Tilburg University

Master Thesis

Beyond Aspirations: Effect of Entrance Exam Results Timing on High School Choice

Theoretical Monte Carlo Simulation of the Deferred Acceptance System

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Abstract

This thesis uses a Monte Carlo simulation of the Deferred Acceptance (DA) school choice mechanism to explore how the timing of entrance exam result disclosure can affect high school choice. Inspired by Czechia's DA mechanism implementation, the study simulates scenarios where students know their achievement levels when submitting applications versus a noisy scenario where they make applications with mistaken beliefs about their achievement. Results show that early disclosure could improve student welfare by aligning choices more closely with true preferences, especially for students from poor backgrounds. However, the outcome heavily depends on the underlying design of the student's utility. Further calibration of the simulation could greatly enhance its evaluation and forecasting capabilities.

Table of content

1. Introduction

Every year, more than a hundred thousand students in Czechia decide which high school to apply to. Their decision is not made in a vacuum, and it will not only influence their own lives. The school choice decision has broader implications, affecting the future economy, social inequality, the quality of the schools, and many other factors. However, a student's decision is not the only factor influencing the outcome. The mechanisms that determine who gets into which school and on what basis also play an important role. Economists have long studied these matching mechanisms and proposed various allocation systems to achieve efficient and equitable outcomes.

In the Czech Republic, school choice is particularly relevant due to the significant inequalities within its educational system. These inequalities often manifest through the reproduction of socio-economic disparities, where students from lower socio-economic backgrounds face barriers that limit their educational opportunities. The impact of these disparities depends largely on the objectives set by policymakers and society. The recent implementation of a new school allocation system in Czechia, based on the Deferred Acceptance (DA) mechanism developed by Gale and Shapley (1962), marks a significant shift in how students are matched to schools. This thesis seeks to explore the effects of this system, particularly focusing on the timing of information disclosure about entrance exam results and how this timing influences the outcomes. Currently, in Czechia, the results are disclosed only after the applications are submitted, meaning that students must guess or approximate their achievement levels, which are the primary determinant of school admission chances.

The central research question guiding this thesis is: **How does the timing of centralized exam result disclosure, before versus after school application submission, affect welfare outcomes?** Specifically, this question examines the role of information, particularly regarding centralized exam results, in the school selection process. The thesis posits that knowing exam results can significantly impact student preferences by informing their aspirations, ambitions, and expectations about school difficulty, potentially altering their perceived fit with different school environments. Currently, students in Czechia have limited means to gauge their standing relative to their peers, relying mainly on classroom exams that offer a narrow and often biased view, comparing them only with their closest peers.

Additionally, the thesis will explore several related questions:

- What is the effect of selecting schools before knowing exam results on different sub-groups of society, specifically on low SES students?
- How sensitive are the outcomes to changes in key parameters, such as the number of available school spots?
- Is the potential welfare improvement significant enough to justify changes in the implementation?

These questions will be addressed through a Monte Carlo simulation of the DA mechanism, allowing for exploring various scenarios and policy implications.

Methodology Overview

The methodology employed in this thesis allows for the testing of multiple changes to the school admission process, including the timing of exam result disclosure, the number of schools students can list in their preferences, and additional admission requirements. By simulating different scenarios, this thesis aims to provide insights into how these changes might affect student outcomes, particularly regarding welfare and equity.

The key innovation in the methodology stems from two information scenarios. The same student will first face a utility optimization problem informed by their true achievement results (simulating the exam results disclosure before school choice). After these outcomes are revealed, students will face a second utility optimization problem, where the only difference will be in their perception of achievement (noisy scenario). All other factors will remain the same; students will make new school choices, which will be compared to the previous ones. The details are introduced in the following chapters, but the primary welfare measure comes from assessing the utility value of the noisy scenario outcomes in the context of the perfectly informed scenario, assuming that students would prefer deciding based on their preferences informed by reality rather than by noise.

This introduction sets the stage for a detailed exploration of how information timing and school choice mechanisms influence educational outcomes. The following chapters will delve into the policy background, review the relevant literature, and present the methodology and findings of the Monte Carlo simulation. Through this analysis, the thesis seeks to contribute to the ongoing debate on school choice and educational equity in Czechia and beyond.

2. Policy and empirical background

The initial motivation for this thesis stems from the recent change in the Czech high school admission process. In 2024, the country adopted the deferred acceptance (DA) system, one of numerous matching systems economists have extensively studied. This thesis continues in that vein, focusing on the specific implementation in the Czech context. Concretely, it concentrates on students' information about their achievements and what different information scenarios change in the final allocation. Understanding the effect is desirable within Czech education's realities and policy goals.

This chapter aims to introduce relevant data and policy objectives and the concrete implementation of the matching algorithm. The next chapter broadens the scope beyond the Czech context, exploring the DA mechanism and how economists have studied it.

2.1. Inequalities in Czech education

The Czech educational system faces significant challenges, including relatively high educational inequalities and early tracking (Straková et al., 2017). These two issues are closely connected. Straková draws on Boudon's (1974) distinction between the primary and secondary effects that contribute to class differences in educational attainment.

The **primary effect arises from the correlation between students' academic performance and their socioeconomic background**, a relationship well-documented in Czechia by empirical studies. In the latest installment of the OECD PISA assessment (2022a), an average of 15% of the differences in student performance across OECD countries can be attributed to differences in socioeconomic status. The Czech Republic and seven other OECD countries exhibit a proportion higher than 20 %. Czech School Inspectorate calculated that the correlation between educational attainment (measured by knowledge test) of 9th graders with their SES background is 0.39 and 0.38 for Czech language and Mathematics, respectively (ČŠI, 2022b)

The secondary effect, on the other hand, manifests in the choices students (or their parents) make when selecting schools. Even if a student from a disadvantaged background has similar academic potential to their peers, they might opt for a less academically ambitious school. In Czechia, a critical educational decision for students is the type of school they attend. The options, in essence, include a general secondary school (gymnasium), which culminates in the state final exam (maturita) and focuses on general education and university preparation, a secondary (vocational) education program that also concludes with a state final exam and a secondary vocational education program without the state final exam. Between districts, the rates in student gymnasium attainment range from 10.6 % to 49.4 %, highlighting the differences between different regions and the local socioeconomic situation (DataPAQ, 2023). Research indicates that family background significantly influences both the aspirations to pursue state exam-level education and the likelihood of being admitted to such programs, even after accounting for students' academic performance (Straková et al., 2019). This means that students from lower socioeconomic backgrounds are disadvantaged when accessing state exam-level education, even if their academic achievements are comparable to those of their peers from more affluent families.

School choice systems directly engage with the decisions made by students and their families regarding which schools to apply to. As a result, the design of these systems has the potential to significantly influence educational outcomes and, consequently, the broader issue of inequality in school attainment.

2.2. Old and New School Allocation Mechanism in Czechia

In the 2023-2024 school year, the Czech Republic implemented a new system for allocating high school applicants to high schools. Previously, students could only select two schools to apply to. Following this, they took a centralized exam (known as the Jednotná přijímací zkouška or JPZ), which served as the primary criterion for most high schools in their admissions decisions. Schools admitted the top-performing applicants until their capacity was reached, leaving everyone else rejected.

For some students, this meant they were accepted to both schools, allowing them to choose which offer to accept. Others received rejections from both schools but could still be admitted later. This was because higher-performing students would take only one of the two offers, thus freeing up places for those rejected initially. After the first round of applications and student responses, schools offered their remaining vacancies to other students on the list.

This system presented some challenges. It allowed for significant strategizing and was susceptible to information asymmetry through, for example, differences in knowledge about the previous year's application numbers (Protivinsky, 2023). Greger et al. (2023) even speak of “hidden demand” as the applications show (due to application strategies) different demands for types of high schools than what parents and children, in reality, want to study. Additionally, the previous system created a substantial time and workload burden for schools and parents, involving administrative tasks and queuing parents to inquire about remaining open spots. Furthermore, some students might be left without a school placement despite their desire to attend one. Schools may also have had empty spots, even in programs that these unplaced students would have been interested in, simply because they hadn't included those schools among their limited choices.

In the 2024 round of applications, the Czech Republic adopted the Deferred Acceptance (DA) mechanism for student allocation, initially developed more than 60 years ago by Gale and Shapley (1962). In this system, students first rank their school preferences, and according to similar priority lists that schools create for their applicants (the metric often includes a standardized test score), an algorithm automatically assigns students to one of the schools on their list. This system promises improved results, greater efficiency, and higher satisfaction for students and schools, and it is strategy-proof (Protivinsky, 2023). More details on the algorithm are in the literature overview, and specific details will be covered in depth in the thesis.

In the Czech context, students rank three schools on their application. That is shy of the ideal requirements for the algorithm, where students theoretically rank all the schools they want to apply to, but it is an improvement nonetheless. Because there is no rational motivation to obscure the true preferences between the three selected schools, the most wanted school should have the first preference. After preference selection, students applying for schools with state exams take the centralized JPZ test, and based on their scores (in combination with optional school-specific measures) and preferences, the algorithm will assign them to a school (details can be found at *Jednotná Přijímací Zkouška 2024│Jednotná Přijímací Zkouška*, n.d.). The school-specific measures usually include past grades or additional school exams. These typically hold less weight in the final decision than the centralized exam results and can be expected to correlate with it.

In practice, the system also allows for a second round of matching for schools with open spots and students not matched in the first round. However, that is irrelevant to this thesis, as it focuses on the first round. In 2024, almost 94 % students, around 94 000, were matched in the first round (MŠMT, 2024), and 75 % of them then to the school with the highest priority.

3. Literature review: Economic view on school choice and allocation mechanisms

3.1. Determinants of school choice (Demand for schools)

The economic approach generally simplifies the complicated school choice decision to a quantifiable set of utility maximization problems on several school characteristics. The following section answers what characteristics are the most important ones that should not be missing in the simulation.

In a private market, the school choice would be mainly seen through the willingness of families or communities to pay tuition fees for school quality or different characteristics. This straightforward approach would help us define the relative importance of school characteristics that students and their families are willing to pay for. However, most high schools are public, and no direct payment for schools is included. One of the workarounds is to look at preferences revealed through other ways, for example, through voting on school spending and taxation or through residential mobility, as a seminal paper by Tiebout (1956) has shown. In the educational context, Black (1999) calculated that when school choice depends on a place of living (as is often the case of primary education), house prices in the district infer the value parents place on school quality. To be precise, parents were willing to pay 2.5 percent more for a 5 percent increase in test scores.

Let's take a closer look at the importance of school quality. There is further strong empirical evidence that families' residential and school choices are related to school quality metrics that can be observed (Corcoran & Cordes, 2017, p. 74). One of the more easily measurable characteristics of a school is the school's test scores. Corcoran and Cordes name several studies that provide evidence for test scores to be strong predictors of applications for inter-district transfers in the USA (Carlson et al., 2011; Reback, 2008) and within districts (Harris & Larsen, 2015). From experimental evidence, research has shown that parents prefer higher-performing schools when given information about which of those are (J. S. Hastings & Weinstein, 2008). This also hints at another input that enters the school choice. Information about school performance is not always perfect, as it can be distorted towards other characteristics discussed below. And the quality of the information itself changes the perception of the characteristic.

School quality is one of many things that are important for students when choosing high schools. Corcoran and Cordes (2017, p. 74) list major non-academic characteristics and the evidence for their importance in the decision. These include school proximity, socioeconomic characteristics of the school (and other students), racial composition of students, and others.

3.2. Estimating weights for school choice factors

Hastings et al. (2009) attempted to estimate the weights parents assign to different school characteristics using a random utility model. They assumed that each parent or student does not have the exact same preference for individual factors but instead shares a similar distribution of preferences. The researchers collected a variety of information on schools and examined how these factors related to parents' choices. Key factors included students' predicted academic achievement at the school and the match between the school and the student's characteristics. Among non-academic factors, distance, and racial composition were particularly significant. Across all groups, racial composition and whether the school was the same as the previous year's school were given the most weight. The study found that white parents who did not receive lunch subsidies (often used as a proxy for income) placed about 5.3 times more emphasis on school quality over distance compared to other factors. However, the results varied significantly depending on income, race, and other demographic variables. For example, white families receiving lunch subsidies placed 1.6 times less weight on school quality compared to those who did not receive subsidies. This suggests that higher-income parents prioritize test scores more heavily.

Similarly, in an earlier study, Glazerman (1998) found that the interaction between a school's average test score and a student's own test score significantly impacted school choice, with a coefficient of 0.39—almost as influential as the distance factor (where the coefficient was -0.55 for bus travel). Like Hastings, Glazerman also highlighted the importance of racial and socioeconomic composition in school choice decisions while placing relatively less emphasis on school quality itself.

These findings are consistent with more recent research. For instance, Agarwal and Somaini (2020) summarize studies that estimate preferences within school choice mechanisms. They identify two consistent findings: first, that student preferences are correlated with proximity to school and selected measures of school performance, and second, that students from disadvantaged backgrounds tend to prioritize achievement less, partly due to a stronger preference for closer schools.

3.2.1. Implications for the Simulation and the Czech Context

The studies mentioned above offer valuable insights into the key determinants of school choice and the relative importance parents and students might assign to them. These determinants include school quality (which is more significant for higher-income students), distance, racial and other relevant school compositions, and additional preferences, such as whether the school was attended the previous year (in case more schools are in the same building, for example). The alignment between the student's abilities and the school's offerings is also crucial.

Another key finding from these studies is that while the set of important parameters remains reasonably consistent, the weight placed on each can vary significantly across different demographic groups. Both academic and non-academic factors play vital roles in these decisions.

To accurately calibrate these factors for the Czech Republic, data specific to Czech school choices, along with information on students and schools, would be required. The only factor that appears less relevant in the Czech context is racial and ethnic composition due to the country's relatively homogenous population. Although in some regions, the proportion of Roma students may influence school choice, data from 2021 show that only 136 primary schools (or 3.2% of the total) had more than 34% Roma students, a percentage that typically indicates school segregation (PAQ Research, 2022). Therefore, racial issues are not as relevant in broader contexts. The recent increase in the student population from Ukraine could potentially impact this, but there is no evidence yet to suggest such a change.

Additionally, at the secondary level, students in the Czech Republic can only continue at a different school than they attended for primary education, which likely reduces the importance of certain other characteristics.

3.3. Role of allocation mechanism in the school choice

The chapter on policy background introduced the importance of self-selection of high schools that propagates educational inequalities. This was further backed in the previous parts by evidence for the presence of income-related importance of school quality in the factors that determine school choice. This section puts these findings in the context of the mechanism that allocates students to high schools.

Economists traditionally study market systems, where buyers and sellers (or supply and demand) match on a perfectly competitive market through prices. However, not all exchanges or matches work this way or suit this matching type. Setting a prize for organ donation matching or seats at public schools is undesirable and would contradict why publicly accessible services exist. For example, public schools ought to offer education to everyone. Starting in the 1960s, scientists like Gale and Shapley (1962) or Alvin E Roth (1982, 1989) and others began publishing about matching algorithms in these applications and many more, for example, in housing (Abdulkadiroğlu & Sönmez, 1999). They also became interested in the outcomes of these different matching systems. Over the years, many different matching algorithms have been developed specifically for school choice (further discussed in the literature), and their usage in practice is widespread.

While the theoretical foundations for matching algorithms have existed for a long time, current researchers focus mainly on these areas:

- **Comparative Analysis:** Comparing different mechanisms based on objective criteria like Pareto efficiency (no one can be made worse off without making someone better off) and stability (how likely to change is the resulting allocation).
- **Empirical Outcomes:** Examining the real-world effects of these algorithms.
- **Influencing Factors:** Understanding how information asymmetry and other variables affect the system's operation.

This thesis focuses on the following aspects relevant to the Czech system:

- **The Effect of Exam Result Knowledge on School Priorities:** How access to exam scores influences student choices and school preferences.
- **Societal Outcomes:** The broader social implications of the new allocation system, including potential benefits for low socioeconomic status (SES) groups.

Both aspects are relevant not only for the Czech system but also for a deeper understanding of the mechanisms themselves. However, the societal outcomes hold particular significance for the Czech context due to the observed correlation between low socioeconomic backgrounds (SES) and students' lower educational aspirations, as explained in the second chapter. Furthermore, low SES students and their parents do not employ considerable effort or do not have enough information to select the optimal strategy (Protivinsky, 2023), theoretically resulting in worse outcomes than for the same students with higher SES.

The DA, in its ideal condition, has the potential to mitigate this effect by eliminating strategizing altogether and offering students the possibility to apply to a good school, even though they don't believe they will get in, without compromising their other chances, as they can still apply to other schools. The introduction of the DA mechanism to the Czech environment in 2024 goes in that direction. However, due to the limitation of three school preferences, it still keeps the strategizing nature of the whole process.

Another way of mitigation is by providing information about student cohort comparisons through the results of the centralized admission exam. The results could contradict their previous low aspirations. However, to have any impact, the exam results must be released before selecting school preferences. This thesis revolves around researching the effects of such a change under the DA mechanism. The DA mechanism works under any student preferences, so the disclosure of the exam information will not impact the algorithm directly. However, it offers an opportunity to observe how big a difference the release of information makes in the outcomes if the DA mechanism determines it.

3.4. The Deferred Acceptance allocation mechanism

The DA mechanism (Gale & Shapley, 1962) works through a series of iterative steps where students apply to schools based on their preferences, and schools conditionally accept or reject these applications based on their priorities until a stable match is found. The process is following (Abdulkadiroğlu & Sönmez, 2003):

In the first step, each student proposes to the first preference on their school priorities list. Schools conditionally accept students according to their priorities until the capacity is available. Everyone else is rejected. In any other step, the remaining unmatched students propose to the highest school on their preference list, to which they have not yet proposed and await the outcome. Every school again considers all its applicants and accepts those according to their priorities. A previously accepted student may be rejected in a subsequent iteration step. This step is repeated until the mechanism converges and students make no further proposals.

The mechanism satisfies, under ideal conditions key objectives (Abdulkadiroğlu et al., 2009):

- **Optimal Stability:** The DA mechanism ensures stable matching, where no student who prefers another school could have been accepted there if not for other lower-priority students. This prevents justified envy, where a student might envy another who was admitted to a preferred school despite lower qualifications.
- **Strategy-Proofness:** The DA mechanism does not encourage strategic behavior. Students can rank schools truthfully based on their real preferences without worrying about being penalized, as long as they can list enough schools.
- **Efficiency and Fairness:** The DA mechanism aims for Pareto efficiency, improving one student's match without harming another. While there can be a trade-off between stability and efficiency, the goal is to achieve the most efficient match that still remains stable. In the DA allocation, no student in ideal conditions wants to switch their place with another after the allocation.

3.5. Comparing allocation mechanisms

Several allocation mechanisms are utilized in school settings. Among the most influential and widely used are apart from the already described Deferred Acceptance (DA), also known as Gale-Shapley (Gale & Shapley, 1962), the Boston mechanism (Abdulkadiroğlu & Sönmez, 2003), and the Top Trading Cycles mechanisms (Shapley & Scarf, 1974). Many researchers focus on comparing the outcomes of these mechanisms during various switches or compared to previous "primitive" systems. For example, De Haan et al. (2023) compared the outcomes of switching from the Boston mechanism to DA in Amsterdam. They concluded that the DA led to higher overall welfare, with low-income students benefiting the most. Their measurement of welfare is particularly relevant to this thesis. They employed three metrics:

- **Rank of Assigned School:** This refers to the position of the assigned school on the student's original preference list. Such a measure indicates the percentage of students placed on their first, second, or other preference.
- **Winners and Losers:** This metric identifies the proportion of students who would be better off (winners) or worse off (losers) under the DA system compared to the Boston mechanism.
- **Mean Welfare:** This represents the average level of welfare change between systems across all students. It essentially calculates the utilities of all students before the change and after, or in practice, how much some students gained from the change minus how much others lose from the change.

The thesis does not compare different mechanisms but outcomes in two different preference settings of the students (with and without the exam information), so the application of similar measures is also possible.

3.6. Relevant empirical and simulation studies on allocation mechanisms and school choice

A study by Kapor et al. (2020) is highly relevant to this thesis by supporting the two achievement information scenarios. They investigated the influence of information asymmetry on student/parent strategies in New Haven, Connecticut. They surveyed 417 households about their school preferences and linked this data to administrative allocation records. They found that families strategized even when the system supposedly incentivized truthful preference revelation. Furthermore, they estimated a model suggesting that families base their decisions on subjective beliefs about admission chances rather than objective information. To be precise, they found that beliefs about the admission chances (of a hypothetical application portfolio) differ by 37 % from the mean of the rational expectations, which the researchers calculated. Their outcome can be compared to the outcome of the simulation.

In the Czech context, Greger et al. (2023) estimated the difference between income groups' approaches to the admission process. Apart from the different time allocated for preparing the students, parents are apparently willing to spend considerable resources. In Prague, parents have, according to questionnaire data, on average spent more than 14 thousand Czech Crowns for preparation and the top 10 % even more than double that amount. For context, the median wage is close to 40 thousand per month. These differences will help guide the setting of the student's utility function and the noise distribution in the simulation design.

Another relevant study (Díaz et al., 2021) utilizes simulations to examine the impact of information on school choice. Their research focuses on the newly introduced "traffic light" system, where schools are assigned green, orange, or red colors according to their performance. This system aimed to inform parents about school quality while transitioning from a neighborhood-based allocation system to a choice-based one. The researchers used an agent-based simulation to explore information asymmetries about school quality between different income levels.

Another relevant simulation is by Munich (2010), which simulates the Czech Republic allocation system recently replaced by the DA mechanism. They explicitly use the perceived level of qualification with a noise error as a determinant in the student's utility function. This thesis further uses their specification of a utility function and some underlying assumptions.

4. Methodology

The primary data for this thesis is simulated, a common approach in research within the field and topic (Díaz et al., 2021; Pathak & Shi, 2013; Utomo et al., 2009). While simulated data allows researchers to address a wide range of questions that might otherwise be too complex, costly, or unethical to explore, it also comes with certain limitations. The validity of simulation results heavily depends on how accurately the underlying assumptions reflect reality. The following sections detail the decisions made in the simulation process and their rationale. Most of the decisions follow the ideas of the literature mentioned above (Agarwal & Somaini, 2020; Glazerman, 1998; J. Hastings, 2009; Münich et al., 2010).

The current model does not incorporate any measure of strategizing and offers students the opportunity to submit enough applications to satisfy the DA dominant strategy of revealing true preferences. In reality, the Czech Republic currently has a system where strategizing plays a crucial role, as such, the simulation currently reflects an idealized condition.

4.1. About the Monte Carlo simulation method

Monte Carlo simulation is chosen for its stochastic nature. Its ambition is not to determine the exact outcome of given parameters but to find the most likely average outcome over repeated random draws from underlying probability distributions. The underlying logic can also be found in the literature that attempts to reverse engineer matching data and find the underlying distributions and preferences. Another advantage of Monte Carlo simulation is its ability to model probable outcomes of different policy decisions. In the Czech context, this could include simulating the effects of increasing or decreasing the number of possible school preferences that can be included in an application.

Monte Carlo simulation typically involves the following four steps (Brandimarte, 2014):

- **Define Input Variables:** Establish the variables used in the simulation.
- **Sample from Them:** Draw random samples from the specified probability distributions.
- **Calculation:** Perform the necessary calculations based on the sampled data.

- **Summarize Results of Interest:** Aggregate the outcomes to identify trends and likely results.

As previously mentioned, selecting appropriate probability distributions is crucial. Therefore, the simulation uses distributions derived from real-world Czech data (such as income distribution), and significant effort is made to justify choices that are based on assumptions or simplifications of reality.

The simulation is programmed in Python, is highly modulable, and accepts changes in many parameters. The complete code, including the data, can be found in the Appendix.

5. Monte Carlo Simulation of the Deferred Acceptance School Allocation

The simulation process consists of six sequential steps:

- **Initial Setup:** First, the simulation establishes the initial parameters, including the number of schools and students, as well as their characteristics.
- **Generating Preferences:** The simulation then calculates each student's preferences for different schools and each school's priorities for admitting students.
- **Initial Matching:** Using the Deferred Acceptance (DA) mechanism, the simulation matches students to available school spots based on these preferences and priorities.
- **Introducing Noise:** Next, the simulation adds variability (noise) to student's perceptions of their academic achievements. This change in perception prompts a recalculation of their school preferences.
- **Re-Matching:** A second round of matching is conducted using the updated preferences influenced by the noise.
- **Outcome Calculation:** Finally, the simulation calculates and records the outcomes of this process.

These six steps are repeated multiple times in the Monte Carlo simulation, ensuring that the results account for randomness and variability. The repeated simulations, using consistent initial settings, are compared to assess the stability and reliability of the outcomes. To ensure the results are robust, an analysis of outcome convergence is conducted, determining the optimal number of simulation iterations. Detailed information on the iteration count and initial settings is provided in the following sections.

5.1. Step one: Synthetic data generation

The simulation begins by creating a specified number of students (i) and schools (s), each with their initial characteristics. The literature research has identified that in the Czech context, the student's utility function has probably the most considerable emphasis on the following academic and non-academic factors:

- School quality (higher for higher-income students)
- Socioeconomic composition of the school
- Ability matching of a student with the school
- Distance
- Various other individual school characteristics

5.1.1. Student characteristics

To approximate these factors, each student (i) is assigned the following characteristics:

- **Location**

  Represented as random x and y locations on a two-dimensional grid of a size specified in the initial settings. This simplifies reality, where students often concentrate in urban locations, which also correlates with income. The random location thus represents the most simplified version of reality. In the future, the simulation can generate locations with one central town or even mimic the real data more closely.

- **Income**

  Generated using a log-normal distribution, which is chosen for its ability to represent the skewed nature of income distribution in real populations. The distribution parameters are imputed from accurate Czech data.

  The Czech School Inspectorate (ČŠI, 2022b) uses for SES the construction measurement method (Vyas & Kumaranayake, 2006), where income plays a vital role, so the income could also be interpreted as a broad SES index.

- **Achievement**

  The CERMAT (institution responsible for the whole application procedure, including the exams) data show for the 2023 exam round that the JPZ results can be assumed as approximately normally distributed within a 0-100 scale with a mean ranging from 38,5 to 60,3 score points out of a 100 and standard deviation ranging from 18,7 to 22,5, depending on the test subject and type of a high school applied by the students (CERMAT, 2023). The simulation assumes a default mean of 50 and a standard deviation of 20, around the middle of the real-world range.

Wage distribution calculation

Wage distribution is typically modeled using a log-normal distribution characterized by a long tail at the high-income end. This distribution also applies in the Czech context (Malá, 2013; Matějka & Duspivová, 2013). However, some research suggests using the Dagum distribution for more precise estimations, particularly in contexts requiring greater accuracy. The Dagum distribution, being more complex, requires three parameters instead of the two required for the log-normal distribution. Given the minor difference in utility outcomes for students, adopting the Dagum distribution may be considered a potential future enhancement to the simulation.

The lognormal distribution requires two parameters, μ (the logarithm of location) and σ (the logarithm of scale). These can be approximated in the following way with their relation to median and mean:

$$\text{Median: } m = e^{\mu}$$

$$\text{Mean: } E[X] = e^{\mu+\sigma^2/2}$$

According to the Czech Statistical Office, the median income for the last quarter of 2023 was 39,685 Kč, and the mean income was 46,013 Kč (ČSU, 2024, p. 7).

By substituting the real data (in thousands) into the equations, we get:

Calculate μ:

$$\mu = ln(median) = ln(39.685) \approx 3.681$$

Calculate σ:

$$E\,[X] \;=\; e^{\mu+\sigma^2/2}$$

$$ln(E\,[X]) = \mu + \sigma^2/2$$

$$\sigma^2 \;=\; 2 \;\times\; ln\,(mean) \;-\; \mu$$

$$\sigma^2 \;=\; 2 \;\times\; ln\,(46.013) \;-\; 3.681 \approx 0.298$$

As the distribution has a long tail towards the high income, which would pose problems in the simulation, the generating process stops differentiating between different income levels after four standard deviations from the mean, which includes roughly 0.02 % of all students. With 10,000 students entering the simulation, 200 students are treated as having the maximum income available. This represents the notion that for high-income outliers, the difference between income no longer meaningfully influences other characteristics, and all are treated the same.

Correlation between achievement and income

Student achievement is generated with a correlation to income to simulate the Czech scenario. This correlation can be adjusted in the simulation settings but is set to 0.39 by default. This figure is derived from an inspection report that calculates the correlation between socioeconomic status and results in Czech language and mathematics tests, which are 0.39 and 0.38, respectively (ČŠI, 2022b). The international PISA assessment (OECD 2018) finds a similar correlation of 0,4.

To establish this correlation between income and achievement:

- The log-normally distributed income is first log-transformed to approximate a normal distribution, making it compatible for introducing a correlation with achievement.
- The variables are then correlated using the following formula:

$$X_1 \;=\; Z_1 \;=\; Income$$

$$X_2 \;=\; \rho Z_1 + \sqrt{1-\rho^2}\,Z_2 \;=\; Achievement$$

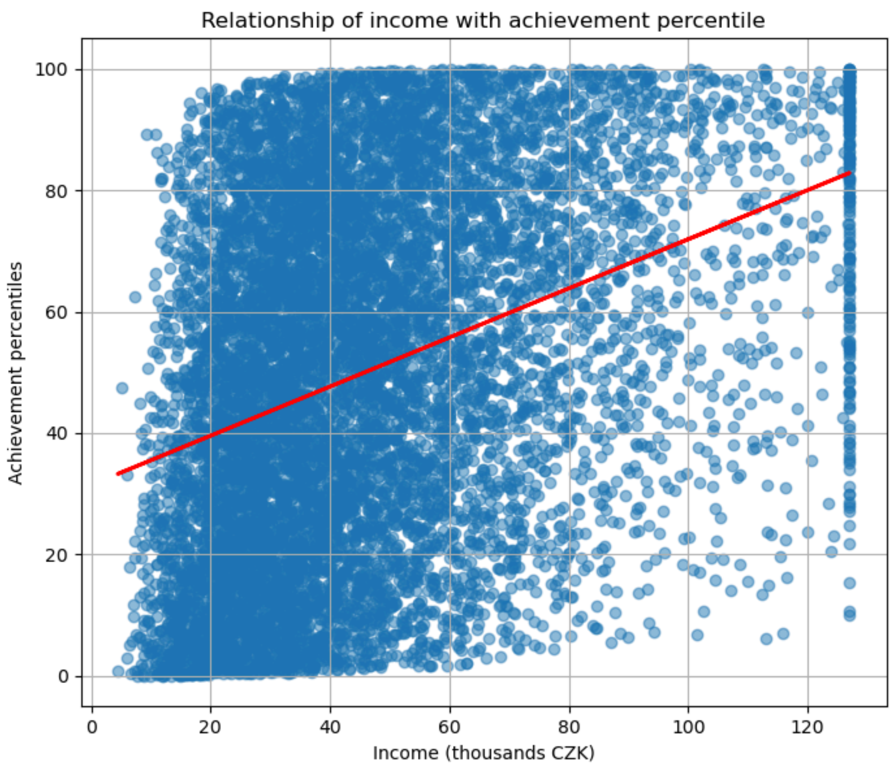
Formula adapted from Brandimarte (2014)

Term explanation:

X_1 previously already generated income transformed to approximately normal distribution

Z_2 previously generated normally distributed achievement

X_2 achievement correlated with income but still with its own distribution



Plot 1.
Example of the generated income and achievement for 10,000 students. The correlation is calculated in the simulation on income data, which are scaled with the same distribution to 0-1 values, explained in detail in the scaling section.

School characteristics

Each school (s) in the simulation is characterized by the following attributes:

- **Location**

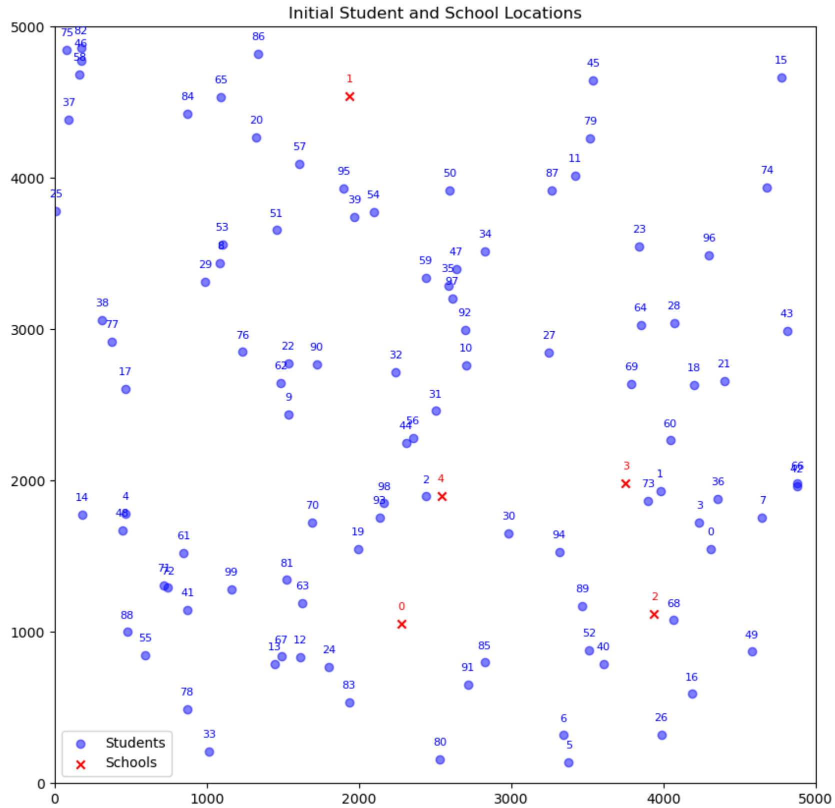
  Schools are assigned locations using the same x and y coordinate system as the students, ensuring that distance calculations are consistent across the simulation.

- **Quality**

  Glazerman (1998) simplified school quality to a measurement of test scores. In this instance, the quality could be assumed to be approximately normally distributed on a 0-100-point scale. Real-world school quality distributions are probably more complex, as is their measurement. As described in the chapter on the Czech context, Czech high school education could be separated into three main school types, representing different quality classes. But Strakova (2010) reminds that the added value of different schools might be similar, and the educational attainment differences can originate mainly from socioeconomic differences. For now, the reality is simplified, but this characteristic offers a possibility for improvements in future versions of the simulation.

- **Capacity**

  School capacity is modeled to reflect actual Czech data, where high schools typically open one, two, or three classes, each consisting of around 25 students (ČŠI, 2024b). School capacity is then determined by drawing from a uniform distribution with a minimum of 25 and a maximum of 75 spots. In reality, schools can have much smaller or larger capacities, depending on the number of subject-specific classes they offer. These are currently combined and simplified as being separate schools altogether. Including real-world capacity data is possible and could be a potential improvement for future versions of the simulation.



Plot 2. Example of an initial location of 100 students and five schools

5.2. Step Two: Calculating Preferences and Priorities

After generating the set of schools $S = \{1, 2, \dots, s_n\}$ and students $I = \{1, 2, \dots, i_n\}$, the simulation proceeds by calculating school preferences for each student (i.e., which schools to apply to) and the priorities of schools (i.e., which students to accept).

5.3. Students preferences

The process for determining student preferences involves selecting schools based on a utility function that reflects the desirability of each school for a given student. The utility function is applied to rank the schools.

For the first school on a student's list:

$$R_{i1} = max_{s \in S, U_{is} > 0} \ U_{is}$$

Subsequent schools in the preference ranking follow this rule:

$$R_{in} = max_{s \in S \setminus \{R_{i1}, R_{i2}, \dots, R_{i(n-1)}\}, U_{is} > 0} \ U_{is}, \ n > 1$$

Where:
n is the preference rank

The approach is adopted from Abdulkadiroglu et al. (2020)

Students follow for each school the following utility function:

$$U_{(i,s)} = z + q * School\ Quality_s - d * Distance_{is} + c * Income_i * School\ Quality_s - a * Aspiration_{is} + \epsilon_{is}$$

Where:

- z represents the baseline utility, a student's inherent preference for attending any school, independent of specific characteristics like quality, distance, or personal aspirations. The purpose of this variable is the ability of the simulation to influence the number of schools students apply to, as it adds the same constant to every student's utility and thus can bring more schools above utility 0, the minimum for listing it in preferences. It is set at 0.05 as default.

- $q * School\ Quality$ reflects the contribution of school quality to the student's utility. q is the weight assigned to school quality, indicating how much a student values the quality of the school. Higher quality schools contribute positively to the utility.

- $-\ d * Distance$ captures the negative impact of Euclidean distance on utility. d is the weight assigned to the distance between the student's home and the school. A larger distance reduces the utility, reflecting the inconvenience or cost of traveling farther.

- $c * Income\ \ * School\,Quality$ represents the interaction between income and school quality. Here, c is the weight that captures how a student's income level affects the importance of school quality in their utility. Literature review shows that higher-income students can be assumed to place more emphasis on school quality.

- $a *\ Aspiration$ denotes the aspirational component of the utility function. a is the weight assigned to the student's aspiration. The aspiration is calculated as the square of the distance between school quality and student achievement. Thus, the component reflects the aspirational mismatch and penalizes bigger mismatches more. In noisy condition, the noisy achievement value enters the calculation, and this component is thus the only one that changes between the two scenarios.

  $$Aspiration_{is} = (School\,Quality_{s} - Achievement_{i})^2$$

  The idea behind this calculation arises from the literature (Agarwal & Somaini, 2020), though this particular method does not.

- ϵ_{is} represents a random noise in the utility function. This term captures unobserved factors or random fluctuations in the student's preferences that are not accounted for by the other variables. It introduces variability in the utility, reflecting the idea that not all student choices are perfectly predictable. In the real world, students might have, for example, a sibling already attending a given school and thus prefer it, independent of other simulated characteristics. The size of the noise is assumed to be normally distributed and, as default, has a standard deviation of 0.01.

The sum of the weights $q + d + c + a = 1$ ensures that the utility components are balanced.

Normalization of Variables:

To ensure that the various factors influencing student preferences are comparable, variables are scaled or normalized to fit within a 0-1 range. This step is crucial for maintaining consistency in how these factors contribute to the overall utility that each student assigns to potential school choices.

- These variables are scaled by dividing each by the highest observed value among all students, which achieves a consistent scale from 0 to 1. This means that these utility components are treated consistently across all students; the same distance, school quality, or income value will contribute equally to the utility function for every student. For instance, a student located in the corner of the grid might have a school with a distance of 1 (representing the maximum distance), while a student located in the center of the grid might have a maximum distance to a school of 0.5.
- Aspiration is normalized relative to each student's maximum aspiration difference. This approach preserves each student's distribution of aspirations for different schools, reflecting the idea that aspiration is more challenging to quantify universally, and students see them in relation to their other choices. By normalizing aspirations this way, the model ensures that each student's aspirations are relative to their own potential, making the utility calculations more personalized.

School preferences

Compared to students, ordering students by preference is more straightforward for schools. In the default settings of the simulation, all schools rank students based solely on their JPZ (entrance exam) achievements. In reality, schools often consider additional factors, such as previous school grades, participation in academic competitions (e.g., knowledge olympiads), and sometimes scores from school-specific entrance exams. These factors are assumed to be highly correlated with the JPZ achievements already generated in the simulation, which is why the simulation currently does not include them separately.

5.4. Step three: Matching students with schools

At the end of the previous step, all schools have established their priority order for students based on JPZ achievement scores, and each student has generated a preference list of schools, ordered by the utility each school provides to them. In the simulation, students submit applications to all schools they want to attend (their utility is positive). This way, the algorithm ensures no room for strategic play. These preferences are ranked from the highest to the lowest utility. The system then applies the Deferred Acceptance (DA) mechanism to sort students into available spots at the schools they prefer.

The DA mechanism operates iteratively. Initially, each student applies to their most preferred school. Schools provisionally accept students based on their priority ranking (up to their capacity) and reject the lowest-ranked students if necessary. Rejected or unmatched students then apply to their next preferred school, and the process repeats until no student wishes to apply to another school or all spots are filled. Details of the algorithm are explained in the Literature chapter.

Once the algorithm completes its iterations, each student is either assigned to a school or left without a placement if certain conditions occur. A student might end up without a match if there are not enough spots at the schools, or if the student without a matched school did not even submit their applications for the schools with empty spots, as their utility for that given school is negative.

5.5. Step four: Introducing noise to student's achievement

Until this point, the simulation has assumed that students are perfectly informed about their academic achievement. To answer the research question, a new set of utilities is constructed for another round of matching, this time introducing uncertainty - or "noise" - into students' perceptions of their own achievement, similar to Münich et al. (2010).

$$Achievement\ (Noisy)_i\ =\ Achievement\ (True)_i\ +\ Noise\ (mean{:}\ 0,\ sd)_i$$

In this new round, the student's utility for each school remains unchanged in all aspects except for the aspiration component, which is directly influenced by their perceived achievement. The idea is that other factors in the utility function, such as distance or base utility from attending a school, should not be influenced by perceived achievement.

To be able to answer research questions about the role of income (or SES) in achievement noise, the simulation adds noise to each student's perceived achievement as follows:

Types of Noise: Each student can experience either a small or large noise in their perceived achievement.

- **Large Noise:** Has a default standard deviation set at 20, representing one standard deviation of the test score results, a significant misinformation about achievement.
- **Small Noise:** Has a default standard deviation set at 5, representing situations where students might have a more accurate estimate of their achievement (e.g., through practice exams or other forms of assessment).

Probability of Noise: The probability that a student receives either large or small noise is determined by their income. The formula is:

$$P(large\, noise) = MAXP - \alpha * Student\, Income$$

Where:

- MAXP is the maximum probability of large noise, set by default at 0.9.
- α is the income scaling factor.
- Student Income is a normalized value between 0 and 1.

This way of simulating offers control over numerous scenarios. If the MAXP is set to 1 and α to 0, each student will experience the same large noise. If MAXP < 0 and α > 0, each student experiences at least a small noise but can also experience a large noise. But this chance decreases with income. By varying α, the simulation can control the strength of the relation, which is unknown in reality. Furthermore, the minimum probability can also be clipped, for example, to 0.1, which reflects that the highest income student can have a chance to have large noise applied.

5.5.1. Context and Justification

The introduction of noise follows the research design and research question. In a baseline scenario for comparison, everyone's noise is kept constant. The underlying assumption is that students do not know their exact achievement levels before submitting their applications, as the JPZ results are available only after applications are submitted.

In reality, students might gather information from their school marks. But those compare them to their classmates, not to the broader cohort. Additionally, students might participate in "practice" JPZ exams offered by private companies, which simulate the real exam environment. These practice exams are more accessible to wealthier families, potentially reducing noise in perceived achievement for higher-income students. However, just being from a higher income family does not guarantee students a visit to practice exams, so they also have a chance of receiving a large noise during the simulation.

Commuting assumption

It is reasonable to assume that most students attend schools within their region. Data shows that, on average, 9.4% of primary and secondary school students commute to schools outside their home region (DataPAQ, 2021). However, the overall net flow of incoming and outgoing daily commuters typically ranges from a few hundred students in either direction for most regions (ČSU, 2023). Since only a portion of these commuters are high school students, the data likely represents the upper boundary of high school commuting. Compared to the total number of high school-aged children, which constituted 5.7% of the population in 2021 (ČŠI, 2021), the net commuting balance accounts for only a small percentage in most regions. The assumption of negligible out-of-region commuting also holds when examining the commuting flows within smaller regional divisions (ČSU, 2023).

An exception to this trend is the Central Bohemian Region and the capital, Prague. In these areas, simplified calculations suggest that around 30% of high school-aged children from the Central Bohemian Region commute to Prague for school.

In future improvements to the simulation, it would be beneficial to incorporate more detailed commuting data for each district to reflect these patterns more accurately.

5.6. Step five: Second matching algorithm

Once the noisy achievements are generated, the new value of aspiration replaces the old one, and new preferences of students are calculated. All other utility values stay the same between the two matchings. The new utility enters in the same fashion the sorting algorithm, and a new set of school-student matches is calculated.

5.7. Step six: Calculating outcomes

After the matching algorithm is run twice—first with students' preferences informed by their true JPZ results and then with preferences based on noisy (partially informed) achievements—the final outcomes are calculated.

The primary goal is to compare the schools to which students are matched in the noisy scenario with the schools they would have been matched to based on their true preferences. The assumption is that while students are matched to a school based on their noisy preferences, they would ideally prefer to be matched according to their preferences as informed by true achievement levels.

5.7.1. Main Measures: Rank and Utility Distance

The primary measure of interest is the "distance" of either a school rank (the rank number on student’s preferences) or utility between the school matched under fully informed preferences and the school matched under noisy preferences. This distance is calculated by:

- **Finding the Matched School:** Identifying the school to which the student is matched in the noisy scenario.
- **Comparing with True Preferences:** Determining where this matched school is in the student's true preference list.
- **Calculating Rank Distance:** the matched school in the noisy scenario is found in the perfectly informed preferences and compared to the matched school under the perfectly informed condition.
- **Calculating Utility Distance:** the process is the same, but this time, the matched school’s utility under perfectly informed preferences is compared to the utility the school matched under noisy preferences has in the perfectly informed preferences

The assumption for welfare measurement is that while the student is matched to a school according to their preferences, they would prefer to be matched according to preferences created from the perfectly informed achievement. Due to changes in preferences for everybody, some students might be lucky and end up in a more preferable school under the noisy preferences. Some students might end up in exactly the same school, for example, if their noise is not large, or they have a strong preference for a particular school and their change in achievement perception does not change that preference (the school is, for example, the only close one).

After finishing all the iterations, the average of the metrics mentioned above, including its confidence intervals, is measured.

In addition to the primary metric, other ones can be studied as well. For example, the proportion of students matched with their 1st to 5th preference and the proportion of students unmatched. Literature on the comparison of matching algorithms often reports this. Ultimately, due to the controlled environment of the simulation, any other measure can be calculated and saved during this step.

5.8. Initial settings

Following the assumption about commuting, the simulation models a region as an approximately independent space unit within which students make decisions on school preferences. In 2018, around 420 thousand students studied in the four high school grades (ČSU, 2018), though this number is expected to be even higher now as stronger population cohorts enter the high school age. The simulation is, by default running with 7,000 students, approximately representing an average region (there are 14 regions in total in Czechia). In 2024, the average region also had around 350 more available spots than students applying (ČŠI, 2024a). To simulate that, the simulation has as a default 150 schools, which are, on average, expected to create 7,500 spots. The grid size is set to 70 000 x 70 000 (meters), again approximately mimicking the real area of an average Czech region, which is, in reality, 75 x 75 km.

The influence of change in these initial settings is tested in the sensitivity analysis.

Overview of default configuration:

Number of Iterations: 2000

Number of Students: 7000

Number of Schools: 150

Grid Size: 70,000

Weights: 0.2 for Distance, 0.2 for Quality, 0.2 for Income Aspiration, and 0.4 for Aspiration

Noise Standard Deviation: 20

Income Scaling Factor: 0

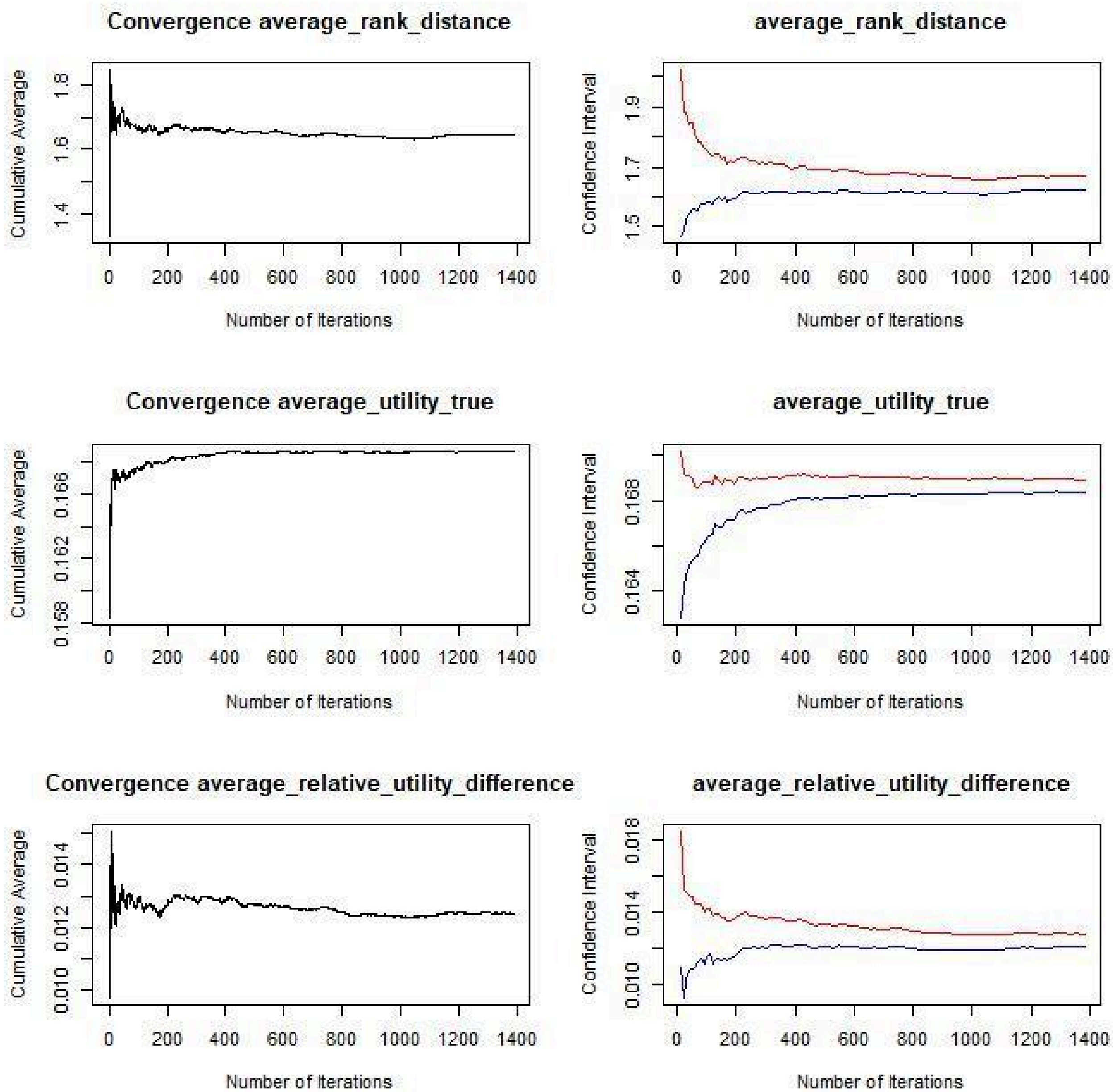
5.9. Determining the number of iterations

5.9.1. Power analysis

The number of iterations required is calculated through two methods. First, the effect size is estimated and used in a power analysis. The primary research question is concerned with welfare measures, which correspond, apart from others, to utility differences. From preliminary research, the medium expected effect size (Cohen's d) is around 0.37, which corresponds, under required power of 0.8 with alfa 0.05 for the significance level, to 116 iterations. With a more modest effect size of 0.3, the number increases to 176 iterations.

5.9.2. Converge analysis

The next option is to look at a graph showing the stabilization of key outcome variables over the number of iterations. From preliminary results, it seems likely that all variables converge to almost final value after around 400 iterations, with already small confidence intervals. However, as the computational power is not an obstacle, the initial iteration number is set to 2000, ensuring fully converged outcomes and narrow confidence intervals.



Plot 3. Convergence outcome of key variables over a number of iterations.

6. Data analysis

The key outcome of all tested simulation configurations can be seen in Attachment 3. All of the variables, even when very small, are due to the robustness of many iterations significantly different from zero and can be taken. The following parts focus on the most important outcomes and deeper analysis.

6.1. Main outcome variables

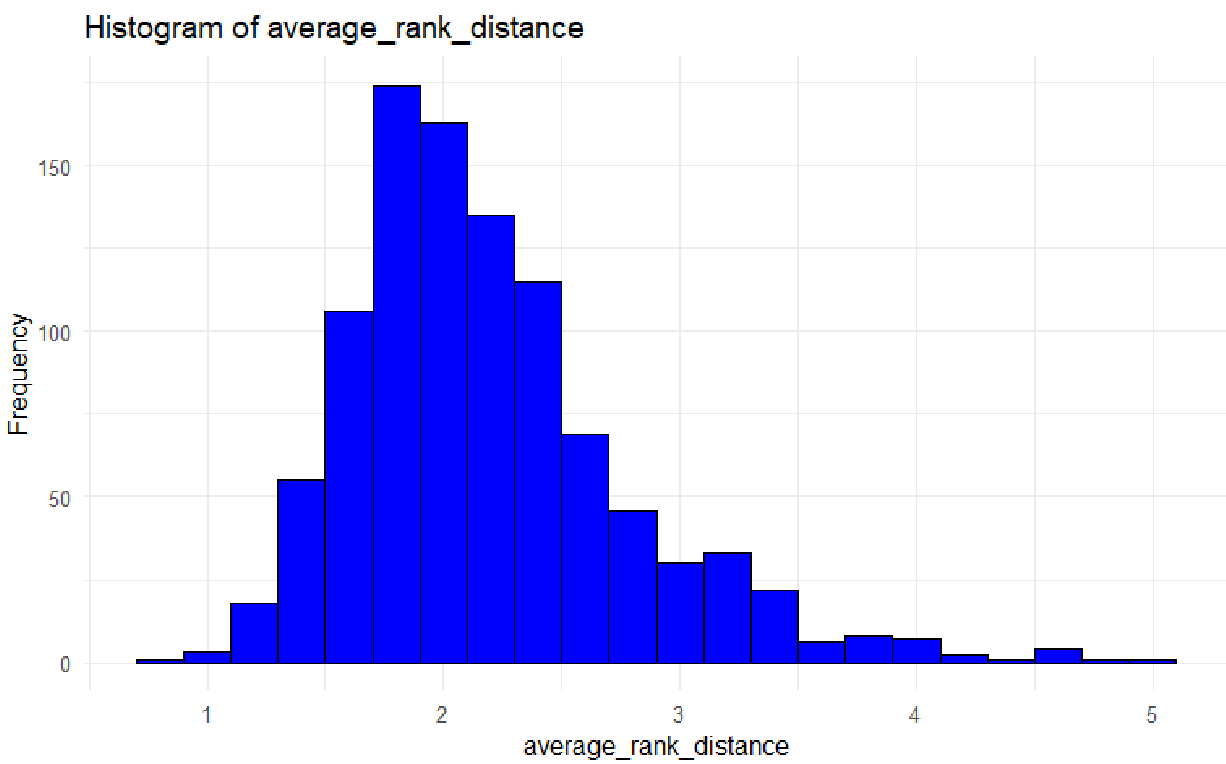
- **Average Rank (True Preferences):** This represents the average rank of schools assigned to students based on their true preferences. Lower values indicate that students generally received higher-ranked schools in their preference lists.
- **Average Utility (True and Noisy Preferences):** Measures the satisfaction or utility derived by students from their assigned schools under both true and noisy preference conditions.
- **Average Utility Distance:** The difference between utilities derived from true and noisy preferences, indicating the impact of noise on student satisfaction.
- **Average Relative Utility Difference:** The relative change in utility between true and noisy conditions, providing insight into the robustness of student assignments against preference noise.
- **Average Rank Distance:** The average difference in rank between schools assigned under true and noisy preferences. Calculated for both from true preferences.
- **Percentage Unchanged Matches:** The proportion of students who received the same school under both true and noisy preferences.

Many of the outcomes are also measured and reported by Income Quintiles, where the first quintile represents the poorest and the fifth quintile the richest students.

6.2. Impact of noise

Under this scenario, the average rank distance between the schools' students were matched to in the true achievement scenario, and the noisy achievement scenario is 2.2. On average, students were matched to their 5.6th preference school in the true scenario and their 7.75th preference school under noisy conditions (all rankings according to preferences in the true scenario). This shift corresponds to a 1.7% decrease in relative utility, indicating that students lost 1.7% of their utility from the true match scenario due to the introduction of noise in their achievement perceptions. Attachment 2 shows the results of paired t-test statistics to test the robustness of the results. The differences are significant at a confidence level of more than 0.99 %. In general, even minimal differences exhibit high significance due to the high number of iterations. Further examples can be seen in Attachment 4.

The distribution of average rank distance is slightly skewed towards higher values and quite widespread, indicating that under unique circumstances, the simulation can have quite different results far from the average of averages.



Plot 4. Histogram of Average Rank Distance between True and Noisy condition

With increasing standard deviation of the noise, the outcomes are also quite rapidly changing. No noise scenario is included as a test of the simulation, and it correctly does not predict any difference between the two noise conditions. But all other configurations show increasing differences. At the highest noise level with a standard deviation of 40, representing that only approximately 68 % of students are in their estimation of achievement within 40 percentile points from their true percentile, only 32 % of students have the same school match after the noise introduction and their relative utility difference averages at 6.23 %. On the other hand, top-income quintile students with noise SD 40 are disadvantaged approximately the same as bottom quintile students with noise with a standard deviation of 25.

Configuration / variable	Mean	Rank (T)	Rel. U dif	Rank (N)	Rank dist.	Unchanged	Rel. U dif (Q1)	Rel. U dif (Q5)	Rank dist. (Q1)	Rank dist. (Q5)
Noise SD 0.0		5,6	*0,00%*	5,6	*0,0*	99,9%	0,00%	*0,00%*	*0,0*	*0,0*
Noise SD 10.0		5,6	0,25%	6,1	0,5	77,3%	0,27%	0,05%	0,6	0,3
Noise SD 20.0		5,6	1,70%	7,7	2,2	56,2%	2,24%	0,66%	2,6	1,3
Noise SD 30.0		5,6	3,86%	10,0	4,5	41,0%	4,74%	2,10%	4,9	3,1
Noise SD 40.0		5,6	6,23%	12,3	6,7	32,4%	7,36%	3,90%	7,0	5,2

Table 1. The outcome of key variables in 5 noise configurations. An overview of used shortcuts is in the attachments.

6.3. Income correlated noise

To examine the possible effects of noise in relation to income closer, let's look at Table 2, which summarizes the situation of two noise levels: small, with a standard deviation of 5, and large, with the same standard deviation of 20. With the income scaling factor (alfa) 0, the income does not play any role, and all students have the same probability of higher noise. Up till income scaling factor 0.2, there are no significant differences in average relative utility differences between the averages of all students. After that, the decrease becomes significant. The result suggests that there might be a 50 % difference between scenarios where income does and does not play a role in achievement information, from 1 % to 1,5 % relative increase in utility differences among matches. The average rank distance decreases similarly, from around 1,9 to 1,5 for income scaling factor 1. However, the decrease is more linear, with each step statistically significant.

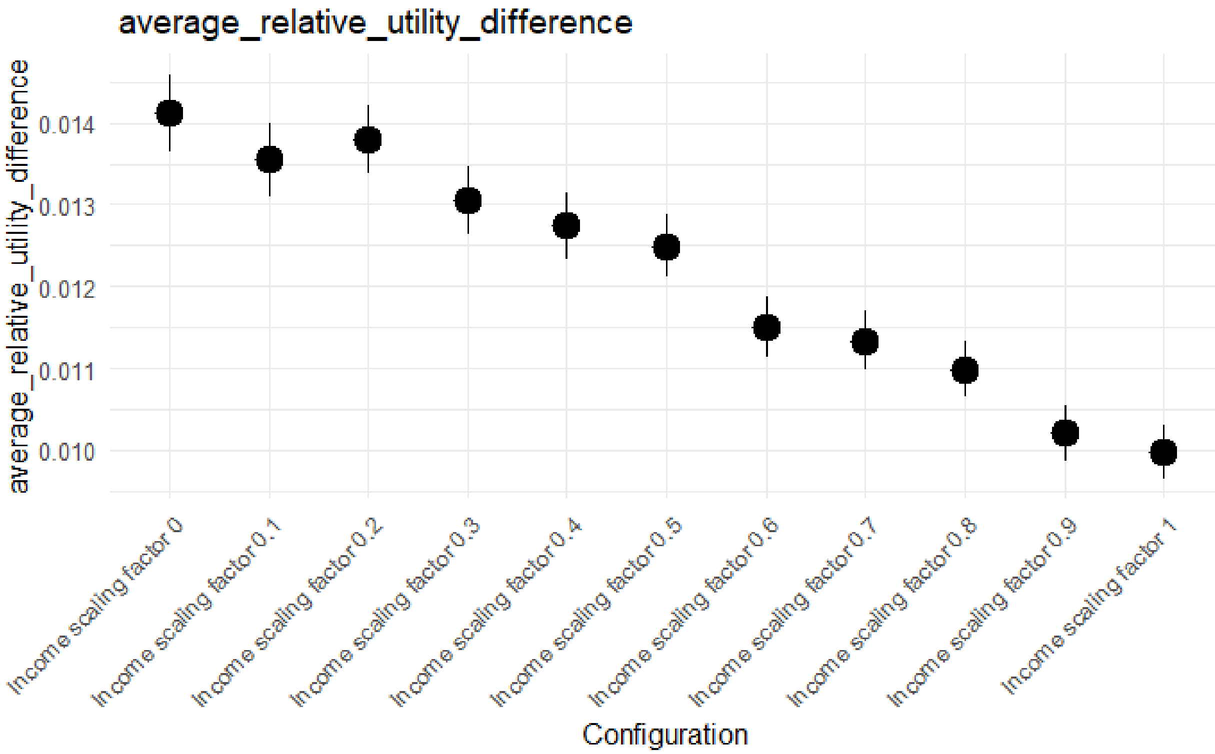
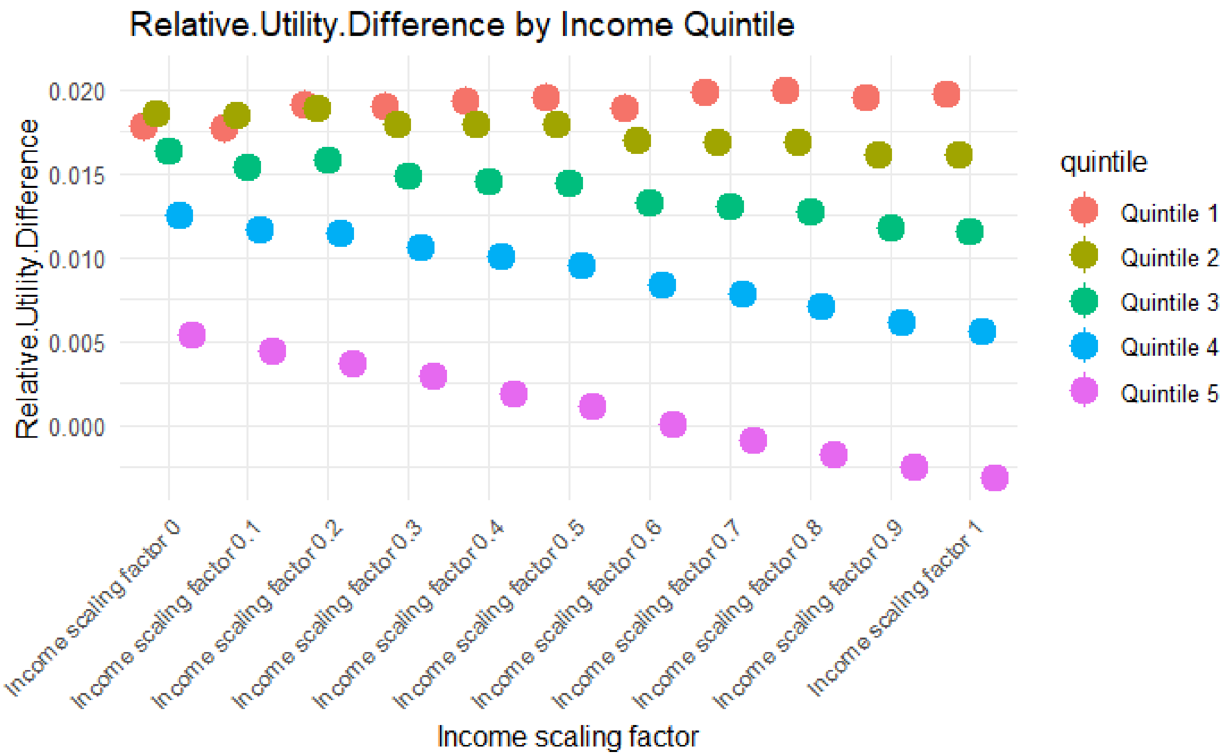


Table 2. Box plot of Averages (and confidence intervals) of Relative Utility Difference of 11 different noise Configurations.

The following plot visualizes the same mean value with a confidence intervals trend but with the distinction between income quintiles (Q1 again representing the poorest students). Looking at the highest income quintile (Q5), the relative utility difference is decreasing (matches under noisy scenario are less and less damaging their utility). With an income scaling factor higher than 0,5, their utility from noisy preferences even becomes better than under a perfectly informed match. This is probably because other students are still significantly affected by noise and, as such, are unintentionally "randomly strategizing" in their preference selection, thus giving an advantage to not strategizing higher-income students, whose preferences are influenced only slightly. In this detailed view, we can see that for the first two income quintiles (40 % of the poorest), the relative utility difference hardly changes among the different configurations.



Plot 5. Box plot of Averages (and confidence intervals) of Relative Utility Difference of 11 different noise Configurations and corresponding income quintiles.

This outcome is interesting, as it suggests that if there is a slight chance (represented by a small income scaling factor) of better information in the real world, the poorest will not benefit from it at all. This is partly the result of how the income scaling factor and probability of large or small noise is created, but not entirely. At alfa = 0, the 100 percentile income faces a large noise probability of 0.9, the same as everyone in the poorest quintile. At alfa = 0.5, the 100th percentile faces a probability of 0.9 - 0.5 * 1 = 0.4, and the person at the 30th percentile with a mean income of 0.32 faces a probability 0.9 - 0.5 * 0.32 = 0.74, yet the decrease compared to 0.9 probability of the alfa = 0 is not significant. Moreover, the relative utility difference is minimal even with alfa = 1 (0.9 - 1 * 0.32 = 0.58). For the poorest quintile, the relative utility difference is even statistically significantly increasing. This means that with a slightly decreasing chance of having a large noise applied, their matches are still getting worse and worse.

6.4. Sensitivity analysis

This analysis is based on Attachment 3.

Considering the number of schools, reducing the available spots for students increases the average rank distance, which is unsurprising. On the other hand, those who are matched experience an increase in their relative utility difference, indicating a better match under the noisy condition.

The sensitivity to aspiration weight is most pronounced. With 0.9 weight, the percentage of students with the same match decreases to 21.4 %, and the relative utility difference increases to 52.8 %, the highest outcome over all configurations. The sensitiveness is most pronounced by the poorest quintile of students. Configurations emphasizing school quality and distance show that increasing the weight of these factors generally leads to better matches and higher proportions of unchanged matches. However, excessive emphasis on these factors, particularly quality (Sensitivity Quality Emphasis), can reduce utility for some students, especially those from lower-income backgrounds. When all weights are balanced, the outcomes are not specifically different from others.

The sensitivity analysis confirms that anything that diminishes the importance of aspiration in general improves outcomes (including higher noise in the utility function) and otherwise. From the other options, it is interesting to point out that increased overall emphasis on quality greatly increases the rank of the school that students are matched to. However, the difference in relative utilities is minimal.

7. Discussion and conclusion

The strength of a simulation lies in the robustness of its underlying assumptions. While most of these assumptions in this simulation are based on existing research, some are estimated arbitrarily due to the lack of precise data. More detailed calibration would be required to enhance the simulation's predictive power, ideally using real-world data.

Specifically, improvements could be made in several areas: better generation of student locations, more realistic distribution of school quality, incorporation of student perceptions of school types, and using actual school capacities along with real-world commuting data. These enhancements are feasible with the current simulation framework. However, calibration is more complex and costly when using real-world data. The current system in Czechia only allows three applications per student, and the full outcomes from CERMAT (the body responsible for the centralized exams) have not yet been released, making detailed calibration challenging.

In its current state, the simulation provides insights into various scenarios and how changes to key characteristics might impact outcomes. The main finding is that the default configuration settings do not drastically alter the results under noisy conditions. While the negative impact of noise is present, it only reduces each student's utility of a matched school under perfectly informed conditions by a few percentage points. On a societal scale, however, this could still lead to a noticeable aggregate welfare improvement, supporting the policy of swapping the timing of exam result disclosures with admission decisions.

One of the reasons for the strong performance of the DA mechanism, even under noisy condition, is probably the relatively high number of applications allowed, which helps mitigate the harmful effects of noise. This is good news for the DA mechanism. A potential next step for the simulation could be to explore what happens when the number of possible applications is further limited and at what point the advantages of the DA mechanism begin to diminish, leading to less favorable outcomes.

Additionally, the simulation highlights the varying effects of noise on different income quintiles. As discussed in the policy chapter, Czechia faces significant educational inequalities. The simulation suggests that the poorest students, who are most disadvantaged by the current system, would benefit the most from the earlier disclosure of exam results. Under the assumption that aspiration accounts for 60% of each student's utility—a relatively high estimate based on existing literature—the impact on the poorest quintile could result in a 7.21% improvement in the relative utility of each student due to the change in school matches.

In conclusion, the simulation supports the swap in examination results and preference selection timing. However, the effect on the utility of students is probably more negligible if students place less emphasis on the aspiration match with their school.

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Attachments

(1) Simulation and analysis code and data depository

The simulation is coded in Python, and subsequent analysis performed in R. Both codes can be found at: https://michalostry.cz/thesis-repository

(2) Results of Paired t-tests

Results of Paired t-tests

Variable Comparison	t-value	df	p-value	Mean Difference	95% CI Lower	95% CI Upper
average_rank_true vs average_rank_noisy	-117.30301	999	0	-2.1916479	-2.2283116	-2.1549842
average_utility_true vs average_utility_noisy_under_true	80.23064	999	0	0.0033176	0.0032364	0.0033987

(3) Complete results of all iterations over key variables

Legend:

Rank (T): Average rank of assigned schools in true preferences

Rel. U dif: Average relative utility difference between true and noisy utility match (in true preferences)

Rank (N): Average rank of assigned schools in noisy preferences

Rank dist.: Average rank distance between true and noisy preferences

Unchanged: Percentage of students without change in match

Rel. U dif (Q1): Relative utility difference for the poorest quintile

Rel. U dif (Q5): Relative utility difference for the wealthiest quintile

Rank dist. (Q1): Rank distance for the poorest quintile

Rank dist. (Q5): Rank distance for the wealthiest quintile

Configuration / Mean variable	Rank (T)	Rel. U dif	Rank (N)	Rank dist.	Unchanged	Rel. U dif (Q1)	Rel. U dif (Q5)	Rank dist. (Q1)	Rank dist. (Q5)
Income scaling factor 0	5,7	1,41%	7,6	1,9	59,0%	1,78%	0,54%	2,3	1,2
Income scaling factor 0.1	5,7	1,35%	7,5	1,9	60,1%	1,77%	0,45%	2,3	1,1
Income scaling factor 0.2	5,6	1,38%	7,4	1,8	61,0%	1,91%	0,37%	2,2	1,0
Income scaling factor 0.3	5,6	1,30%	7,3	1,8	62,0%	1,90%	0,29%	2,2	0,9
Income scaling factor 0.4	5,6	1,27%	7,3	1,7	62,9%	1,93%	0,19%	2,2	0,8
Income scaling factor 0.5	5,6	1,25%	7,2	1,7	64,0%	1,95%	0,11%	2,2	0,7
Income scaling factor 0.6	5,6	1,15%	7,2	1,6	65,2%	1,89%	0,00%	2,2	0,5
Income scaling factor 0.7	5,6	1,13%	7,1	1,5	66,2%	1,98%	-0,09%	2,2	0,4
Income scaling factor 0.8	5,5	1,10%	7,0	1,4	67,4%	2,00%	-0,17%	2,1	0,3
Income scaling factor 0.9	5,6	1,02%	7,0	1,4	68,4%	1,95%	-0,25%	2,1	0,2
Income scaling factor 1	5,6	1,00%	6,9	1,3	69,5%	1,98%	-0,32%	2,1	0,1
Noise SD 0.0	5,6	*0,00%*	5,6	*0,0*	99,9%	*0,00%*	*0,00%*	*0,0*	*0,0*
Noise SD 10.0	5,6	0,25%	6,1	0,5	77,3%	0,27%	0,05%	0,6	0,3
Noise SD 20.0	5,6	1,70%	7,7	2,2	56,2%	2,24%	0,66%	2,6	1,3
Noise SD 30.0	5,6	3,86%	10,0	4,5	41,0%	4,74%	2,10%	4,9	3,1
Noise SD 40.0	5,6	6,23%	12,3	6,7	32,4%	7,36%	3,90%	7,0	5,2
Num Schools 120	12,9	-3,74 %	13,5	0,5	48,4%	-6,18%	-2,30%	0,8	-0,1
Num Schools 130	9,9	-1,26 %	11,2	1,2	52,4%	-2,50%	-0,77%	1,6	0,5
Num Schools 140	7,4	0,41%	9,2	1,8	55,4%	-0,17%	0,26%	2,1	1,1
Num Schools 150	5,6	1,67%	7,7	2,2	56,2%	2,19%	0,65%	2,6	1,3
Sensitivity Aspiration 01	24,5	-0,06 %	24,7	0,1	89,7%	-0,08%	-0,10%	0,2	0,0
Sensitivity Aspiration 02	15,7	-0,04 %	16,0	0,3	80,3%	-0,01%	-0,19%	0,5	0,1
Sensitivity Aspiration 03	8,4	0,24%	9,2	0,8	68,5%	0,39%	-0,15%	1,1	0,3

Sensitivity Aspiration 04	5,6	1,69%	7,8	2,2	56,2%	2,24%	0,64%	2,6	1,3
Sensitivity Aspiration 05	4,9	5,25%	9,5	4,6	46,1%	6,89%	2,91%	5,0	3,4
Sensitivity Aspiration 06	4,8	11,83 %	12,9	8,1	37,7%	15,94%	7,21%	8,8	6,4
Sensitivity Aspiration 07	5,0	21,63 %	17,6	12,5	29,6%	29,25%	13,72%	14,2	9,8
Sensitivity Aspiration 08	5,5	34,14 %	22,6	17,0	22,8%	44,02%	23,61%	19,8	13,3
Sensitivity Aspiration 09	5,9	52,84 %	27,2	21,4	17,2%	64,51%	39,63%	25,3	16,7
Sensitivity Aspiration Emphasis	5,6	1,63%	7,8	2,1	56,3%	2,15%	0,60%	2,5	1,3
Sensitivity Balanced	11,6	0,04%	12,1	0,5	74,7%	0,12%	-0,21%	0,7	0,1
Sensitivity Distance	2,4	-0,58 %	2,5	0,2	80,0%	-0,85%	-0,46%	0,2	0,1
Sensitivity Distance Emphasis	6,3	-0,17 %	6,6	0,3	77,0%	-0,33%	-0,26%	0,4	0,2
Sensitivity Income	38,6	0,04%	38,8	0,2	87,9%	0,22%	-0,11%	0,5	-0,1
Sensitivity Income Emphasis	17,2	0,06%	17,6	0,4	78,8%	0,30%	-0,25%	0,7	0,0
Sensitivity No Aspiration Balanced	30,3	-0,02 %	30,3	0,0	98,1%	-0,04%	-0,01%	0,0	0,0
Sensitivity Original	5,6	1,67%	7,7	2,2	56,1%	2,17%	0,65%	2,5	1,3
Sensitivity Quality	61,1	0,01%	61,2	0,0	94,0%	0,03%	-0,02%	0,1	0,0
Sensitivity Quality Emphasis	33,4	0,03%	33,7	0,3	83,6%	0,16%	-0,10%	0,5	0,1
Student Data Income scaling factor 0.2	5,6	1,37%	7,4	1,9	60,9%	1,96%	0,37%	2,3	1,0
Student Data Income scaling factor 0.4	5,6	1,24%	7,3	1,7	63,1%	1,84%	0,20%	2,2	0,8
Student Data Income scaling factor 0.6	5,5	1,18%	7,1	1,6	65,0%	1,86%	0,02%	2,2	0,5
Student Data Income scaling factor 0.8	5,6	1,04%	7,0	1,4	67,4%	1,83%	-0,18%	2,1	0,3
Utility Noise 005	2,7	1,37%	3,9	1,2	68,9%	1,85%	0,63%	1,2	0,9
Utility Noise 01	3,7	1,30%	5,1	1,4	65,8%	1,64%	0,61%	1,6	0,9

Note: T-test of significance shows that all numbers, except those <u>underlined,</u> are on a 95 % confidence interval significantly different from 0. Variables in *italics* (NOISE SD 0) have not been tested for significance from 0.

(4) T-test of significance on a 95 % confidence level for selected variables

Configuration: Income scaling factor 0.5

Config.Name	t_value	p_value	mean_diff	ci_low	ci_high
Rank (T)	11,23376	5,21493E-28	0,001081	0,000893	0,00127
Rel. U dif	45,16427	2,0561E-268	0,019532	0,018684	0,02038
Rank (N)	176,8413	0	5,554576	5,492956	5,616195
Rank dist.	63,04935	0	0,012493	0,012104	0,012882
Unchanged	275,7204	0	7,225667	7,174255	7,277079
Rel. U dif (Q1)	143,5073	0	1,662861	1,64013	1,685593
Rel. U dif (Q5)	867,3571	0	63,98389	63,83917	64,12861

Rank dist. (Q1)	151,9395	0	2,236949	2,208066	2,265831
Rank dist. (Q5)	69,55723	0	0,65972	0,641113	0,678327