a survey of a random set of individuals each year, and the data is representative at the state level.
  - Each year you'll have a different set individuals, so this would be a cross section.
  - However, I can aggregate all the observations at the state-year level. (I could get the average for each variable).
  - In each year I have 50 observations (50-states) across several years.
  - I am making a state-year panel data.

Example of aggregation

	intvid	idate	_state	genhlth	hlthplan	asthma
748808	191	05152002	1	Very Good	1	.
748809	191	05152002	1	Very Good	1	.
748810	191	05152002	1	Very Good	1	.
748811	191	05152002	1	Fair	1	.
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748815	191	05192008	1	Good	1	.
748816	191	05202002	1	Very Good	2	.
748817	191	05202002	1	Fair	2	.
748818	191	05202008	1	Excellent	1	.
748819	191	05212002	1	Excellent	1	.
748820	191	05212002	1	Good	1	.
748821	191	05222006	1	Fair	1	.
748822	191	05232006	1	Poor	1	.
748823	191	06012002	1	Fair	1	.
748824	191	06072006	1	Excellent	1	.
748825	191	06072006	1	Very Good	1	.
748826	191	06072006	1	Very Good	2	.
748827	191	06082006	1	Excellent	2	.
748828	191	06202006	1	Excellent	1	.
748829	191	07292008	1	Good	1	.
748830	191	07302008	1	Very Good	1	.
748831	191	08062003	1	Poor	1	.
748832	191	09302008	1	Poor	1	.
748833	191	10222007	1	Excellent	1	.
748834	191	11182008	1	Good	1	.
748835	191	11242008	1	Excellent	1	.
748836	191	12172008	1	Fair	1	.
748837	191	08012000	5	Good	1	2
748838	191	03162007	10	Excellent	1	.

STATA

- `recode hlthplan (2=0)`
- `replace hlthplan=.  if hlthplan>1`
- `replace genhlth=.  if genhlth>5`
- `collapse (mean) genhlth hlthplan, by( _state iyear)`

After Collapsed

	_state	iyear	genhlth	hlthplan
1	1	2000	2.636194	.8504194
2	1	2001	2.6618142	.85959673
3	1	2002	2.6847754	.8760469
4	1	2003	2.7149937	.86928308
5	1	2004	2.7637773	.85170341
6	1	2005	2.7853925	.86356342
7	1	2006	2.7638845	.87077779
8	1	2007	2.8512747	.880198
9	1	2008	2.8409843	.88365126
10	1	2009	2.8382664	.87869644
11	1	2010	2.8896754	.87828994
12	1	2011	2.8679903	.
13	1	2012	2.886801	.
14	5	2000	2.6259117	.84460634
15	5	2001	2.6381767	.85158759
16	5	2002	2.6725855	.83960235
17	5	2003	2.6212673	.83361632
18	5	2004	2.7129211	.84226418
19	5	2005	2.7299922	.85026217
20	5	2006	2.7194047	.8500827
21	5	2007	2.7360244	.85637629
22	5	2008	2.7720058	.86914206
23	5	2009	2.8119419	.87150127
24	5	2010	2.7921119	.87858778
25	5	2011	2.8764143	.
26	5	2012	2.8554649	.
27	5	2013	2	.
28	10	2000	2.3677812	.92009133
29	10	2001	2.4392273	.91805226
30	10	2002	2.4609456	.9236896
31	10	2003	2.4177802	.91827291
32	10	2004	2.4432225	.92297852
33	10	2005	2.4683638	.92650336
34	10	2006	2.4523871	.92897725
35	10	2007	2.5298991	.93347138
36	10	2008	2.5321758	.94156671
37	10	2009	2.5135071	.93303359
38	10	2010	2.5503745	.93712211
39	10	2011	2.555001	.
40	10	2012	2.5266225	.

Collapse Command

- To take the average of lots of variables by certain metrics (e.g. state and year). We use the command collapse:
- `collapse (statistic) var1 var2 .... , by (group1 group2....)`
- `collapse (mean) genhlth hlthplan medcost checkup exerany, by (_state iyear)`

Overview

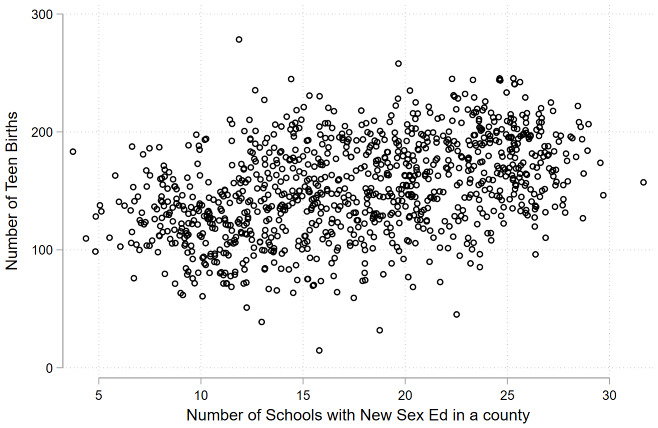
- ~~What is Panel Data/Longitudinal Data?~~
- ~~What is cross-sectional data?~~
- ~~Can cross-sectional data become panel data?~~
- What are things that I can do in panel data that I can't do in cross-sectional data?
  - We are about to find out. Let's look at an example.
  - Teen Pregnancy and Sex Ed
  - Police and Crime

Example: Teen Pregnancy and Sex Ed

State	County	Year	# Schools with *New* Sex Ed Curricula	# Teen Pregnancy Births	Population i 10K
TN	Nashville	2001	12	20	103
TN	Nashville	2002	14	35	104
TN	Nashville	2003	14	38	104
VA	Smallerville	2001	3	8	3.2
VA	Smallerville	2002	4	8	3.3
VA	Smallerville	2003	5	7	3.3

- Trying to disentangle the effect of sex-ed on teen pregnancy.

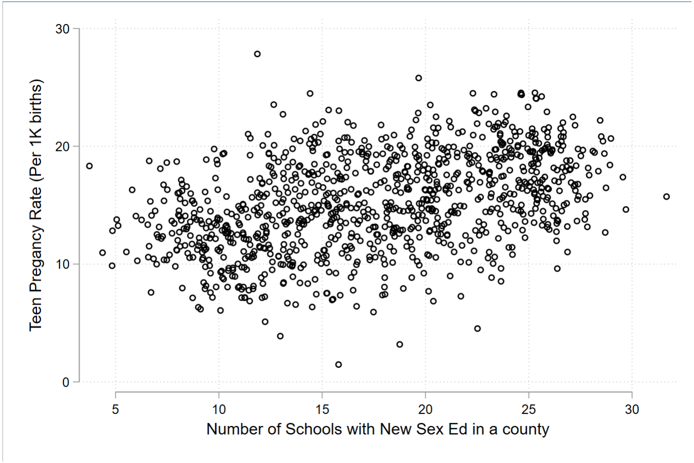
Plotting Data



What are some issues with this relationship?

- Population adjustment

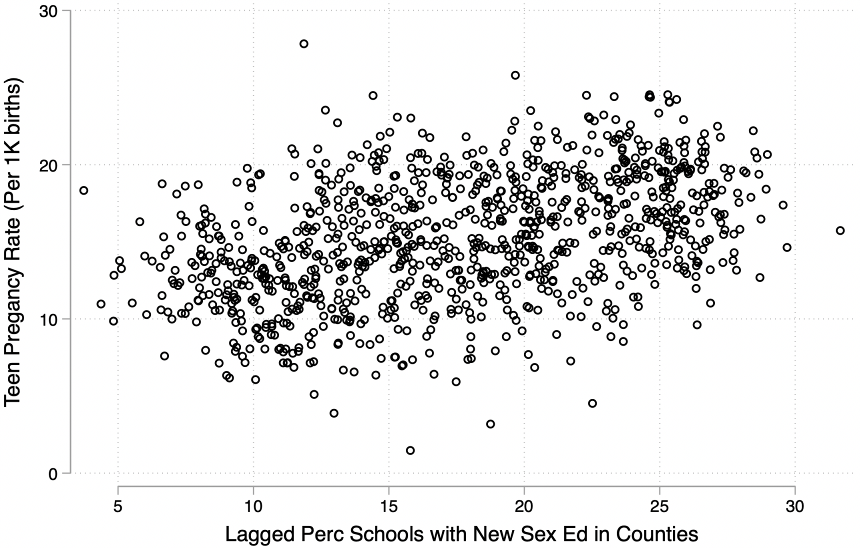
Plotting Data



Practicing calling BS

- Percent of School instead of Number.
- Measurement error (how is the data recorded)
- Reverse causality: want to use lag-Sex ED
- What's an OVB story?

Plotting Data



Sex Ed on Teen Pregnancy

- The model associated with this relationship is:

$$TeenPregnancy\ \ per\ 1K_{sct} = \beta_0 + \beta_1 PercSchoolNewCurriculum_{sct-1} + \varepsilon_{sct}$$

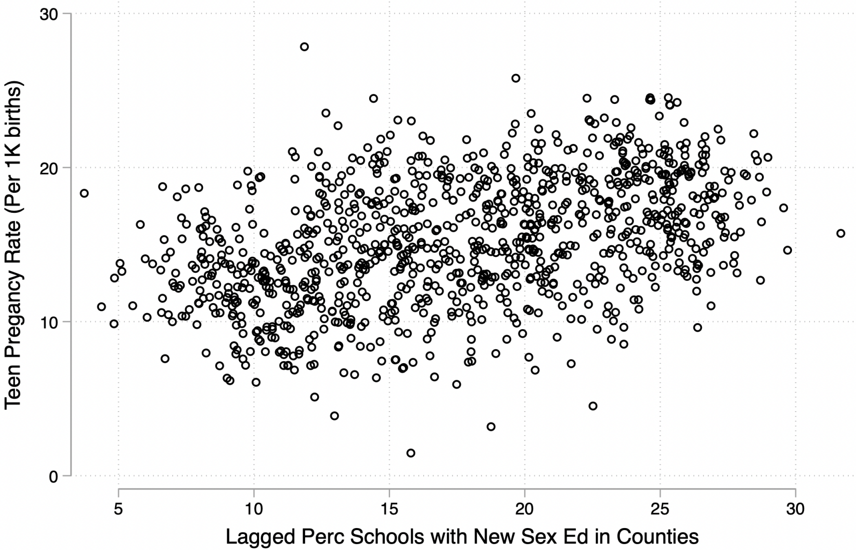
- We use the data we have, pooling observation from all cities together
- This is the regular set-up we have seen. We are going to call this a "Pooled Regression"
- Let's run that regression with the data!
  - What should we get? Positive, neutral or negative relationship?

Regression Results

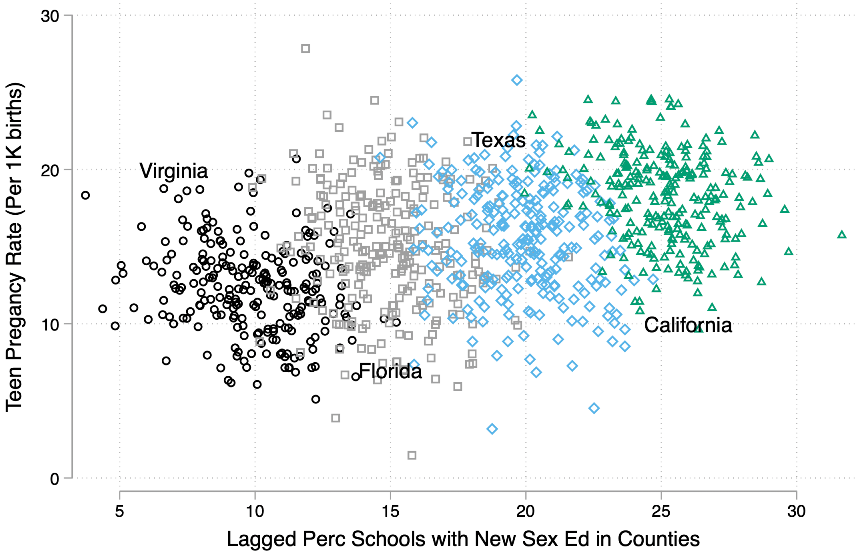
```
. reg teen_pregnancy_rate perc_sexed

      Source |       SS           df       MS      Number of obs   =     1,000
-------------+----------------------------------   F(1, 998)       =    217.32
       Model |  2923.0915         1   2923.0915   Prob > F        =    0.0000
    Residual |  13423.7985      998  13.4506999   R-squared       =    0.1788
-------------+----------------------------------   Adj R-squared   =    0.1780
       Total |   16346.89       999  16.3632532   Root MSE        =    3.6675

------------------------------------------------------------------------------
teen_pregn~e |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
  perc_sexed |    .287133   .0194775    14.74   0.000     .2489113    .3253546
       _cons |    10.1774   .3611061    28.18   0.000     9.468782    10.88601
------------------------------------------------------------------------------
```

- What's the interpretation of that coefficient?
- What is the t-statistic for a null of 0?
- What is the conclusion from this regression?
  - Note that we can add controls to this and the relationship would still look positive.

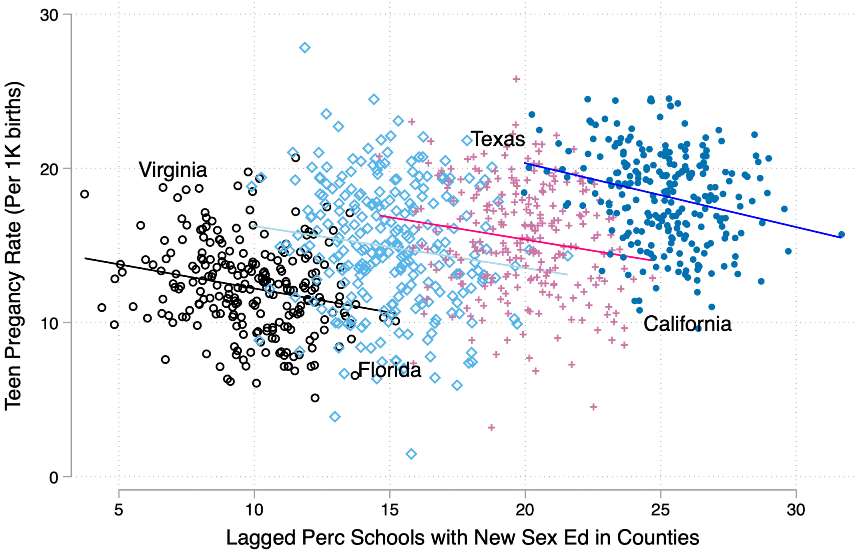
Plotting the Data



What if we look by states?



What if we look by states?



What happened?

- When looking at overall trends of sex-ed classes and teen pregnancy, there was a positive relationship.
- The regression

$$TeenPregnancy\ \ per\ 1K_{sct} = \beta_0 + \beta_1 PercSchoolNewCurriculum_{sct-1} + \varepsilon_{sct}$$

- Gave us the same intuition.
- However, once **we look at the same data**, by states, in each city the trend is declining?
- How could this be? In each state, sex-ed curricula and teen-pregnancy are negatively correlated but overall I see a positive relationship of sex-ed curricula and teen-pregnancy.

What happened?

- This is because the graph was conflicting the fact some states start with (1) higher teen pregnancy and (2) higher Perc of new sex-ed schools.
- Notice that even when we accounted for population this was still true.
- The graph or pooled analysis, did not account for the relationship in each state, it just pooled all the information together.
- By pooling the information together, this "*masks*" a pattern in the data.

Framework to think about this

- Think of different states having different "starting points" of the x and y variables.
- We like to think of these different "starting points" as some characteristics that are "fixed" about the state. That is, that don't change much over time.
- In each state, sex-ed schools and teen-pregnancy have a negative relationship, but when pooling things together, it was a positive relationship essentially ignoring these "starting points" and treating all the observations as coming from the same unit.

How does panel data helps us here?

- If we didn't have several points per state, how would a graph look?
- We wouldn't even be able to test this "negative relationship"
- Panel data allows us to disentangle these "overall" relationships.
- How to deal with this in a regression?

$$TeenPregnancy\ \ per\ 1K_{sct} = \beta_0 + \beta_1 PercSchoolNewCurriculum_{sct-1} + \varepsilon_{sct}$$

Regression Format

$$TeenPregnancy\ per\ 1K_{sct} = \beta_0 + \beta_1 PercSchoolNewCurriculum_{sct-1} + \varepsilon_{sct}$$

- Why did the the pooled regression showed ***biased*** coefficients?
  - Intuition: It was not accounting for those "starting points" or "fixed" items.
  - Math: Recall that if our coefficient of interest is biased, this implied that $corr(PercSchoolNewCurriculum_{sct-1}, \varepsilon_{sct}) \neq 0$.
    - Since we know there is bias in our estimate, this means that main-explanatory variable and the error terms are somewhat related.

Fixed-effects in math

- This relationship was about those "fixed" unobservables:

$$\varepsilon_{sct} = \alpha_s + \nu_{sct}$$

- The α_s are what we called "fixed effects". They represent the tendency for unit s to be higher or lower than a given base unit.
- This term in the error is capturing ***all*** time-invariant *observable and unobservable* characteristics of each state.

Fixed-Effects in our Example

- In our example, this could be things like:
  - A different culture of "Age of first sex encounter"
  - The price of condoms
  - These are things that have stayed relatively constant.

Fixed-Effects in our Example

- In the pooled regression, we found a "reverse" sign of our main β because our estimate was "biased".
- This state-specific baseline levels (or generally, time-invariant fixed effect) were violating the exogeneity condition and making our estimate biased. $corr(PercSchoolNewCurriculum_{sct-1}, \alpha_s)$
- So, how to fix it? (Yes, it is a Pun).

How to fix it? (Pun)

- How do we account for these unobservable (and observable characteristics) baseline levels of each unit (state, city, etc)?
- How do we tell our regression to account for this relationship within each unit, like we did in the graph, as oppose to use the data all together?
  - We use a fixed-effects approach.
- There are two ways to go about it or interpret it
  - **Least Squares Dummy Variable Approach (LSDA)**
  - De-meaned approach (will not review)

Least Square Dummy Approach

$$TeenPregnancy\ \ per\ \ 1K_{sct} = \beta_0 + \beta_1 PercSchoolNewCurriculum_{sct-1} + \varepsilon_{sct}$$

$$\varepsilon_{sct} = \alpha_s + \nu_{sct}$$

- The LSDA just means including a dummy variable for each single unit of the panel.
- In our case, if the panel is a state-year panel, we include a dummy for each state.

$$TeenPregnancy\ \ per\ \ 1K_{sct} = \beta_0 + \beta_1 PercSchoolNewCurriculum_{sct-1} + \overrightarrow{\alpha_s} + \varepsilon_{sct}$$

$$TeenPregnancy\ per\ 1K_{sct} = \beta_0 + \beta_1 PercSchoolNewCurriculum_{sct-1} + \alpha_{va} VA + \alpha_{tx} TX + \alpha_{fl}$$

Least Square Dummy Approach

- Think about the intuition, these dummies are telling the regression to compare "within" states (within units).
  - This is why some people call this the within-estimator
- How does this look more generally and in the data?

How does this look?

$$Y_{st} = \beta_0 + \beta_1 X_{st} + \alpha_1 S_1 + \alpha_2 S_2 + \alpha_3 S_3 + \ldots + \alpha_{49} S_{49} + v_{st}$$

State	Year	# Schools with *New* Sex Ed Curricula	# Teen Pregnancy	Population in 10K	VA Dummy	TX Dummy	CA Dummy
VA	2001	12	20	103	1	0	0
VA	2002	14	35	104	1	0	0
VA	2003	14	38	104	1	0	0
TX	2001	3	8	3.2	0	1	0
TX	2002	4	8	3.3	0	1	0
TX	2003	5	7	3.3	0	1	0

How to do this in STATA?

- Method 1: using the "i." approach.

```
estimates clear
eststo:  reg teen_pregnancy_rate perc_sexed
eststo:  reg teen_pregnancy_rate perc_sexed i.fips
```

- Method 2: using xtset

```
xtset fips year
eststo:  xtreg teen_pregnancy_rate perc_sexed, fe
```

How to do this in STATA?

- Method 3: areg command

```
eststo:  areg teen_pregnancy_rate perc_sexed,
a(fips)
```

Applying FE

Table 1. OLS vs. FE Model

	Pooled OLS	Fixed-Effect (i.fips)	Fixed-Effect (xtreg)	Fixed-Effect (areg)
Perc. New Sex Ed Schools	0.287***	-0.318***	-0.318***	-0.318***
	(0.0195)	(0.0550)	(0.0550)	(0.0550)
N	1,000	1,000	1,000	1,000
Number of States	4	4	4	4

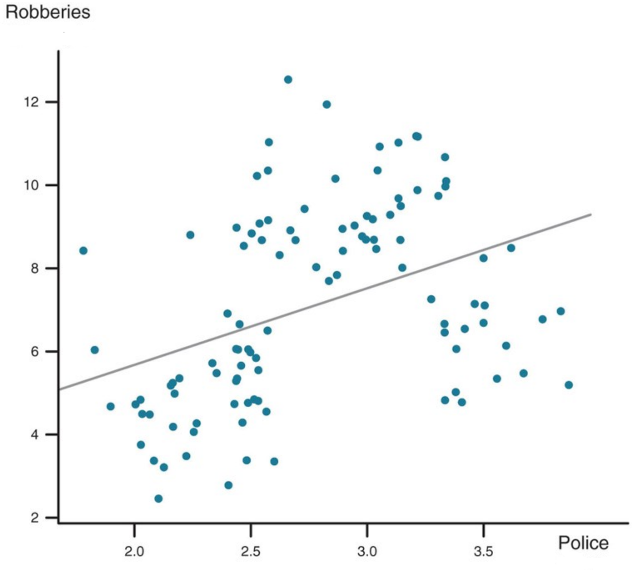
Standard errors in parentheses.* $p<0.05$, ** $p<0.01$, *** $p<0.001$

Example: Crime and Police

City	Year	# Police	# Robberies	Population in 10K
Nashville	2001	1,289	4,452	103
Nashville	2002	1450	3,507	104
Nashville	2003	1455	3,804	104
Smallerville	2001	200	845	3.2
Smallerville	2002	250	801	3.3
Smallerville	2003	244	759	3.3

- Trying to disentange the effect of police on crime.

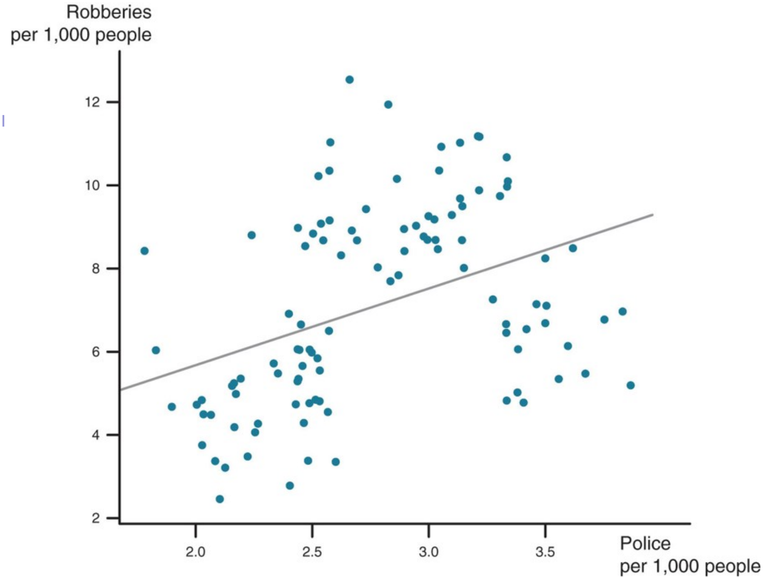
Plotting Data



What are some issues with this relationship?

- Population adjustment

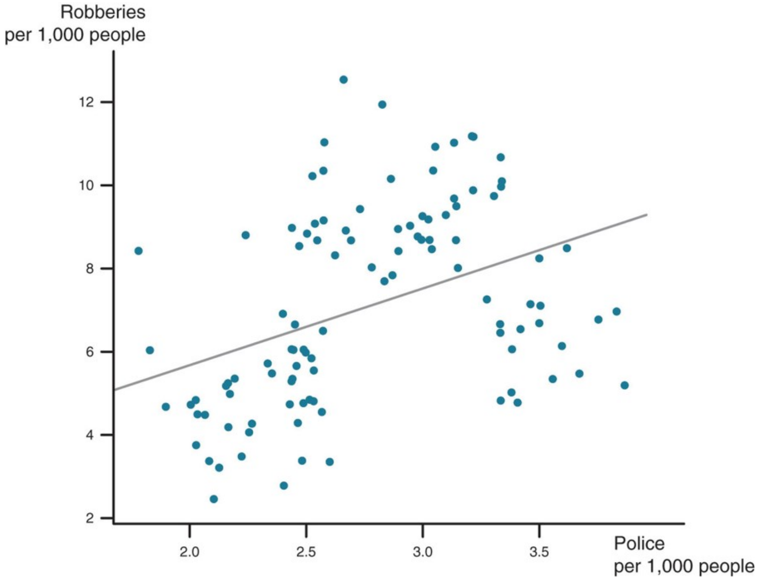
Plotting Data



Practicing calling BS

- Population adjustment
- Measurement error (how is the data recorded)
- Reverse causality: want to use lag-police counts

Plotting Data



Police on Crime

- The model associated with this relationship is:

$$Robberies\ per\ 1K_{ct} = \beta_0 + \beta_1 LagPolice\ per1K_{ct-1} + \varepsilon_{ct}$$

- We use the data we have, pooling observation from all cities together
- This is the regular set-up we have seen. We are going to call this a "Pooled Regression"
- Let's run that regression with the data!
  - What should we get? Positive, neutral or negative relationship?

Regression Results

TABLE 8.1 Basic OLS Analysis of Burglary and Police Officers

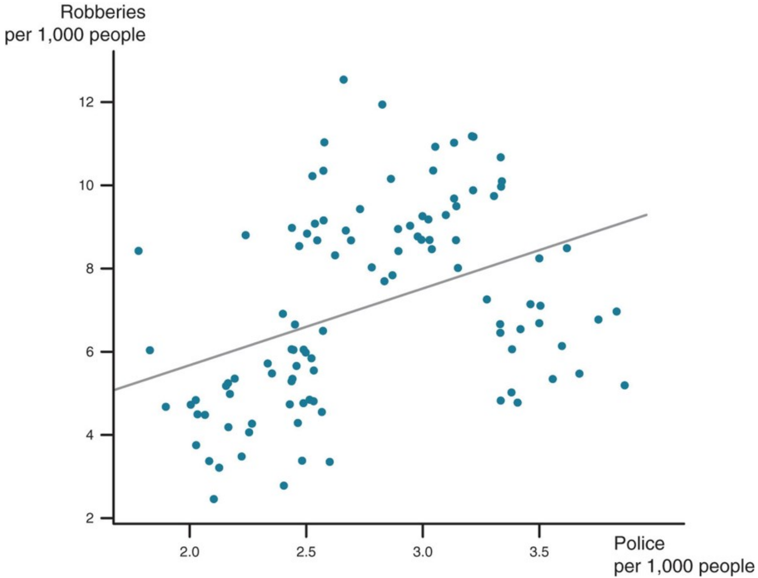
	Pooled OLS
Lag police, per capita	2.37*
	(0.07)
	[$t = 32.59$]
N	1,232

Standard errors in parentheses.

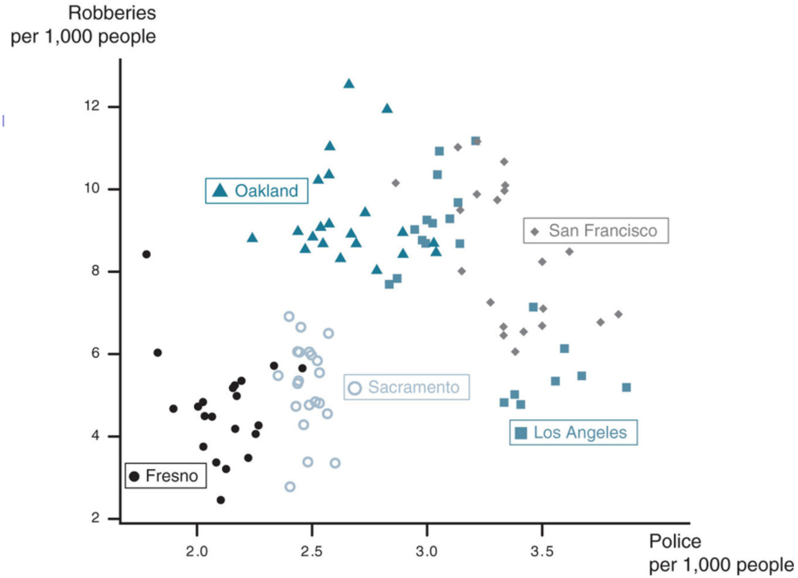
* *indicates significance at* $p < 0.05$*, two-tailed.*

- What's the interpretation of that coefficient?
- What is the conclusion from this regression?
  - Note that we can add controls to this and the relationship would still look positive.

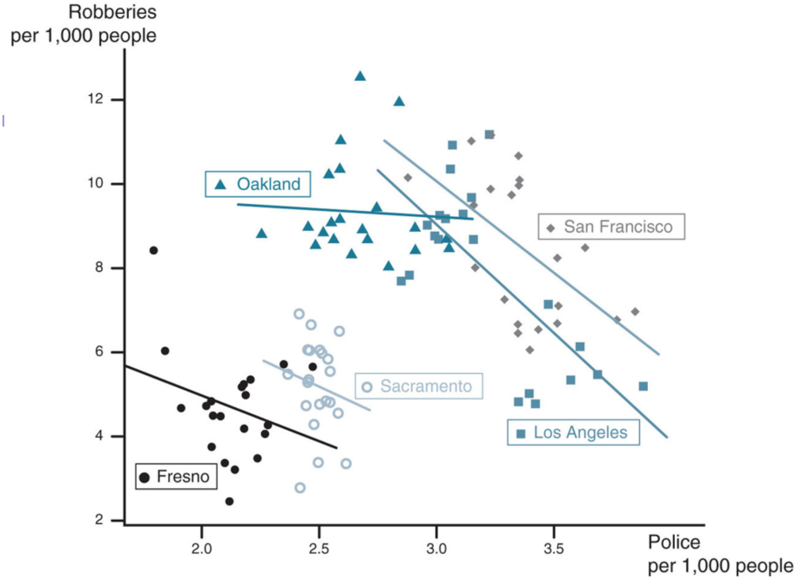
Plotting the Data



What if we look by cities?



What if we look by cities?



What happened?

- When looking at overall trends of police and crime, there was a positive relationship.
- The regression

$$Robberies\ per\ 1K_{ct} = \beta_0 + \beta_1 LagPolice\ per1K_{ct-1} + \varepsilon_{ct}$$

- Gave us the same intuition.
- However, once **we look at the same data**, by cities, in each city the trend is declining?
- How could this be?: In each city crime and police and negatively related but overall I see a positive relationship of crime and police?

What happened?

- This is because the graph was conflicting the fact the bigger cities will have (1) more crime and (2) more police than smaller cities.
- Notice that even when we accounted for population this was still true.
- The graph or pooled analysis, did not account for the crime-police relationship in each city, it just pooled all the information together.
- By pooling the information together, this “masks” that each unit of observation (cities) may have a totally different relationship.

Framework to think about this

- Think of different cities having different "starting points" of the crime-police ratio.
- We like to think of these different "starting points" as some characteristics that are "fixed" about the city. That is, that don't change much over time.
- In each city, this police and crime have a negative relationship, but when pooling things together, it was a positive relationship essentially ignoring these "starting points" and treating all the observations as coming from the same unit.

How does panel data helps us here?

- If we didn't have several points per city, how would a graph look?
- We wouldn't even be able to test this "negative relationship"
- Panel data allows us to disentangle these "overall" relationships.
- How to deal with this in a regression?

$$Robberies\ per\ 1K_{ct} = \beta_0 + \beta_1 LagPolice\ per 1K_{ct-1} + \varepsilon_{ct}$$

Regression Format

$$Robberies\ per\ 1K_{ct} = \beta_0 + \beta_1 LagPolice\ per 1K_{ct-1} + \varepsilon_{ct}$$

- Why did the the pooled regression may showed ***biased*** coefficients?
  - It was not accounting for those "starting points" or "fixed" items. Let's write it in math to understand a bit better what this means.
  - Since we know there is bias in our estimate, this means that X (police) and the error terms are somewhat related.
  - This relationship was about those "fixed" unobservables. In math:

$$\varepsilon_{ct} = \alpha_c + \nu_{ct}$$

  - The α_c are what we called "fixed effects". They represent the tendency for unit c to be higher or lower than a given base unit.
  - This term in the error is capturing ***all*** time-invariant *observable and unobservable* characteristics of each city.

Fixed-Effects in our Example

- In our example, this could be things like:
  - A "culture of crime"
  - The presence of drugs
  - The presence of gangs.
  - These are things that have stayed relatively constant, and places like SF have had more police than places like Fresno where there is less crime.

Fixed-Effects in our Example

- Therefore, the reason in the pooled regression, we found “reverse” signs because our estimate was “biased”
- This city-specific baseline levels are what was violating the exogeneity condition.
- In a more general sense, you can think each panel unit of observation (e.g. city, individual) having a different baseline level of the outcome variable Y, and these levels are correlated with X.
- Therefore it is going to be very important to account for those time-invariant, fixed effects.
- So, how to fix it? (Yes, it is a Pun).

How to fix it? (Pun)

- How do we account for these unobservable (and observable characteristics) baseline levels of tendencies of each city?
- How do we tell our regression to account for this relationship within each city, like we did in the graph, as oppose to use the data all together?
  - We use a fixed-effects approach.
- There are two ways to go about it or interpret it
  - Least Squares dummy variable approach
  - De-meaned approach

Least Square Dummy Approach

$$Robberies\ per\ 1K_{ct} = \beta_0 + \beta_1 LagPolice\ per1K_{ct-1} + \varepsilon_{ct}$$

$$\varepsilon_{ct} = \alpha_c + \nu_{ct}$$

- The LSDA, just means including a dummy variable for each single city.

Least Square Dummy Approach

$$Y_{it} = \beta_0 + \beta_1 X_{it} + \alpha_{\boldsymbol{i}} + \nu_{it}$$

- Where $\alpha_i$is a set of dummies for each city in our data. Say we had 50 cities, then we would have 49 dummies. Each dummy represents a city.
- Why 49 dummies?
  - Because of multicolinearity
  - Just as before, if we had three races (White, black and Hispanic) we couldn't include all the dummies. Similarly we can't include all the 50 cities dummies.
  - If we do, then STATA will just drop it.

How does this look?

$$Y_{ct} = \beta_0 + \beta_1 X_{ct-1} + \alpha_1 C_1 + \alpha_2 C_2 + \alpha_3 C_3 + \ldots + \alpha_{49} C_{49} + \nu_{ct}$$

TABLE 8.2 Example of Robbery and Police Data for Cities in California

City	Year	Robberies per 1,000	Police per 1,000 (lagged)	D_1 (Fresno dummy)	D_2 (Oakland dummy)	D_3 (San Francisco dummy)
Fresno	1991	6.03	1.83	1	0	0
Fresno	1992	8.42	1.78	1	0	0
Oakland	1991	10.35	2.57	0	1	0
Oakland	1992	11.94	2.82	0	1	0
San Francisco	1991	9.50	3.14	0	0	1
San Francisco	1992	11.02	3.14	0	0	1

Applying FE

TABLE 8.4 Burglary and Police Officers, Pooled versus Fixed Effects Models

	Pooled OLS	Fixed effects (one-way)
Lag police (per capita)	2.37*	1.49*
	(0.07)	(0.17)
	[$t = 32.59$]	[$t = 8.67$]
N	1,232	1,232
Number of cities	59	59

Standard errors in parentheses.

* *indicates significance at* $p < 0.05$*, two-tailed.*

What happened?

- It seems that the relationship still seems positive? Even when accounting for city fixed effects?
- Let's go back to our regression:

$$Robberies\ per\ 1K_{ct} = \beta_0 + \beta_1 LagPolice\ per1K_{ct-1} + \alpha_{\boldsymbol{C}} + \nu_{ct}$$

- We could include other controls...
- or we could do the same exercise with the other component of the panel: time.
- We've included fixed effects for Cities (C) but we could include "Time fixed effects"
- In this case: year FE.
- What do they mean?

How to do this in STATA?

- Method 1: using the "i." approach.

```
estimates clear
eststo: reg crime lag_police_per_capita
eststo: reg crime lag_police_per_capita i.city
```

- Method 2: using xtset

```
xtset city year
eststo: xtreg crime lag_police_per_capita, fe
```

How to do this in STATA?

- Method 3: areg command

```
eststo: areg crime lag_police_per_capita, a(city)
```

Applying FE

Table 1. OLS vs. FE Model

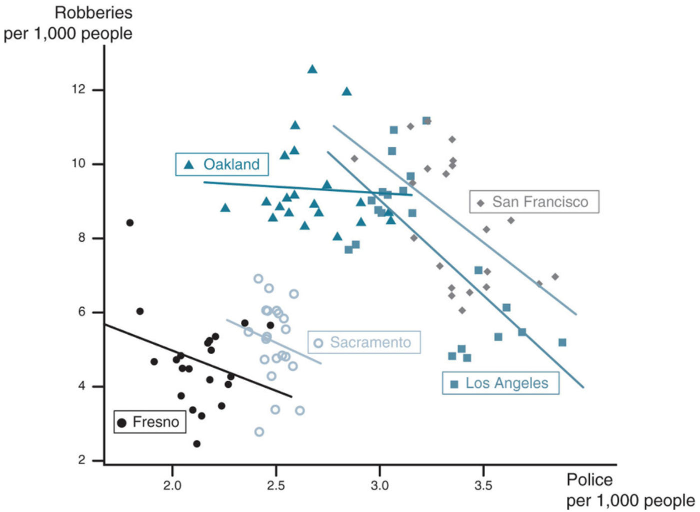
	Pooled OLS	Fixed-Effect (i.city)	Two way Fixed-Effect (i.city i.years)	+ 1 Control
Lag Police Per Capita	0.237*	1.49*	0.147	-0.443**
	(0.0729)	(0.172)	(0.170)	(0.161)
N	1,232	1,232	1,232	1,232
Number of Cities	59	59	59	29

Standard errors in parentheses.* p<0.05, ** p<0.01, *** p<0.001

De-mean approach

- This is another approach to including fixed-effect
- This approach also helps us with an alternative interpretation of the fixed-effects
- Let's go back to our city example to understand the de-mean approach

City and Crime



De-mean approach

- The de-mean approach will tell you to, to calculate the mean robberies per 1000 people for each city, across all years. (Here you can see the within-city component).

City	Year	Police PC (Lag)	Crime PC
Nashville	2001	2.33	8
Nashville	2002	2.54	7
Nashville	2003	2.60	6
Smallerville	2001	1.33	4
Smallerville	2002	1.47	3
Smallerville	2003	1.48	2
...	...	...	...

De-mean approach

City	Year	Police PC (Lag)	Crime PC	Aver Pol PC
Nashville	2001	2.33	8	2.49
Nashville	2002	2.54	7	2.49
Nashville	2003	2.60	6	2.49
Smallerville	2001	1.33	4	1.43
Smallerville	2002	1.47	3	1.43
Smallerville	2003	1.48	2	1.43
...	...	...	...	

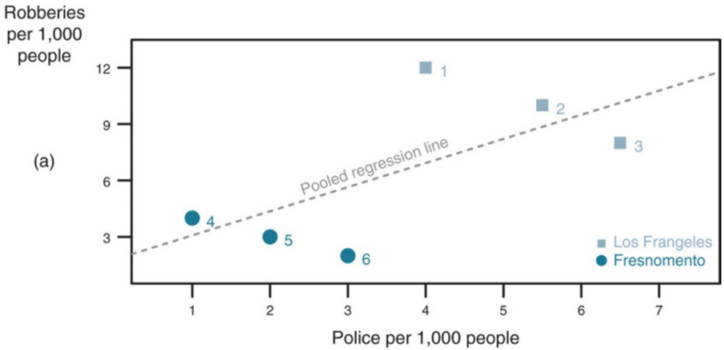
De-mean approach

City	Year	Police PC (Lag)	Crime PC	Aver Pol PC	Ave Cri PC
Nashville	2001	2.33	8	2.49	7
Nashville	2002	2.54	7	2.49	7
Nashville	2003	2.60	6	2.49	7
Smallerville	2001	1.33	4	1.43	3
Smallerville	2002	1.47	3	1.43	3
Smallerville	2003	1.48	2	1.43	3
...	...	...	...		

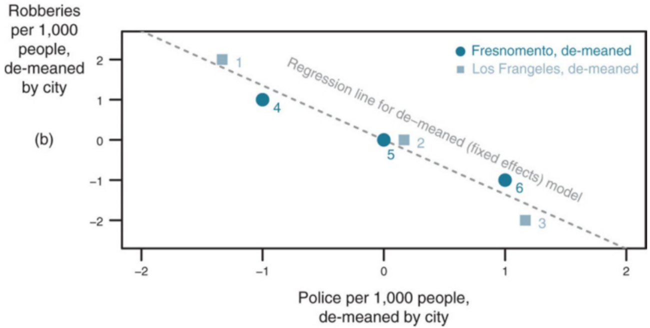
De-mean approach

- Once we've obtain the mean for the outcome variable Y and all the X's, we "de-mean" each variable by its mean.
- This means, we subtract the value of each variable by its mean. Essentially calculating deviations from the mean
- Once each variable had been de-mean:
  - Run the regression using the "de-mean variable".
  - That would give you the same approach as the dummy approach

Graphical Example



Graphical Example



Deviation from the mean

- This give us some intuition about what the fixed effect model is doing.
- We are observing the relationship between police and robberies using within city-variation.

Generalization of Fixed-effects

- Whenever we have panel data or we observe a unit in multiple occasions, we can make within-unit comparison.

$$Y_{ij} = \beta_0 + \beta_1 X_{ij} + \varepsilon_{ij}$$

$$Y_{ij} = \beta_0 + \beta_1 X_{ij} + \overrightarrow{\alpha_{\mathbf{i}}} + \nu_{ij}$$

- Where $\overrightarrow{\alpha_i}$ is a set of dummies for each unit in our data.
- Say we had 50 states, then we would have 49 dummies. Each dummy represents a state.
- Let's go through some examples: firms FE? country FE? individual FE?

More than one Fixed-effect?

$$Y_{ij} = \beta_0 + \beta_1 X_{ij} + \vec{\alpha_i} + v_{ij}$$

- We can also add more than one set of FE:

$$Y_{ij} = \beta_0 + \beta_1 X_{ij} + \vec{\alpha_i} + \vec{\gamma_j} + v_{ij}$$

- This usually represents the second dimension in our data: like time.
  - What would Year FE be capturing? Quarter FE? Month FE? What's the difference between year-month FE and month FE?

Handy Questions when using FE

- What type of FE could one use?
- What is each FE capturing? What are some potential main concerns or sources of bias?
- Where is the variation of the main explanatory variable coming from, after controlling for FE?
  - Is that plausibly exogenous?

Birthweight on Adult Outcomes

- We want to understand the relationship between birthweight and long-term outcomes (Do they dissapear as you age?)
- Say we have data by birth, birth records information, this includes who is the mother, the birthweigth and so on.
- We have linked these data with long-term outcomes of these births (Educational, labor outcomes).
- What's the simple regression we want to run?

$$Y_{ij} = \alpha_0 + \beta_1 BirthWeigth_{ij} + \varepsilon_{ij}$$

Birthweight on Adult Outcomes

- If we want to include a type of fixed-effect, which one would you include?
- Thing of what other type of within-comparison we could make?
- We could include:
  - Mother FE or Sibling FE or Family FE (How would this be coded?)

$$Y_{ip} = \alpha_0 + \beta_1 Birthweight_i + \alpha_1 SibPair_1 + \alpha_2 SibPair_2 + ... + \alpha_P SibPair_P + \varepsilon_{ip}$$

  - What would this FE be capturing?
  - Where is the variation coming from?
  - Is it plausibly exogenous?

Birthweight on Adult Outcomes

- We could also include
  - Twin FE (How could this be coded?)

$$Y_{it} = \alpha_0 + \beta_1 Birthweight_i + \alpha_1 Twin_1 + \alpha_2 Twin_2 + ... + \alpha_T Twin_T + \varepsilon_{it}$$

  - We would need a lot of data!

Example I

From the Cradle to the Labor Market? The Effect of Birth Weight on Adult Outcomes
Sandra E. Black, Paul J. Devereux, and Kjell G. Salvanes
NBER Working Paper No. 11796
November 2005
JEL No. J1, I1

ABSTRACT

Lower birth weight babies have worse outcomes, both short-run in terms of one-year mortality rates and longer run in terms of educational attainment and earnings. However, recent research has called into question whether birth weight itself is important or whether it simply reflects other hard-to-measure characteristics. By applying within twin techniques using a unique dataset from Norway, we examine both short-run and long-run outcomes for the same cohorts. We find that birth weight does matter; very small short-run fixed effect estimates can be misleading because longer-run effects on outcomes such as height, IQ, earnings, and education are significant and similar in magnitude to OLS estimates. Our estimates suggest that eliminating birth weight differences between socio-economic groups would have sizeable effects on the later outcomes of children from poorer families.

Other common FE

- Examples from readings:
  - When estimating the effect of birth weight on long-run and short-run outcomes (height, IQ, earnings, and education), Black et al (2007) use "Twin FE".
    - This means they think that different set of twins have different "starting points" or some unobserved time-invariant component.
    - In the case of twins, what's the most obvious "unobserved time invariant" component?
    - Genetic component
  - Another important question: Where is the variation coming from?

Twin FE

- The variation is coming from "within twin pair". This is variation in birthweight conditional on each twin pair is plausibly exogenous.
  - This variation is what we use to identify the effects on BW on Y

Sibling FE

Longer-Term Effects of Head Start

By ELIANA GARCES, DUNCAN THOMAS, AND JANET CURRIE*

Specially collected data on adults in the Panel Study of Income Dynamics are used to provide evidence on the longer-term effects of Head Start, an early intervention program for poor preschool-age children. Whites who attended Head Start are, relative to their siblings who did not, significantly more likely to complete high school, attend college, and possibly have higher earnings in their early twenties. African-Americans who participated in Head Start are less likely to have been booked or charged with a crime. There is some evidence of positive spillovers from older Head Start children to their younger siblings. (*JEL* J24, I38)

Alcohol Policies

State alcohol policies, teen drinking and traffic fatalities

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Received 17 January 1998; received in revised form 19 June 1998; accepted 2 July 1998

Abstract

This empirical study evaluates the policy responsiveness of teen drinking in models that can condition on the unobserved state-specific attributes that may have biased conventional evaluations. The results demonstrate that cross-state heterogeneity can be important and that beer taxes have relatively small and statistically insignificant effects on teen drinking. Models of youth traffic fatalities also indicate that the conventional beer tax elasticities are not robust to additional controls for omitted variables. The importance of these omitted variables is illustrated by a counterfactual which compares models of nighttime fatalities to those that occur in the daytime when the rate of alcohol involvement is substantially lower.

Keywords: Alcohol use; Traffic fatalities; Alcohol taxes; Minimum legal drinking age; Young adults

Overall questions

- When including FE one should keep it mind:
  - What are the observable things the FE are capturing?
  - What are the unobservable things that FE are capturing?
  - Where is the variation coming from?
    - Why would we expect variation within this group? Is this variation plausibly random?