Public Investment Criteria for Underpriced Public Products

By GENE E. MUMY AND STEVE H. HANKE*

Procedures for the design and operation of public undertakings have traditionally separated the evaluation of project benefits and the pricing of project outputs. Benefit-cost calculations have been based upon standard efficiency criteria without regard to the pricing policy utilized during the operating phase. An undertaking, whose scale has been appropriately determined by equating incremental benefits and costs, is deemed worthy of investment if its total benefits exceed costs. However, if the pricing policy that governs the sale of output is not explicitly taken into account in the benefit-cost analysis, serious errors in the estimation of benefits and, hence, the scaling of facilities will be encountered.

In practice, no widespread recognition of the effects of ultimate public project pricing policies on prior benefit calculations is evident (see Hanke). For example, the Water Resources Council has recently provided new rules for benefit-cost analysis that are far more comprehensive than any previous federal effort, but the new standards do not mention pricing policies or how they might alter the performance of public undertakings.

In the academic literature, very few have noted the dilemma created by not explicitly taking into consideration pricing policies when conducting benefit-cost analyses. John Krutilla is one of the few economists to explicitly recognize the general implications of separating evaluation and pricing procedures. In their classic benefit-cost study of the Victoria Line, Christopher Foster and Michael E. Beesley assumed that the fares imposed by the Victoria Line would make no difference in the calculated net rate of return for the project. Both Christopher Winsten and Alan Walters, in discussing the Foster-Beesley paper, correctly criticized this assumption. They argued that net benefits will be reduced by charging underground fares that exceed the appropriate marginal cost, adjusted for nonoptimal prices that existed in competing transport modes. In order to maintain consistency between evaluation and pricing criteria, Krutilla, Winsten, and Walters have recommended that standard evaluative criteria should be used in conjunction with marginal cost pricing. Roland McKean also expressed a preference for the use of standard evaluative criteria and marginal cost pricing. McKean realized, however, that prices would be set below marginal costs in some cases. In these situations, he suggested that project benefits be conservatively estimated. One of the few economists who has analytically dealt with the problem of evaluative criteria modification in the face of prices that are below marginal cost is William Vickrey. In a discussion of peak-load pricing, he recognizes that there are institutional, technical, and economic reasons why prices cannot always be set equal to the peak-load marginal cost. To maintain consistency between pricing and evaluation, Vickrey develops benefit measures for various types of nonprice rationing assumptions. Joseph Seneca has also dealt analytically with the problem of separating pricing and evaluation. He developed a measure of benefits for a given output in the extreme case in which no price is charged.

In our reading of the benefit-cost literature, we have found that most analysts ignore the problem of maintaining consistency

* Assistant professor, Virginia Polytechnic Institute and State University, and associate professor, The Johns Hopkins University and University of California, Berkeley, respectively. This paper was prepared with the assistance of a research grant from the National Science Foundation, NSF-RANN, GI-34869. We wish to thank John Boland, George Borts, Gardner Brown, Paul Bugg, Anthony Fisher, Jack Knetsch, John Krutilla, Joseph Seneca, Roger Sherman, Ralph Turvey, Michael Visscher, Richard Walker, and an anonymous referee for their helpful suggestions.

between evaluation and pricing criteria. The few who recognize the problem prefer to maintain consistency by recommending that standard evaluative criteria be used in conjunction with marginal cost pricing. We believe that the insistence on the use of standard evaluative criteria and marginal cost pricing is unrealistic. Evaluative criteria that are consistent with the zero pricing and underpricing of public products must be developed if efficiency within the public sector is to be improved.

In this paper, we reformulate the zero-price evaluative criterion developed by Seneca and build upon it to derive appropriate investment criteria. We then extend the analysis to the general case of underpricing, which was touched upon by Vickrey. Welfare levels for zero pricing and underpricing are compared for standard and reformulated investment criteria. Equity effects are not considered.[1] The policy implications derived from our analysis are also presented.

I. The Zero-Pricing Case

We begin our analysis with a discussion of the limiting case in which a public product's price is zero. For the output of the project in question, we have an aggregate demand price function $P=P(Q)$, where $dP/dQ<0$, $P(0)$ is finite, and there is a finite output Q_m, where $P(Q_m)=0$. If prices are used to allocate capacity, it is assumed that the definite integral of the demand price function is an adequate measure of total benefits. We further assume that operating costs are zero, and capacity is measured in terms of possible output Q for a given project scale. Hence, we have a capacity cost function represented by $C=C(Q)$. Our analysis is simplified by focusing on a single time period in which capacity depreciates instantaneously at the end of the demand period (for example, one year) for which our demand price function is relevant. Lastly, we assume that congestion costs do not exist.

Our assumption regarding congestion costs indicates that there is a direct relation between the number of production and consumption units.[2] This assumption excludes the possibility that for a given number of production units there exist alternative consumption units of varying quality, where the quality is a function of use. Our model, therefore, is developed for the case in which there is a one-to-one correspondence between output and consumption. For example, if an irrigation project is being considered, the amount of water consumed cannot be greater or less than the net amount (gross water diverted less evaporation and leakage) of water produced. Our assumption implies that, regardless of the method used to allocate the output, individuals' valuations of the output will not be altered. This is necessary to maintain independence between the demand function and the price charged.

The problem for project planners is to determine the level of output ($\hat{Q}$) (capacity) that maximizes the appropriate welfare (W) function. The welfare function used in standard analysis is represented by the following equation:

$$(1) \qquad W(Q) = \int_0^Q P(Q')dQ' - C(Q)$$

To maximize welfare, we differentiate equation (1) with respect to Q and set the results equal to zero, obtaining the first-order condition:

$$(2) \qquad P(Q) = C'(Q)$$

This condition represents the standard investment criterion; capacity should be chosen so that the value of the marginal unit of capacity equals its marginal cost. However, this investment criterion leads to a welfare maximum only if capacity is allocated so that only points on the demand price function with values greater than or equal to $P(\hat{Q})$ are served. The familiar marginal-cost pricing rule in which a price of $\hat{P}=P(\hat{Q})=C'(\hat{Q})$ is charged would lead to a welfare maximum, since users who do not value capacity at levels equal to or greater than the price $\hat{P}$ would be excluded. But many times no price is charged for public products and, lacking omniscient bureau-

[1] For a discussion of equity arguments for and against the underpricing of public products, see Krutilla.

[2] This relation is discussed by William Oakland.

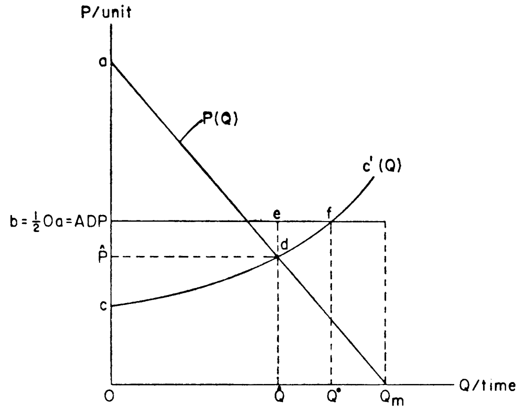


FIGURE 1. CAPACITY DETERMINATION WITH STANDARD AND NEW INVESTMENT CRITERIA ZERO-PRICING CASE

crats, there is no practical way of allocating capacity to its highest value uses.

In the case where price is not used to allocate capacity, we assume that for any given capacity, each of the demanded consumption units from zero to Q_m has an equal probability of being satisfied. Therefore, the number of demanded consumption units is always greater than capacity, except for the case in which capacity equals Q_m. In this case, each demanded consumption unit has a probability of one that it will be satisfied.[3] Given this assumption, the expected value of a demanded consumption unit that is satisfied is given by the average demand price (ADP) for all demanded consumption units (see Figure 1). The average demand price can be written as:

$$ADP = \frac{1}{Q_m} \int_0^{Q_m} P(Q')dQ', \quad \text{for } 0 \leqq Q \leqq Q_m$$

For any given capacity that is less than or equal to Q_m, the expected total benefits B are determined by multiplying ADP times the number of demanded consumption units satisfied (capacity). Expected total benefits are represented by:

$$(3) \qquad B = \frac{Q}{Q_m} \int_0^{Q_m} P(Q')dQ' \qquad \text{for } 0 \leqq Q \leqq Q_m$$

Note that the above benefit function is only defined for values of Q in the range from zero to Q_m. For Q greater than or equal to Q_m, all demand is satisfied and there is no rationing problem. Hence, for Q greater than or equal to Q_m, the benefit function is the constant $\int_0^{Q_m} P(Q')dQ'$ and marginal benefits are zero. If we subtract costs $C(Q)$ from equation (3), we obtain the correct welfare measure (expected net benefits) when prices are not used to ration capacity. The new welfare function is represented by the following equation:

$$(4) \quad W = \frac{Q}{Q_m} \int_0^{Q_m} P(Q')dQ' - C(Q), \qquad \text{for } 0 \leqq Q \leqq Q_m$$

Differentiating equation (4) with respect to Q and setting the results equal to zero gives the first-order condition for maximum expected net benefits:

$$(5) \qquad \frac{1}{Q_m} \int_0^{Q_m} P(Q')dQ' = C'(Q)$$

This condition represents the new investment criterion for this case in which no price is charged for the output from a public undertaking. Optimum capacity occurs at the level where the marginal cost of capacity equals the average demand price.

To complete our discussion of the investment criterion for the zero-pricing case, we must also consider the situations in which the marginal cost function does not intersect the average demand price function in the range of output from zero to Q_m (see Figure 1). First, if the marginal cost function is everywhere greater than ADP, the optimum capacity is zero since, for any proposed capacity, the expected benefits are less than costs. Secondly, if the marginal cost function is below ADP in the range of output from zero to Q_m or, because of decreasing costs,

[3] This assumption seems most appropriate, unless there is a definite reason for suspecting that the majority of output will go to either high willingness-to-pay users or to low willingness-to-pay users (see Gardner Brown and M. Bruce Johnson and Michael Visscher). The effect of various rationing assumptions upon our analysis will be discussed later in the paper.

the marginal cost function intersects ADP from above instead of from below,[4] the optimum capacity is Q_m.[5] In this second case, the expected marginal gross benefit function is ADP. Therefore, expected marginal benefits exceed marginal costs until Q_m is reached. At Q_m all demand is satisfied, marginal benefits are zero and the optimum capacity is reached.

We can now see that when prices are not charged for public products, the standard investment criterion (2) and the new investment criterion (5) will not usually lead to the same choice of capacity. It would be strictly coincidental to find $P(Q)=C'(Q)=ADP$, which are the necessary conditions for the same capacity to be selected using both criteria.

The potential welfare gain from the use of equation (5) rather than equation (2) as an investment criterion can be easily illustrated. When it is assumed that price will ration capacity to its highest-valued uses, net benefit is the difference between the area beneath the demand curve and the area beneath the marginal cost curve at the level of Q chosen. Using the standard evaluative criterion of equation (2), optimum capacity in Figure 1 is $\hat{Q}$, where the demand and marginal cost curves intersect. If price $\hat{P}$ equal to marginal cost $C'(\hat{Q})$ is used to allocate $\hat{Q}$, net benefits equal the area *acd*. But if no price is charged, the expected net benefit from capacity $\hat{Q}$ is not the area *acd*. Assuming all consumption units from zero to Q_m have an equal probability of being satisfied, expected net benefit is the area beneath the average demand price curve (here a constant (1/2) Oa since demand is drawn as a linear function) minus the area beneath the marginal cost curve at the level of Q chosen. At capacity $\hat{Q}$, then, expected net benefit at a zero price equals the area *bcde*. Expected net benefit can be increased by using the investment criterion in equation (5) and extending capacity from $\hat{Q}$ to Q^*. At Q^*, average demand price equals marginal cost, and an offer of capacity Q^* rather than $\hat{Q}$ increases welfare by the amount *edf*.

[4] Note that the marginal cost function must intersect ADP from below to satisfy second-order conditions for a maximum.

[5] This assumes that the total conditions are met under conditions of decreasing costs.

II. The Under-pricing Case

By extending our analysis of welfare measures and investment criteria for the zero-pricing case, we develop a measure of welfare and investment criteria for the more general case of underpricing. In the underpricing case, a positive price $\overline{P}$ is levied for a given output and capacity of $\overline{Q}$, such that $\overline{P}<P(\overline{Q})$.

We assume $Q=Q(P)$ to be the demand function which is the unique inverse of our demand price function $P=P(Q)$. For the underpriced case, where $Q=Q(\overline{P})>\overline{Q}$ and thus greater than capacity, we are again interested in determining the expected value of the demanded consumption units that are satisfied. If we assume each of the $\bar{\bar{Q}}$ units has an equal probability of being satisfied, the expected value (ADP) of a satisfied unit is given by:

$$ADP = \frac{1}{\bar{\bar{Q}}}\int_0^{\bar{\bar{Q}}=Q(\overline{P})} P(Q')dQ', \qquad \text{for } 0 \leqq Q \leqq \bar{\bar{Q}}$$

This expected value is the average demand price for all consumption units from zero to $\bar{\bar{Q}}$. For any output Q expected total gross benefits B are:

$$B = \frac{Q}{\bar{\bar{Q}}}\int_0^{\bar{\bar{Q}}=Q(\overline{P})} P(Q')dQ' \qquad \text{for } 0 \leqq Q \leqq \bar{\bar{Q}}$$

For reasons analogous to those given for the zero price case, this benefit function is appropriate only for values of Q that are in the range from zero to $\bar{\bar{Q}}$. Capacity above $\bar{\bar{Q}}$ will not be used because the price $\overline{P}$ will restrict use to $\bar{\bar{Q}}$. The benefit function at this point becomes a constant, $\int_0^{Q} P(Q')dQ'\cdot\bar{\bar{Q}}$. Subtracting costs $C(Q)$, we obtain the welfare function that we wish to maximize:

$$(6)\quad W(Q) = \frac{Q}{\bar{\bar{Q}}}\int_0^{\bar{\bar{Q}}=Q(\overline{P})} P(Q')dQ' - C(Q)$$

This case is analogous to the zero price case. The only difference is that, in the underpricing case, the average demand price is the average over the quantities from zero to $\bar{\bar{Q}}$, instead of zero to Q_m.

If $\overline{P}=0$, then $\bar{\bar{Q}}=Q_m$ and equation (6) is identical to equation (4). Now suppose we impose the marginal cost-pricing rule, i.e., $\overline{P}=P(\overline{Q})=C'(\overline{Q})$. This implies that $Q(\overline{P})=\overline{Q}$, which means that Q and $\bar{\bar{Q}}$, in the first term of equation (6), cancel out and the upper limit of the definite integral becomes $\overline{Q}$. Thus, equation (6) becomes a specific example of the generalized equation (1), and marginal cost pricing and zero pricing are special cases of underpricing. It could be shown, in a manner analogous to that used in the first section of the paper, that welfare losses from using an investment criterion derivable from equation (6) will generally be less than welfare losses resulting from using (2), if a price less than marginal cost is charged.

III. Implications of the Analysis for Project Scale and the Imposition of User Fees

Now that evaluation criteria have been developed, we examine whether the new criteria will yield capacities that are greater than, less than, or equal to those determined by the standard criterion when public products are underpriced. Without information concerning the cost and demand functions, it is impossible to say a priori whether capacity determined by the new criterion is greater than, equal to, or less than capacity determined by the standard criterion. However, if no prices are charged, if increasing costs exist, and if P represents the price indicated by the marginal cost price criterion, the relevant relationships are: if $P=P(Q)=C'(Q)\gtreqless ADP$, then $Q^*\gtreqless\hat{Q}$ where Q^* and $\hat{Q}$ are the capacities determined by the new and standard criteria, respectively. This relationship is shown graphically in Figure 2, which is the same as Figure 1, except that now we have three alternative marginal cost curves. In the case of $C_1'(Q)$, $P(Q)=C_1'(Q)>ADP$ and $Q_1^*<\hat{Q}_1$; for $C_2'(Q)$, $P(Q)=C_2'(Q)=ADP$ and $Q_2^*=\hat{Q}_2$; and for $C_3'(Q)$, $P(Q)=C_3'(Q)<ADP$ and $Q_3^*>\hat{Q}_3$.

The same capacity relationships hold for conditions of constant costs, except when $C'(Q)$ is everywhere equal to ADP; if the new criterion is used, capacity is indeterminate. Furthermore, we know that under conditions of constant costs the new evaluative criterion will determine capacity to be zero, if $C'(Q)$ is greater than ADP, and capacity will be Q_m, if $C(Q)'$ is less than ADP.

Under conditions of decreasing costs, if the total condition (total benefits exceed total costs) is met, the new criterion will always select capacity Q_m, which is greater than the capacity that will be selected by the standard criterion. However, the e are cases in which the total condition will be met by the standard welfare measure, but not by the new welfare measure, because total benefits calculated for any given capacity are greater when using the standard measure rather than the new one except at Q_m, where both measures yield the same benefits. In these situations, the standard criterion results in a greater capacity, since there is no investment if the new criterion is used.

Another result of our analysis is a criterion for the choice of pricing policy when the exclusion of nonpayers is possible but costly. High exclusion costs are frequently used to justify the practice of not charging prices for public products. Suppose that by incurring

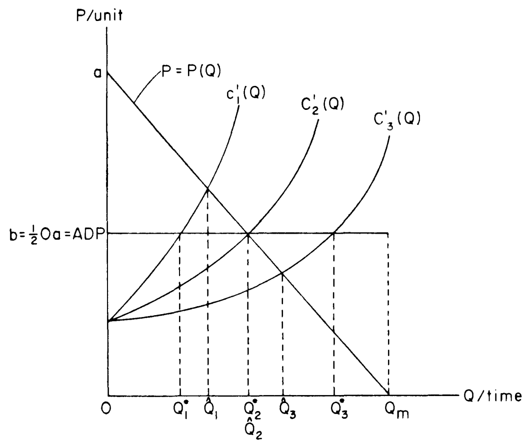


Figure 2. Capacity Determination with Standard and New Investment Criteria Zero-Pricing Case

an exclusion cost (EC) all nonpayers could be excluded from a project's benefits. If one uses the standard investment criterion for choosing capacity, the incurring of exclusion costs may erroneously appear to be worthwhile in many cases. For example, in Figure 1 capacity Q^* maximizes net benefits when Q goes unpriced, so at zero price Q^* offers greater net benefits than capacity $\hat{Q}$. Therefore, for some values of EC, the potential gain in net benefits from excluding nonpayers at capacity $\hat{Q}$ (equal to the difference between area *acd* and area *bcde*) will exceed EC. However, if an increase in capacity to Q^* is considered, the potential gain from exclusion (equal to the difference between area *acd* and area *bcf*) will be less than EC. Now if we calculate zero pricing benefits at capacity Q^* the pricing policy criterion is as follows: If the gain from exclusion exceeds EC, charge marginal cost and choose capacity $\hat{Q}$ (from standard investment criterion); if the gain from exclusion is less than EC, price at zero and choose capacity Q^* (from the new zero-pricing investment criterion).

IV. Alternative Nonprice Rationing Assumptions

When effective demand exceeds capacity, it becomes necessary to make some sort of assumption regarding the mechanism used to determine which consumers will be successful in acquiring the output. We believe that the most reasonable assumption, and the assumption employed in our analysis, is one in which each of the demanded consumption units at a given price has an equal probability of being satisfied. Other rationing assumptions, however, are possible. To illustrate how different assumptions would alter our results, we briefly analyze and compare the marginal benefit function for the zero price case under two alternative rationing assumptions: 1) the highest willingness-to-pay consumption units demanded are satisfied first; and 2) the lowest willingness-to-pay units demanded are satisfied first.

In some situations, the first rationing assumption might be the most reasonable. For example, if reservations are used to allocate capacity, it might be reasonable to suppose that the consumers who would be willing to pay the most would be the ones who obtained reservations first or if a lottery in which all consumers had an equal chance of obtaining a reservation were instituted, a perfect speculators market in reservations might develop and the reservations of low-valued consumers would be transferred to high-valued consumers. If queuing and transactions costs were absent in these situations, the result would be the same as if a price were charged to allocate output to its highest willingness-to-pay users. In this case, the marginal benefit function is given by the standard demand-price function $P=P(Q)$, rather than the average demand-price function.

It is also possible to posit situations in which the lowest willingness-to-pay units are satisfied first. Let us assume that the low willingness-to-pay users obtain reservations well in advance of actual consumption and that the high willingness-to-pay users attempt to make reservations on short notice. If no transfers are allowed or if transfer costs are too high, the lowest willingness-to-pay units will obtain reservations first. In this case, the consumption units that are satisfied by any output Q_1 are those from (Q_m-Q_1) to Q_m on the demand-price function. The total gross benefits for output Q_1 are given by the area under the demand-price function $[P=P(Q)]$, from (Q_m-Q_1) to Q_m as shown in

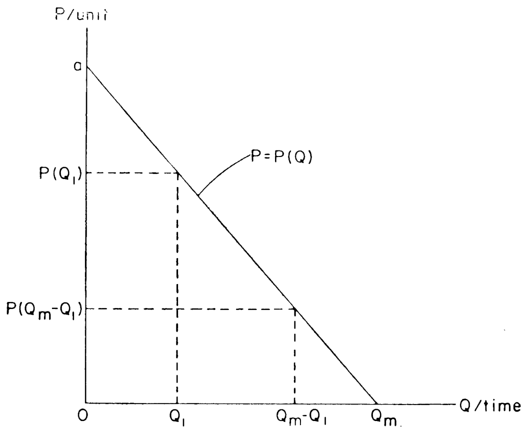


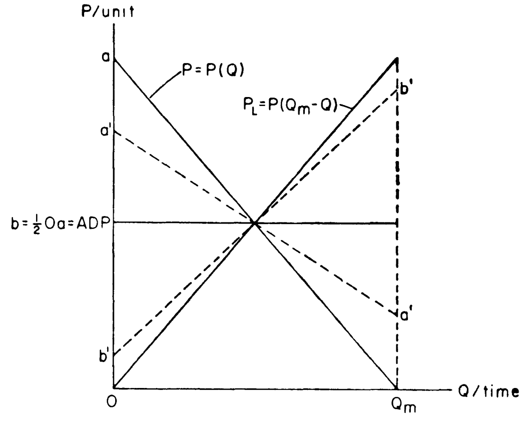
FIGURE 3. VALUATION OF BENEFITS FOR LOW VALUED USERS

Figure 3. The marginal benefit is given by the demand-price of the last unit satisfied and is $P_L = P(Q_m - Q_1)$. Since $P(Q)$ is a decreasing function with respect to Q, $P(Q_m - Q)$ is an increasing function with respect to Q (see Figure 4). The three marginal benefit functions associated with the assumptions mentioned in this section are summarized for the case of a linear demand function in Figure 4.

Variations of the above assumptions can also be incorporated into our analysis. In the case where higher willingness-to-pay units have a greater probability (but not equal to one) of being satisfied before lower willingness-to-pay units, the slope of the expected marginal benefit curve will be somewhere between that of $P(Q)$ and ADP, for example, curve $a'a'$ in Figure 4. Alternatively, if lower willingness-to-pay units have a greater probability of being satisfied than higher willingness-to-pay units, the expected marginal benefit curve will have a slope somewhere between that of $P(Q_m - Q)$ and ADP, for example, curve $b'b'$ in Figure 4.

To determine optimum capacity in these cases, one would find the point where the marginal cost curve intersects the appropriate marginal benefit curve from below, and then determine whether or not total benefits are greater than total costs at this point. If total benefits are greater than total costs then this intersection determines the optimum capacity, but if total benefits are less than total costs then the optimum capacity is zero. However, note that all of the marginal benefit curves, except for the demand price curve, have discontinuities at Q_m. These discontinuities must be treated as vertical segments of the marginal benefit curves so that the marginal cost curve passing through the discontinuity represents an intersection of the marginal benefit function from below. This feature of the marginal benefit functions increases the importance of second-order and total conditions in determining the optimum capacity, and gives rise to many situations where the optimum capacity is either zero or Q_m. This is illustrated in Figure 5.

Figure 5 represents a case where the lowest willingness-to-pay units are satisfied first, so the marginal benefit curve is given by the curve labelled P_L. Marginal cost is constant



$P = P(Q)$ = highest willingness-to-pay units satisfied first
ADP = each unit has an equal probability of being satisfied
$P_L = P(Q_m - Q)$ = lowest willingness-to-pay units satisfied first

FIGURE 4. MARGINAL BENEFIT FUNCTIONS FOR DIFFERENT RATIONING ASSUMPTIONS

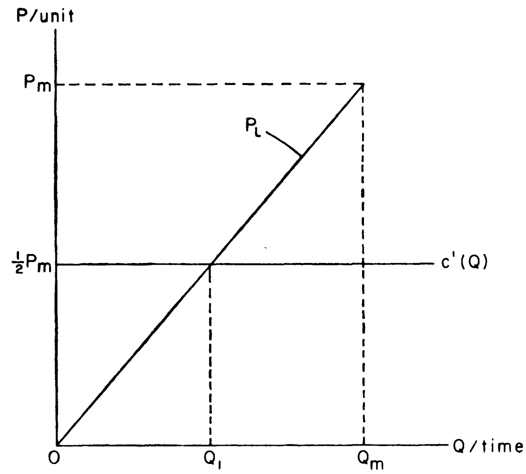


FIGURE 5. CAPACITY DETERMINATION WITH LOWEST WILLINGNESS-TO-PAY UNITS SATISFIED FIRST AND CONSTANT MARGINAL COST

at level $1/2\ P_m$ where P_m is the highest willingness-to-pay for any demanded consumption unit. The marginal cost curve intersects the marginal benefit curve at points associated with capacities Q_1 and Q_m. But capacity Q_1 minimizes net benefits because the marginal cost curve intersects the marginal benefit curve from above. Therefore, the correct intersection is the one associated with capacity Q_m since here the marginal cost curve intersects the marginal benefit curve from below. However, the area under the marginal cost curve from 0 to Q_m is just equal, by construction, to the area under the marginal benefit curve from 0 to Q_m, so at capacity Q_m net benefits are zero. This means we should just be indifferent between a capacity of zero or a capacity of Q_m. On the other hand, for any marginal cost curve that is constant at a level that is less than $1/2\ P_m$, the area under the marginal cost curve will be less than that enclosed by the marginal benefit curve, so net benefits are greater than zero, and the optimum capacity is Q_m. Similarly, for any marginal cost curve that is constant at a level that is higher than $1/2\ P_m$ the optimum capacity is zero.

V. Conclusions

Theoretical as well as practical developments in benefit-cost analysis have assumed that marginal cost pricing will be used in conjunction with standard evaluative and investment criteria. However, political and institutional realities often tend to support policies in which public products are priced below marginal cost. In this paper, we have developed evaluative and investment criteria appropriate for the cases in which public products are underpriced (for example, zero priced). Utilization of the new criteria that assume underpricing leads to an increase in efficiency in these situations. Our results are based upon the assumption that consumers will be drawn from a random sample of all consumers who would like to obtain the output at the set price. Even though this assumption is probably the most reasonable and was used in our calculations, others could also have been made. Several alternatives were reviewed to illustrate how they might be incorporated into our analysis. Although we have ignored possible equity benefits that might be obtained by deviating from marginal cost pricing, our analysis does establish the proper measure of efficiency benefits against which equity benefits can be compared in cases of zero or underpricing.

Our analysis suggests that benefit-cost analysis cannot be conducted independently of the pricing policy chosen to allocate an undertaking's capacity. Either prices must be set equal to marginal cost and the standard evaluative criteria used, or if prices are set below marginal cost, then alternate evaluative criteria must be used.

REFERENCES

G. M. Brown, Jr. and M. G. Johnson, "Welfare-Maximizing Price and Output with Stochastic Demand: Reply," *Amer. Econ. Rev.*, Mar. 1973, *63*, 230–31.

C. D. Foster and M. E. Beesley, "Estimating the Social Benefit of Constructing an Underground Railway in London," *J. Royal Statist. Soc.*, Part 1, 1963, *126*, 46–78.

S. H. Hanke, "The Political Economy of Water Resources Development," *Transactions of the Thirty-Eighth North American Wildlife and Natural Resources Conference*, 1973, *38*, 377–89.

J. V. Krutilla, "Efficiency Goals, Market Failure, and the Substitution of Public for Private Action," *The Analysis and Evaluation of Public Expenditures: The PPB System*, Vol. 1, Washington 1969, 277–89.

R. McKean, *Efficiency in Government Through Systems Analysis*, New York 1958.

W. H. Oakland, "Congestion, Public Goods and Welfare," *J. Publ. Econ.*, Nov. 1972, *1*, 339–57.

J. J. Seneca, "The Welfare Effects of Zero Pricing of Public Goods," *Publ. Choice*, Spring 1970, *8*, 101–10.

W. S. Vickrey, *Microstatics*, New York 1964.

M. L. Visscher, "Welfare-Maximizing Price and Output with Stochastic Demand: Comment," *Amer. Econ. Rev.*, Mar. 1973, *63*, 224–29.

A. A. Walters, Discussion of Foster and Beesley, "Estimating the Social Benefit of Constructing and Underground Railway in London," *J. Royal Statist. Soc.*, Part 1, 1963, *126,* 81–83.

C. B. Winsten, Discussion of Foster and Beesley, "Estimating the Social Benefit of Constructing an Underground Railway in London," *J. Royal Statist. Soc.*, Part 1, 1963, *126,* 79–81.

Water Resources Council, "Principles and Standards for Planning Water and Related Land Resources," in *Federal Register,* Sept. 10, 1973, *38,* 24778–869.