

STRATEGIES FOR SOCIAL INQUIRY

# Set-Theoretic Methods for the Social Sciences

A Guide to Qualitative  
Comparative Analysis

CARSTEN Q. SCHNEIDER  
CLAUDIUS WAGEMANN

CAMBRIDGE

CAMBRIDGE

more information - [www.cambridge.org/9781107013520](http://www.cambridge.org/9781107013520)

# **Set-Theoretic Methods for the Social Sciences**

---

A Guide to Qualitative Comparative Analysis

**Carsten Q. Schneider  
and  
Claudius Wagemann**



**CAMBRIDGE**  
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS

Cambridge, New York, Melbourne, Madrid, Cape Town,  
Singapore, São Paulo, Delhi, Mexico City

Cambridge University Press  
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

[www.cambridge.org](http://www.cambridge.org)

Information on this title: [www.cambridge.org/9781107601130](http://www.cambridge.org/9781107601130)

Translated and adapted from *Qualitative Comparative Analysis (QCA) und Fuzzy Sets: Ein Lehrbuch für Anwender und alle, die es werden wollen* published in German by Verlag Barbara Budrich 2007, © Verlag Barbara Budrich 2007.

First published in English by Cambridge University Press 2012 as *Set-Theoretic Methods for the Social Sciences: A Guide to Qualitative Comparative Analysis* © Cambridge University Press 2012.

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

Printed and bound in the United Kingdom by the MPG Books Group

*A catalogue record for this publication is available from the British Library*

*Library of Congress Cataloguing in Publication data*

Schneider, Carsten Q., 1972– author.

Set-theoretic methods for the social sciences : a guide to qualitative comparative analysis / Carsten Q. Schneider and Claudius Wagemann.  
pages cm. – (Strategies for social inquiry)

Includes bibliographical references and index.

ISBN 978-1-107-01352-0 (hardback) – ISBN 978-1-107-60113-0 (paperback)

1. Social sciences – Comparative method. 2. Social sciences – Mathematical models.

3. Set theory. I. Wagemann, Claudius, author. II. Title.

H61.S379 2012

300.72–dc23 2012015930

ISBN 978-1-107-01352-0 Hardback

ISBN 978-1-107-60113-0 Paperback

Additional resources for this publication at [www.cambridge.org/schneider-wagemann](http://www.cambridge.org/schneider-wagemann)

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication, and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

# Contents

<i>List of figures</i>	<i>page</i> xii
<i>List of tables</i>	xiv
<i>Acknowledgements</i>	xvi
Introduction	1
Set-theoretic approaches in the social sciences	1
Qualitative Comparative Analysis as a set-theoretic approach and technique	8
Variants of QCA	13
Plan of the book	16
How to use this book	19
<hr/>	
<b>Part I Set-theoretic methods: the basics</b>	21
1 Sets, set membership, and calibration	23
1.1 The notion of sets	24
1.1.1 Sets and concepts	24
1.1.2 The pros and cons of crisp sets	24
1.1.3 Properties of fuzzy sets	27
1.1.4 What fuzzy sets are not	30
1.2 The calibration of set membership	32
1.2.1 Principles of calibration	32
1.2.2 The use of quantitative scales for calibration	33
1.2.3 The “direct” and “indirect” methods of calibration	35
1.2.4 Does the choice of calibration strategy matter much?	38
1.2.5 Assessing calibration	40
2 Notions and operations in set theory	42
2.1 Conjunctions, Boolean and fuzzy multiplication, intersection, logical AND	42

# Figures

0.1	Venn diagram for relation of sufficiency	page 5
0.2	Set-theoretic approaches in the social sciences	10
1.1	Membership in fuzzy set of <i>Länder</i> with underdeveloped all-day schools plotted against percentage of pupils enrolled in all-day schools	36
3.1	Two-by-two table – sufficiency	59
3.2	Venn diagram – sufficiency	60
3.3	XY plots in crisp-set analysis – distribution of cases for sufficient conditions	66
3.4	XY plot – distribution of cases for sufficient condition X	67
3.5	XY plot – fully consistent sufficiency solution	69
3.6	Two-by-two table – necessity	71
3.7	Venn diagram – necessity	72
3.8	XY plot – distribution of cases for necessary condition X	76
3.9	XY plot – non-consistent necessary condition	77
3.10	Two-by-two table – necessity and sufficiency	84
3.11	XY plot – contrasting perfect set relation with perfect correlations	86
4.1	Venn diagram with three conditions	94
4.2	Three-dimensional property space	98
4.3	Logical minimization of primitive expressions to prime implicants	110
4.4	Venn diagram with logically redundant prime implicant	112
5.1	Venn diagrams – consistent and inconsistent sufficient conditions	124
5.2	XY plot – consistent and inconsistent sufficient conditions	125
5.3	Venn diagrams – different levels of coverage sufficiency	130
5.4	XY plot – different levels of coverage sufficiency	132
5.5	Venn diagram – equifinal solution term and types of coverage	135
5.6	XY plot – condition STOCK, outcome EXPORT	142
5.7	Venn diagrams – trivial and non-trivial necessary conditions	145
5.8	XY plot – condition MA+STOCK, outcome EXPORT	147
5.9	XY plot – the tension between consistency and coverage of sufficient conditions	149
6.1	Conservative, intermediate, and most parsimonious solution terms	172
6.2	Venn diagram – types of counterfactuals in Standard Analysis procedure	176

# Tables

1.1	Verbal description of fuzzy-set membership scores	page 29
1.2	Calibration of condition “many institutional veto points”	34
1.3	QUALITATIVE versus direct method of calibration for set “many institutional veto points”	39
2.1	Important operations in set-theoretic methods	43
2.2	Determining membership in complex sets	52
2.3	Basic operations and notations in set-theoretic approaches	54
3.1	Sufficiency: stylized data matrix	59
3.2	Hypothetical data matrix with ten cases and set-membership scores in three conditions and the outcome	61
3.3	Hypothetical data matrix with complements of three conditions	63
3.4	Hypothetical data matrix with some conjunctions	64
3.5	Hypothetical data matrix with fuzzy-set membership scores	68
3.6	Data matrix – necessity	71
3.7	Hypothetical data matrix with all complements of single conditions and conjunction $\sim A+C$	73
4.1	Data matrix with ten cases, three conditions, and outcome	95
4.2	Hypothetical truth table with three conditions	96
4.3	Hypothetical data matrix with fuzzy-set membership scores	97
4.4	Fuzzy-set data matrix with two cases	100
4.5	Fuzzy-set membership in ideal types for hypothetical data matrix	101
4.6	Fuzzy-set ideal types for hypothetical data matrix	101
4.7	Fuzzy-set membership in rows and outcome	102
4.8	Truth table derived from hypothetical fuzzy-set data	104
4.9	Example of hypothetical truth table	106
4.10	Prime implicant chart	111
5.1	Two-by-two tables – consistent and inconsistent sufficient conditions	124
5.2	Two-by-two tables – different levels of coverage sufficiency	131
5.3	Fuzzy-set membership in solution and outcome (Vis 2009)	136
5.4	Fuzzy-set membership in path PS and outcome (Vis 2009)	138
5.5	Two-by-two tables – consistent and inconsistent necessary conditions	140
5.6	Analysis necessity, single conditions (Schneider <i>et al.</i> 2010: 255)	142

# Introduction

---

## Easy reading guide

The Introduction presents an overview of the book. We spell out what this book contains, what it is good for – and what it is not! Rather than starting with technical details of set-theoretic methods, we put the content of the book into a broader context of current methodological debates. The Introduction will help the reader to find out whether, in general, this book might be interesting and, if so, which chapters in particular are most relevant for him/her.

In a first step, we show that notions of sets and their relations are more common in the social sciences than might probably be known. Then we describe Qualitative Comparative Analysis (QCA) as the most developed form of set-theoretic method. We spell out the defining features of QCA and how they differentiate it from other set-theoretic methods. In the next section, we explain the differences and similarities among the various forms of QCA. In the following section, we not only explain the structure of the book, but also provide details on how to use the book by addressing some of its features, such as the Easy reading guides, At-a-glance boxes, the Glossary, or the online material which contains chapter-by-chapter “how-to” sections and exercises.

In short, by reading this Introduction, readers should get a better understanding of what to expect from this book and how to use it in order to maximize its utility.

---

## Set-theoretic approaches in the social sciences

---

Arguments about set relations are pervasive in the social sciences, but this is not always obvious. Take, for example, Brady’s (2010) intriguing deconstruction of the widely debated claim that, in the 2000 US Presidential Election, George W. Bush lost about 10,000 votes because Al Gore had been declared the winner before the closure of the polling stations in those western counties of Florida that are on Central Standard Time (i.e., the Panhandle). This claim is made by Lott (2000), who arrived at this inference by estimating a “‘difference-in-differences’ form of regression analysis, based on data-set

dimensions – labor market protection and transfer payments – then there are four different ideal-typical forms of the welfare state: high labor market protection with high transfer payments; high labor market protection with low transfer payments; low labor market protection with high transfer payments; and low labor market protection with low transfer payments. As Kvist (2006) shows, a set-theoretic approach to forming and arguing about typologies can be very helpful, especially if we – as Kvist does – go beyond dichotomous (crisp) sets and work with fuzzy sets in which cases can have degrees of membership in each dimension.

Notions of set theory are also useful for those more ambitious social science practices that are designed to give a causal interpretation to patterns found in the data. Prominent examples are John Stuart Mill's methods (see, e.g., Mahoney 2003). The possibility of interpreting them in a set-theoretic manner is an aspect that has not received enough attention so far (Mahoney 2007: 134).

#### **At-a-glance: set-theoretic approaches in the social sciences**

Set-theoretic methods operate on membership scores of elements in sets; causal relations are modeled as subset or superset relations; **necessity**<sup>3</sup> and **sufficiency** or **INUS** and **SUIN** conditions are at the center of attention.

The use of set theory focuses attention on unraveling causally complex patterns in terms of **equifinality**, **conjunctural causation**, and **asymmetry**.

Set theory can be useful for concept formation, the creation of typologies, and causal analysis.

## **Qualitative Comparative Analysis as a set-theoretic approach and technique**

Qualitative Comparative Analysis, commonly known under its acronym QCA, is the methodological tool that is perhaps most directly associated with set theory. QCA distinguishes itself from other set-theoretic approaches by the combined presence of the following features. First, it aims at a causal interpretation. This is not necessarily true for other set-theoretic approaches – just think of concept formation or the creation of typologies, which typically do not include any reference to an outcome (for two exceptions, Elman 2005 and George and Bennett 2005). Second, QCA makes use of so-called truth tables.

<sup>3</sup> All the terms that are further defined in the Glossary are printed in bold in the At-a-glance boxes.



of “robustness” in regards to QCA results and what robustness tests should look like; spell out the logic of theory evaluation in set-theoretic methods, as opposed to hypothesis testing in statistical approaches; and present the principles of case selection for within-case studies after a QCA.

The conclusion attempts at a general evaluation of QCA as a social science method and offers an outlook on further developments in set-theoretic methods.

---

## How to use this book

---

Before we enter the debate, let us give some useful hints on how to read this book. We suggest starting at the beginning. While in later chapters we at least briefly reiterate crucial points, it remains the case that issues raised in later chapters can best be understood by thoroughly reading the preceding chapters.

This book is explicitly designed to cater to both beginners and very advanced readers. In order to allow all readers to better navigate through the book and to easily identify the chapters that are most relevant to their current needs and interests, we employ several devices. First, each main chapter starts with an “Easy reading guide.” This presents the content and main points made in the chapter in question. The Easy reading guides can help both more advanced readers to move directly to specific sections and beginners to identify those sections that are fundamental for understanding the method and which ones contain additional arguments and debates. The second device is “At-a-glance” boxes at the end of most sections. They summarize the key points of the respective section and are directly connected to the “Glossary,” our third didactic device. It contains definitions of all key terms in set-theoretic methods that are used and introduced in the book. Terms printed in bold in the At-a-glance boxes are those that are contained in the Glossary. Finally, we provide online learning material for each chapter. The “How to” sections contain practical guidance on how to use the currently available software packages (fsQCA, 2.5, Tosmana 1.3.2, Stata, and R) in order to perform the analytic operations described in the respective chapter. The exercises and solutions are subdivided into conceptual questions, exercises that require calculations by hand, and exercises practicing the use of the software by reanalyzing published QCA.

Throughout the book, we make use of published examples of set-theoretic analyses. In the early chapters, however, when we need to separate specific

# Part I

## Set-theoretic methods: the basics

## 1.1 The notion of sets

### 1.1.1 Sets and concepts

The use of the term “set” is not very broadly diffused in social science methodology. However, a good part of our conceptual reasoning, as Mahoney (2010) shows, is at least based on an implicit idea of sets. According to Mahoney, there are two basic modes of looking at concepts: if we define concepts “as a mental representation of an empirical property” (Mahoney 2010: 2), then we will measure cases “according to whether or the extent to which they are in possession of the represented property” (Mahoney 2010: 2). Measurement theory provides us with many useful techniques for doing this. This ultimately results in the use of variables when defining a concept (Mahoney 2010: 13). If, however, we refer to concepts as sets, defined in terms of “boundaries that define zones of inclusion and exclusion” (Mahoney 2010: 7), then “[c]ases are measured according to their fit within the boundaries of a set” (Mahoney 2010: 2). Sets work as “data containers” (Sartori 1970: 1039). Although this seems to be a subtle and often overlooked differentiation, these two views of concepts are fundamentally different. When we measure a concept by means of traditional measurement theory, it represents a property or a group of properties. The set-theoretic view, instead, uses set membership in order to define whether a case can be described by a concept or not. Therefore, in the framework of set-theoretic methods, issues of concept formation have a somewhat different connotation than in traditional measurement theory, by focussing on whether a case belongs to a concept (i.e., a set) or not. This process of assigning set membership is also called “calibration” (see [section 1.2](#)).

### 1.1.2 The pros and cons of crisp sets

When QCA was first discussed in the 1980s and 1990s, it was limited to crisp sets. This required a decision whether a case is a member of a set or not. As such, this also corresponds to how sets are generally perceived, namely as boxes into which cases can be sorted or not. However, as argued in the [Introduction](#), it is not always easy to make such clear-cut decisions, above all when dealing with more fine-grained social science concepts for which detailed and nuanced information is available. Not surprisingly, the need for “dichotomization” has triggered some serious criticism of crisp-set QCA (Bollen, Entwisle, and Alderson 1993; Goldthorpe 1997; for an overview and

# Glossary

<b>Addition, Boolean/fuzzy</b>	See <i>logical OR</i> .
<b>Arithmetic remainder</b>	<i>Logical remainder</i> that occurs when the number of logically possible combinations of <i>conditions</i> (see also <i>configuration</i> ) exceeds the number of cases at hand.
<b>Associativity</b>	The sequence in which single sets are combined (when the operator remains the same) is unimportant. $(A * B) * C = A * (B * C) = (A * C) * B$ $A + B) + C = A + (B + C) = (A + C) + B.$
<b>Assumption</b>	Claim that a given <i>logical remainder</i> is sufficient for the <i>outcome</i> , which therefore is subsequently included into the <i>logical minimization</i> process. See also <i>counterfactual</i> and <i>simplifying assumption</i> .
<b>Asymmetry</b>	Implies that (a) a causal role attributed to a <i>condition</i> always refers to only one of the two qualitative states – presence or absence – in which the <i>condition</i> set can be found and (b) any solution term always refers to only one of the two qualitative states – presence or absence – in which the <i>outcome</i> set can be found. Both forms of asymmetry are the consequence of the fact that, in set-theoretic methods, the presence of a set and its negation denote two qualitatively different phenomena. <i>Sufficiency</i> and <i>necessity</i> are typical asymmetric relations.
<b>Calibration</b>	Process in which set <i>membership scores</i> are assigned to cases.

## Bibliography

- Abbott, Andrew 2001. *Time Matters: On Theory and Method*. University of Chicago Press.
- Achen, Christopher 2005. "Let's put garbage-can regressions and garbage-can probits where they belong." *Conflict Management and Peace Science* 22: 327–39.
2008. "Registration and voting under rational expectations: the econometric implications." Paper presented at the Summer Meeting of the Society for Political Methodology, Ann Arbor, MI.
- Adcock, Robert 2007. "Who's afraid of determinism? The ambivalence of macro-historical inquiry." *Journal of the Philosophy of History* 1(3): 346–64.
- Adcock, Robert, and Collier, David 2001. "Measurement validity: a shared standard for qualitative and quantitative research." *American Political Science Review* 95: 529–46.
- Altman, David, and Perez-Linan, Anibal 2002. "Assessing the quality of democracy: freedom, competitiveness and participation in eighteen Latin American countries." *Democratization* 9: 85–100.
- Amenta, Edwin, and Poulsen, Jane D. 1994. "Where to begin. A survey of five approaches to selecting independent variables for qualitative comparative analysis." *Sociological Methods & Research* 23: 22–53.
- Amenta, Edwin, Caren, Neal, and Olasky, Sheera J. 2005. "Age for leisure? Political mediation and the impact of the pension movement on US old-age policy." *American Sociological Review* 70: 516–38.
- Barton, Allen H. 1955. "The concept of property space in social research." In Paul F. Lazarsfeld and Morris Rosenberg (eds.), *The Language of Social Research: A Reader in the Methodology of the Social Sciences*. New York and London: The Free Press, pp. 40–53.
- Baumgartner, Michael 2008. "Uncovering deterministic causal structures: a Boolean approach." *Synthese* 170(1): 71–96.
2009. "Inferring causal complexity." *Sociological Methods & Research* 38(1): 71–101.
- Bennett, Andrew, and Elman, Colin 2006. "Complex causal relations and case study methods: the example of path dependence." *Political Analysis* 14: 250–67.
- Berg-Schlusser, Dirk, and De Meur, Gisèle 1997. "Reduction of complexity for small-N analysis: a stepwise multi-methodological approach." *Comparative Social Research* 16: 133–62.
2008. "Comparative research design: case and variable selection." In Rihoux and Ragin (eds.), pp. 19–32.
- Berg-Schlusser, Dirk, De Meur, Gisèle, Rihoux, Benoit, and Ragin, Charles C. 2008. "Qualitative comparative analysis (QCA) as an approach." In Rihoux and Ragin (eds.), pp. 1–18.

# Index

- Achen, C., 157  
Adcock, R., 317  
addition, Boolean, 51  
additivity, 86  
algorithm, 92, 104, 105, 112, 115, 120, 152,  
177–179, 182, 190, 191, 193, 197, 242, 243,  
281, 318  
analytic moment, 11, 91, 305  
antecedent condition, 267, 268  
anticipated effects, 265  
arithmetic remainder, 153, 154, 157  
arrow, 53, 70, 82, 267  
associativity, 48, 55  
assumption, 86, 122, 151, 157, 158, 160–162, 164–  
169, 173–176, 188, 197–204, 209, 210, 212, 214,  
215, 217–219, 221–225, 228, 231, 239, 277–279,  
282, 283, 285, 298, 304, 305, 314, 315, 318  
asymmetry, 6, 8, 53, 54, 78, 79, 81, 112–114, 300,  
307, 316, 317
- Berg-Schlosser, D., 255–257, 260, 262, 276  
binomial probability test, 128  
Black Raven Paradox, 85  
Boolean algebra, 9, 17, 46, 47, 49, 54, 55, 92, 104,  
115, 313  
Boolean calculator, 284  
Boolean methods, 6  
Brady, H. E., 1, 2, 297  
Braumoeller, B., 80, 88, 128, 297  
Bush, G. W., 1, 2
- calibration, 23, 24, 32–35, 37–41, 121, 128, 182,  
266, 277, 284–289, 291, 294, 316, 318, 319  
Caren, N., 9, 215, 268–271  
case selection, 11, 17, 19, 70, 123, 160, 275, 286,  
293, 306, 307, 312, 316, 318, 319  
causal analysis, 8, 13  
causal complexity, 4, 6, 9, 17, 56, 60, 77–81, 83,  
86–90, 297, 313, 316–318, 336  
causal mechanism, 189, 254, 301, 308, 309  
causal relation, 6, 8, 53, 70, 77, 79, 81, 155  
causality, 6, 58, 78, 79, 89, 90, 215, 267, 313  
Clark, W. R., 87  
clustered remainder, 154  
commutativity, 48, 50, 53, 55  
Compasses, 13  
complement, 18, 43, 47, 48, 50, 54, 55, 65, 67, 82,  
91, 101, 123, 203, 226, 238  
complex solution term, 162, 166  
concept, 3, 6, 7, 11, 14–16, 23–26, 28–35, 37, 40,  
41, 45, 46, 53, 74, 113, 121, 260, 261, 263, 266,  
277, 291, 293, 312, 321  
concept formation, 7, 8, 24, 319  
conceptual boundaries, 3, 27  
condition, 11, 52, 53, 55, 60, 66, 103, 128, 133,  
188, 201, 206, 228, 230, 237, 240, 261, 270, 272,  
278–281, 286, 288, 289, 298, 302, 307–311, 314,  
319, 325  
configuration, 7, 91, 92, 95, 103, 115, 122, 155,  
179, 190, 199, 223, 254, 279, 299, 305  
Configurational Comparative Methods, 6, 203  
confirmed most likely case, 299  
conjunction, 43, 44, 48, 54, 55, 63–65, 68, 72, 73,  
79, 92, 95, 100–102, 104–109, 122, 134, 149,  
151, 155, 168, 169, 171, 173, 197, 215–217, 229,  
241, 243, 248, 257, 258, 266, 279, 284, 288, 297,  
301, 303, 307, 309  
conjunctural causality, 6, 79  
conjunctural causation, 8, 9  
conservative solution term, 162, 166, 169, 171,  
172, 202, 212, 225, 228, 230, 285  
consistency, 18, 119, 120, 122–131, 133–137,  
139–141, 143–151, 178, 182, 185–189, 191, 193,  
201, 202, 204, 206, 210, 214, 216, 221, 224–233,  
235–237, 239–244, 247, 254, 278–282, 284–294,  
300–303, 305, 307, 308, 310, 314, 316, 318–320  
consistency level, 128, 143, 148, 279, 286, 292  
consistent least likely case, 303, 305  
consistent most likely case, 303, 305  
contradictory (simplifying) assumption, 204, 205